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Mathematics
for the international student

7 MYP 2

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for use with
IB Middle Years
Programme

MATHEMATICS FOR THE INTERNATIONAL STUDENT 7 (MYP 2)

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FOREWORD

This book may be used as a general textbook at about 7th Grade (or Year 7) level in classes where students are expected to complete a rigorous course in Mathematics. It is the second book in our Middle Years series 'Mathematics for the International Student'.

In terms of the IB Middle Years Programme (MYP), our series does not pretend to be a definitive course. In response to requests from teachers who use 'Mathematics for the International Student' at IB Diploma level, we have endeavoured to interpret their requirements, as expressed to us, for a series that would prepare students for the Mathematics courses at Diploma level. We have developed the series independently of the International Baccalaureate Organization (IBO) in consultation with experienced teachers of IB Mathematics. Neither the series nor this text is endorsed by the IBO.

In regard to this book, it is not our intention that each chapter be worked through in full. Time constraints will not allow for this. Teachers must select exercises carefully, according to the abilities and prior knowledge of their students, to make the most efficient use of time and give as thorough coverage of content as possible.

We understand the emphasis that the IB MYP places on the five Areas of Interaction and in response there are links on the CD to printable pages which offer ideas for projects and investigations to help busy teachers (see p. 5).

Frequent use of the interactive features on the CD should nurture a much deeper understanding and appreciation of mathematical concepts. The inclusion of our new **Self Tutor** software (see p. 4) is intended to help students who have been absent from classes or who experience difficulty understanding the material.

The book contains many problems to cater for a range of student abilities and interests, and efforts have been made to contextualise problems so that students can see the practical applications of the mathematics they are studying.

We welcome your feedback. Email: info@haesemathematics.com.au
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The publishers wish to make it clear that acknowledging these individuals does not imply any endorsement of this book by any of them, and all responsibility for the content rests with the authors and publishers.

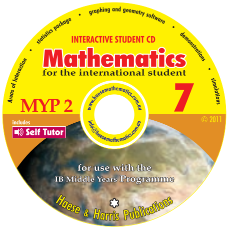
USING THE INTERACTIVE CD

The interactive CD is ideal for independent study.

Students can revisit concepts taught in class and undertake their own revision and practice. The CD also has the text of the book, allowing students to leave the textbook at school and keep the CD at home.

By clicking on the relevant icon, a range of new interactive features can be accessed:

- Self Tutor
- Areas of Interaction links to printable pages
- Interactive Links – to spreadsheets, graphing and geometry software, computer demonstrations and simulations



INTERACTIVE LINK

NEW!

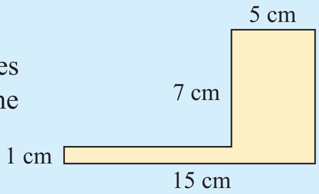
SELF TUTOR is a new exciting feature of this book.

The **Self Tutor** icon on each worked example denotes an active link on the CD.

> Simply 'click' on the **Self Tutor** (or anywhere in the example box) to access the worked example, with a teacher's voice explaining each step necessary to reach the answer.
>
> Play any line as often as you like. See how the basic processes come alive using movement and colour on the screen.
>
> Ideal for students who have missed lessons or need extra help.

Example 7 **Self Tutor**

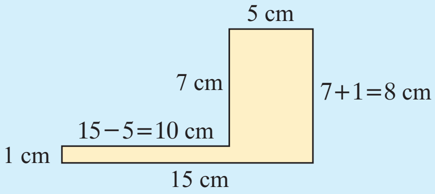
Find the lengths of the unknown sides and hence calculate the perimeter of the figure:



We use the known lengths to calculate the other side lengths:

Now $P = 5 + 8 + 15 + 1 + 10 + 7$ cm

$\therefore$ $P = 46$ cm



See **Chapter 9**, **Length and area**, p. 181

AREAS OF INTERACTION

The International Baccalaureate Middle Years Programme focuses teaching and learning through five Areas of Interaction:

- Approaches to learning
- Community and service
- Human ingenuity
- Environments
- Health and social education

The Areas of Interaction are intended as a focus for developing connections between different subject areas in the curriculum and to promote an understanding of the interrelatedness of different branches of knowledge and the coherence of knowledge as a whole.

In an effort to assist busy teachers, we offer the following printable pages of ideas for projects and investigations:

Click on the heading to access a printable 'pop-up' version of the link.

LINKS
click here

STAINED GLASS WINDOWS

Areas of interaction:
Human ingenuity, Approaches to learning

Links to printable pages of ideas for projects and investigations

Chapter 2: Angles, lines and parallelism p. 53	**STAINED GLASS WINDOWS** Human ingenuity, Approaches to learning
Chapter 3: Properties of numbers p. 74	**MATCHSTICK MATHEMATICS** Approaches to learning
Chapter 6: Decimal numbers p. 133	**LEAP YEARS** Human ingenuity, Environment
Chapter 7: Percentage p. 155	**ELECTIONS** Approaches to learning
Chapter 8: Algebra: Expressions and evaluation p. 170	**BARYCENTRES IN SPACE** Human ingenuity
Chapter 9: Length and area p. 195	**POPULATION DENSITY** Health and social education
Chapter 11: Further measurement p. 230	**HOW MUCH WATER IS LOST WHEN A TAP IS LEFT DRIPPING?** Environments, Community and Service
Chapter 17: Line graphs p. 347	**HOW ARE TAXI FARES CALCULATED?** Human ingenuity, Approaches to learning
Chapter 18: Circles p. 364	**FLAG RATIOS** Human ingenuity
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Chapter 1

Whole numbers

Contents:

OPENING PROBLEM

A sixteen storey hotel with floors G, 1, 2, 3, ..., 15 has no accommodation on the ground floor. On the even numbered floors (2, 4, 6, ...) there are 28 guest rooms. On the odd numbered floors there are 25 guest rooms.

Room cleaners work for four hours each day, during which time each cleaner can clean 12 guest rooms. Each cleaner is paid at a rate of €16 per hour.

Consider the following questions:

a How many floors are odd numbered?

b In total, how many guest rooms are on all the odd numbered floors?

c If each guest room has three chairs, how many chairs are on each even numbered floor?

d How many guest rooms are in the hotel?

e How many cleaners are required to clean all guest rooms assuming the hotel was 'full' the previous night?

f What is the total cost of hiring the cleaners to clean the guest rooms of the hotel?

All over the world, people use numbers. They are a vital part of our lives, and have been important to humans for thousands of years. We need to understand the properties of numbers and the operations between them.

Over the ages, different people have created their own **number systems** to help them count. The Ancient Egyptians, Romans, and Greeks all used different symbols for their numbers, and helped to developed the more efficient systems we use today.

There are still many number systems in use around the world, but the most common is the **Hindu-Arabic** system used in this course. An early form of this system was established in ancient India around 3000 BC, and the first of the modern characters was developed about 2000 years ago. Use of the system slowly spread westwards, and in the 7th century AD it was adopted by the Arabs.

RESEARCH HISTORY OF THE HINDU-ARABIC SYSTEM

Divide your class into small groups. Each group should write a report on a particular aspect of the history of the Hindu-Arabic system. Topics you should include are:

- the Indus valley civilization
- the Bakhshali manuscript
- Al-Uqlidisi
- Brahmi numerals
- Aryabhata
- the Codex Vigilanus

A THE NUMBER SYSTEM

When we write any number, we write some combination of the ten symbols: 1, 2, 3, 4, 5, 6, 7, 8, 9 and 0. These symbols are called **digits**. When we write these digits together, we form **numerals** that represent numbers.

The numbers we use for counting, such as 1, 2, 3, 10, 100, ... are usually called the **counting numbers** or **natural numbers**. The set of natural numbers is endless. There is no largest natural number, so we say the set of all natural numbers is **infinite**.

If we include the number **zero** or 0, then our set becomes the set of **whole numbers**.

There are three features of the Hindu-Arabic system that make it efficient:

- it uses only **10 digits** to construct all the **natural numbers**
- it has a **place value system** where digits represent different numbers when placed in different place value columns
- it uses the digit **0** to show an empty place value.

PLACE VALUES

The **place** or position of a digit in a number determines its value.

For example, 5378 is really

5 thousand 3 hundred and seventy eight

5000 + 300 + 70 + 8

You should already be familiar with the place values:

units	1	ten thousands	10 000
tens	10	hundred thousands	100 000
hundreds	100	millions	1 000 000
thousands	1000	ten millions	10 000 000

When a number is written out as a sum, we call it **expanded form**.

Example 1 Self Tutor

a Write in numeral form the number "three thousand, two hundred and seven".
b What number is represented by the digit 6 in the numeral 1695?

a 3207 **b** six hundred or 600

EXERCISE 1A

1 Write in numeral form:

a forty seven **b** six hundred and forty eight **c** seven hundred and one
d three thousand, four hundred and forty eight
e six hundred and twenty five thousand, nine hundred and ninety
f three million, six hundred thousand, nine hundred and seventy three.

2 When writing out a *cheque* to pay a debt, the amount must be written in both numbers and words. Write the following amounts in words:

a \$91 **b** €362 **c** \$4056 **d** £9807 **e** \$43 670 **f** €507 800

3 What number is represented by the digit 7 in the following?

a 47 **b** 67 **c** 372 **d** 702
e 4709 **f** 17 000 **g** 3067 **h** 370 000
i 175 236 **j** 5 700 000 **k** 67 000 000 **l** 146 070

4 Write the following numbers:

a one less than nine **b** two greater than ten **c** one less than 200
d 2 more than 3000 **e** the largest two digit number.

5 Write the following quantities in order, beginning with the smallest:

a Kylie 57 kg, Amanda 75 kg, Sarah 49 kg, Lindy 60 kg
b Wan 173 cm, Xiang 148 cm, Hao 138 cm, Gan 174 cm
c \$1100, \$1004, one thousand and forty dollars
d Barina 708 kg, Laser 880 kg, Excel 808 kg, Corolla 890 kg
e forty dollars, forty four dollars, fourteen dollars, fifty four dollars, forty five dollars.

Example 2 **Self Tutor**

a Express $50\,000 + 6000 + 70 + 4$ in simplest form.
b Write 6807 in expanded form.

a $50\,000 + 6000 + 70 + 4 = 56\,074$
b $6807 = 6000 + 800 + 7$

6 Express in simplest form:

a $90 + 7$ **b** $400 + 30 + 6$
c $8000 + 4$ **d** $5000 + 600 + 8$
e $70\,000 + 60 + 5$ **f** $4\,000\,000 + 900 + 8$

7 Write in expanded form:

a 730 **b** 4871 **c** 68 904 **d** 760 391

8 **a** Use the digits 7, 1, and 9 once only to make the largest number you can.
b Write the largest number you can using the digits 3, 1, 0, 4, 5, and 7 once only.

B ROUNDING NUMBERS

Often we do not need to know the exact value of a number, but rather we want a reasonable **estimate** or **approximation** of it.

For example, when we measure the height of a tree, we do not need to know an *exact* value. We only need an approximation, so we **round off** the number on the tape measure.

ROUNDING TO A POWER OF 10

We can round off numbers to the nearest power of ten. For example, we can round off to the nearest 10, 100 or 1000.

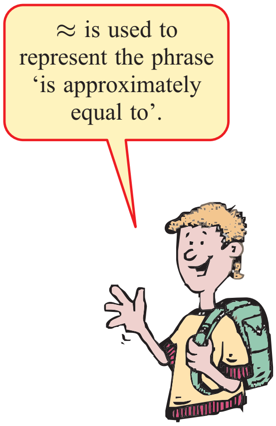
157 is closer to 160 than to 150, so to round to the nearest 10, we **round up** to 160.

153 is closer to 150 than to 160, so to round to the nearest 10, we **round down** to 150.

We use the symbol $\approx$ to mean "is approximately equal to".

so, $157 \approx 160$ and $153 \approx 150$.

When a number is **halfway** between tens we **always round up**. For example, $155 \approx 160$.



$\approx$ is used to represent the phrase 'is approximately equal to'.

ROUNDING TO A NUMBER OF FIGURES

We round to a number of **significant figures** if we believe this number of digits are important.

For example, to round 3482 to **two** significant figures, we notice that 3482 is closer to 3500 than it is to 3400. So, $3482 \approx 3500$ (to 2 significant figures).

The rules for rounding off are:

- If the digit **after** the one being rounded off is **less than 5**, i.e., 0, 1, 2, 3 or 4, then we **round down**.
- If the digit **after** the one being rounded off is **5 or more**, i.e., 5, 6, 7, 8 or 9, then we **round up**.

Example 3 **Self Tutor**

Round off:

a 769 to the nearest 10
b 6705 to the nearest 100
c 3143 to one significant figure
d 15 579 to two significant figures

a $769 \approx 770$ {to the nearest 10}
b $6705 \approx 6700$ {to the nearest 100}
c $3143 \approx 3000$ {to one significant figure}
d $15\,579 \approx 16\,000$ {to two significant figures}

EXERCISE 1B

1 Round off to the nearest 10:

a 43
b 65
c 98
d 147
e 199
f 451
g 797
h 9995

2 Round off to the nearest 100:

- **a** 87
- **b** 369
- **c** 442
- **d** 650
- **e** 991
- **f** 1426
- **g** 11 765
- **h** 34 037

3 Round off to the nearest 1000:

- **a** 784
- **b** 5500
- **c** 7435
- **d** 9987
- **e** 12 324
- **f** 23 497
- **g** 53 469
- **h** 670 934

4 Round off to one significant figure:

- **a** 69
- **b** 197
- **c** 293
- **d** 347
- **e** 963
- **f** 2555
- **g** 6734
- **h** 39 500

5 Round off to two significant figures:

- **a** 891
- **b** 166
- **c** 750
- **d** 238
- **e** 561
- **f** 5647
- **g** 9750
- **h** 23 501

6 Round off to the accuracy given:

- **a** €35 246 (to the nearest €1000)
- **b** a distance of 3651 km (to the nearest 100 km)
- **c** a donation of $375 (to one significant figure)
- **d** a house sold for £237 629 (to the nearest £10 000)
- **e** the attendance at a rock concert is 16 723 (to the nearest thousand)
- **f** the crowd is 35 381 people (to two significant figures)

C ESTIMATION

To help find errors in a calculation, it is useful to be able to accurately **estimate** the answer. The estimate will tell us if the computed answer is **reasonable**.

When estimating we usually **round** each number to **one significant figure** and evaluate the result. We call this a **one figure approximation**.

Example 4 **Self Tutor**

Find the approximate value of 7235×591.

We round each number to one significant figure.

$$\begin{aligned} 7235 \times 591 &\approx 7000 \times 600 \\ &\approx 4\,200\,000 \end{aligned}$$

The estimate tells us the correct answer should have 7 places in it.

We expect the answer to be about 4 million.

Example 5

Self Tutor

Estimate $3946 \div 79$.

$$\begin{aligned} & 3946 \div 79 \\ \approx\ & 4000 \div 80 && \{\text{rounding to one significant figure}\} \\ \approx\ & 400 \div 8 && \{\text{dividing each number by } 10\} \\ \approx\ & 50 \end{aligned}$$

EXERCISE 1C

1 Estimate using a one figure approximation:

a 389×63
b 4619×22
c 4062×638
d 389×2178
e $588 \times 11\,642$
f $29 \times 675\,328$

2 Estimate the following using a one figure approximation:

a $641 \div 59$
b $2038 \div 49$
c $5899 \div 30$
d $2780 \div 41$
e $85\,980 \div 299$
f $36\,890 \div 786$

3 In the following questions, round the given data to one significant figure to estimate the value asked for.

a Tracy delivers 405 papers on a paper round. She does this every week for a year. Estimate the number of papers delivered in the year.

b In the school dining hall there are 18 benches, each with 12 chairs.
Estimate the number of chairs in the dining hall.

c It took me 11 hours to drive the 1057 km from Calgary to Vancouver. Estimate my average speed in kilometres per hour.

d In 2006, an average of 1071 flights were scheduled to arrive in or depart from Dubai each day. If they each carried an average of 84 people, estimate the number of passengers handled by Dubai International Airport each day.

e Tim counted 42 jelly beans in the bottom layer of a jar. He thinks that there are 38 layers in the jar. Estimate the number of jelly beans in the jar.

f Sally earned €404 per week for 7 months of the year. Estimate the total amount of money she earned.

D OPERATING WITH NUMBERS

There are four **basic operations** that are carried out with numbers:

Addition + to find a **sum**
Subtraction − to find a **difference**
Multiplication × to find a **product**
Division ÷ to find a **quotient**

SUMS AND DIFFERENCES

To find the **sum** of two or more numbers, we *add* them.

For example, the sum of 3 and 16 is $3+16=19$.

To find the **difference** between two numbers, we *subtract* the smaller from the larger.

For example, the difference between 3 and 16 is $16-3=13$.

When we add or subtract **zero (0)**, the number remains unchanged.

For example, $23+0=23$, $23-0=23$.

When **adding** several numbers, we do not have to carry out the addition in the given order. Sometimes it is easier to change the order.

Example 6 **Self Tutor**

Find: **a** the sum of 187, 369 and 13 **b** the difference between 37 and 82.

a $187+369+13$
$=187+13+369$
$=200+369$
$=569$

b The difference between 37 and 82
$=82-37$
$=45$

EXERCISE 1D.1

1 Simplify the following:

a $3+0$ **b** $0+3$ **c** $5-0$ **d** $423+0+89$
e $1-0$ **f** $23+47-0$ **g** $20+0-8$ **h** $53-0+47$

2 Simplify the following, choosing the easiest order to perform the addition:

a $8+259+92$ **b** $137+269+63$ **c** $987+241+159$
d $163+979+21$ **e** $567+167+33$ **f** $364+779+636$
g $978+777+22$ **h** $99+899+1901$ **i** $89+75+25+11$

3 Find:

a the sum of 5, 7 and 8 **b** the difference between 19 and 56
c the sum of the first 10 natural numbers **d** by how much 639 exceeds 483.

4 What number must be increased by 374 to get 832?

5 What number must be decreased by 674 to get 3705?

6 Mount Cook in New Zealand is 3765 m above sea level, whereas Mount Manaslu in Nepal is 8163 m high. How much higher is Mount Manaslu than Mount Cook?

7 In a golf tournament Vijay won the first prize of $463 700. Aaron came second, winning $238 000. What was the difference between the two prizes? What would they each have won if they had tied?

8 My bank account balance was €7667. If I withdrew amounts of €1379, €2608 and €937, what is my bank balance now?

9 Sally stands on some scales with a 15 kg dumbbell in each hand. If the scales read 92 kg, what does she weigh?

PRODUCTS

The word **product** is used to represent the result of a multiplication.

For example, the product of 3 and 5 is $3 \times 5 = 15$.

When we **multiply**, changing the order can often be used to simplify the process.

Multiplying by **one** (1) does not change the value of a number.

For example, $17 \times 1 = 17$, $1 \times 17 = 17$.

Multiplying by **zero** (0) produces zero.

For example, $17 \times 0 = 0$.

Example 7 **Self Tutor**

Find the products: **a** 7×8 **b** 7×80 **c** 70×800

a 7×8
$= 56$

b 7×80
$= 7 \times 8 \times 10$
$= 56 \times 10$
$= 560$

c 70×800
$= 7 \times 10 \times 8 \times 100$
$= 56 \times 1000$
$= 56\,000$

QUOTIENTS

The word **quotient** is used to represent the result of a division. The number being divided is the **dividend** and the number we are dividing by is called the **divisor**.

For example,

$$\underset{\substack{\uparrow \\ \text{dividend}}}{15} \div \underset{\substack{\uparrow \\ \text{divisor}}}{3} = \underset{\substack{\uparrow \\ \text{quotient}}}{5}$$

Dividing by **one** (1) does not change the value of a number.

For example, $38 \div 1 = 38$.

Division by **zero** (0) is meaningless. We say it is **undefined**.

For example, $0 \div 4 = 0$ but $4 \div 0$ is undefined.

Neither the Egyptians nor the Romans had a symbol to represent nothing. The symbol 0 was called *zephirum* in Arabic. Our word zero comes from this.

EXERCISE 1D.2

1 Find the product:

a 8×9	**b** 80×9	**c** 80×90
d 5×6	**e** 50×6	**f** 50×600
g 7×13	**h** 7×1300	**i** $70 \times 13\,000$

2 Find the quotient:

a $8 \div 4$	**b** $80 \div 4$	**c** $8000 \div 40$
d $36 \div 9$	**e** $360 \div 90$	**f** $3600 \div 9$
g $56 \div 8$	**h** $560 \div 80$	**i** $56\,000 \div 800$

Example 8 **Self Tutor**

Simplify: **a** $4 \times 37 \times 25$ **b** $17 \times 8 \times 125$

a
$$\begin{aligned} & 4 \times 37 \times 25 \\ &= 4 \times 25 \times 37 \\ &= 100 \times 37 \\ &= 3700 \end{aligned}$$

b
$$\begin{aligned} & 17 \times 8 \times 125 \\ &= 17 \times 1000 \\ &= 17\,000 \end{aligned}$$

To simplify these products, we notice that $4\times\ 25 = 100$ and $8\times\ 125 = 1000$.

3 Simplify the following, taking short cuts where possible:

a $5 \times 41 \times 2$	**b** $25 \times 91 \times 4$	**c** $20 \times 113 \times 5$
d $50 \times 200 \times 19$	**e** $57 \times 125 \times 8$	**f** $789 \times 250 \times 40$
g $4 \times 8 \times 125 \times 250$	**h** $8 \times 2 \times 96 \times 125 \times 50$	**i** $5 \times 57 \times 8 \times 125 \times 200$

4 Simplify, if possible:

a 6×0	**b** $6 \div 0$	**c** 0×6
d $0 \div 11$	**e** 11×0	**f** 0×11
g 0×1	**h** 0×0	**i** $0 \div 1$
j 0×37	**k** 87×0	**l** $87 \div 0$

Check these results on your calculator!

Example 9 **Self Tutor**

Simplify the following: **a** 87×15 **b** $456 \div 19$

a
$$\begin{array}{rl} 8\ 7 & \\ \times\quad 1\ 5 & \\ \hline 4\ 3\ 5 & \{\text{multiplying 87 by 5}\} \\ 8\ 7\ 0 & \{\text{multiplying 87 by 10}\} \\ \hline 1\ 3\ 0\ 5 & \{\text{adding}\} \qquad \therefore\ 87 \times 15 = 1305 \end{array}$$

b

$$\begin{array}{rrrrl}
 & & 2 & 4 & \\ \cline{2-4}
19 & \big)\,4 & 5 & 6 & \quad \{\text{19 goes into 45 twice}\} \\
 & 3 & 8 & & \\ \cline{2-3}
 & & 7 & 6 & \quad \{\text{bring 6 down}\} \\
 & & 7 & 6 & \quad \{\text{19 goes into 76 four times}\} \\ \cline{3-4}
 & & & 0 & \quad \therefore \quad 456 \div 19 = 24
\end{array}$$

5 Simplify the following:

- **a** 39×13
- **b** 107×9
- **c** 117×17
- **d** $98 \div 7$
- **e** $507 \div 13$
- **f** $1311 \div 23$

6 Find:

- **a** the product of 17 and 32
- **b** the quotient of 437 and 19
- **c** the product of the first 5 natural numbers.

7 Solve the following problems:

- **a** What must I multiply $25 by to get $1375?
- **b** What answer would I get if I start with 69 and add on 8, 31 times?
- **c** I planted 400 rows of cabbages and each row contained 250 plants. How many cabbages were planted altogether?
- **d** Ian swims 4500 m in a training session. If the pool is 50 m long, how many laps does he swim?
- **e** A contractor bought 34 loads of soil, each weighing 12 tonnes. If the soil cost €23 per tonne, what was the total cost?
- **f** All rooms of a motel cost £78 per day to rent. The motel has 6 floors and 37 rooms per floor. What is the total rental received per day if the motel is fully occupied?
- **g** How many 38-seat buses are needed to transport 646 students to the athletics stadium?

8 Answer the questions in the **Opening Problem** on page **10**.

ACTIVITY 1 CALCULATOR USE

Over the next few years you will be performing a lot of calculations in mathematics and other subjects. You can use your calculator to help save time with your calculations.

Warning:

- A calculator will not necessarily give you a correct answer unless you *understand* what to do.
- Not all calculators work the same way. You will need to check how *your calculator* performs each type of operation.

What to do:

1 Press the keys in this order and check that you get the correct answer:

a $8\ \boxed{+}\ 6\ \boxed{=}$ **b** $11\ \boxed{-}\ 5\ \boxed{=}$ **c** $3\ \boxed{\times}\ 8\ \boxed{=}$ **d** $16\ \boxed{\div}\ 2\ \boxed{=}$

2 Press the keys in this order and check that you get the correct answer:

$15\ \boxed{\times}\ 12\ \boxed{\times}\ 3\ \boxed{\div}\ 27\ \boxed{\times}\ 128\ \boxed{+}\ 26943\ \boxed{=}$ *Answer*: 29 503.

3 Test yourself on these problems. To find out if you have the correct answer, turn your calculator upside down to find the correct word.

a $4378 - 51\,095 + 657 \times 1376 \div 6 \div 3$

b $2+2+2+2+2+2+2+3+3+3+3+3+4+4+4+4+773$

c $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 - 3\,628\,462$

Answers: **a** lose **b** bib **c** bee

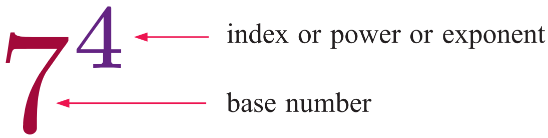
Perhaps you can invent more of these problems?

E INDEX OR EXPONENT NOTATION

A convenient way to write a product of *identical numbers* is to use **index notation**.

For example, instead of writing $7 \times 7 \times 7 \times 7$, we can write 7^4.

In 7^4, the 7 is called the **base number** and the 4 is called the **index** or **power** or **exponent**. The index is the number of times the base number appears in the product.



The following table demonstrates correct language when talking about index notation:

Natural number	*Factorised form*	*Index form*	*Spoken form*
3	3	3^1	three
9	3×3	3^2	three squared
27	$3 \times 3 \times 3$	3^3	three cubed
81	$3 \times 3 \times 3 \times 3$	3^4	three to the fourth
243	$3 \times 3 \times 3 \times 3 \times 3$	3^5	three to the fifth

Example 10 **Self Tutor**

Write in index form:
$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= 2^4 \times 3^3$
{4 factors of 2, and 3 factors of 3}

Example 11 **Self Tutor**

Write as a natural number:
$2^3 \times 3^2 \times 5$

$$\begin{aligned} & 2^3 \times 3^2 \times 5 \\ &= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \\ &= 8 \times 9 \times 5 \\ &= 40 \times 9 \\ &= 360 \end{aligned}$$

The **power key** of your calculator may look like $\boxed{\wedge}$, $\boxed{x^y}$ or $\boxed{y^x}$. It can be used to enter numbers in index form into the calculator.

Example 12 **Self Tutor**

Use your calculator to convert $2^3 \times 3^4 \times 11^2$ into natural number form.

Key in 2 $\boxed{\wedge}$ 3 $\boxed{\times}$ 3 $\boxed{\wedge}$ 4 $\boxed{\times}$ 11 $\boxed{\wedge}$ 2 $\boxed{=}$ *Answer*: 78 408

EXERCISE 1E

1 Write each number in index form:

- **a** $2 \times 2 \times 3 \times 3 \times 3$
- **b** $2 \times 5 \times 5$
- **c** $2 \times 3 \times 3 \times 3 \times 5$
- **d** $5 \times 5 \times 7 \times 7$
- **e** $2 \times 2 \times 5 \times 5 \times 5 \times 7$
- **f** $3 \times 3 \times 7 \times 7 \times 11 \times 11$
- **g** $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
- **h** $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$

2 Convert each product into natural number form:

- **a** $2 \times 3 \times 5$
- **b** $2^2 \times 5$
- **c** $2^3 \times 7$
- **d** $2 \times 3^3 \times 5$
- **e** $2^2 \times 3^2 \times 11$
- **f** $2^3 \times 5^2 \times 11^2$

3 Use your calculator to convert each product into natural number form:

- **a** $2^5 \times 3^7$
- **b** $2^3 \times 3^4 \times 7^3$
- **c** $2^3 \times 3^2 \times 11^4$
- **d** $2^5 \times 5^3 \times 7^2 \times 11$
- **e** $3^4 \times 5^3 \times 13^2$
- **f** $2^8 \times 5^2 \times 15^3$

4 Write the following in index form with 2 as a base:

- **a** 2
- **b** 4
- **c** 16
- **d** 64

5 Write the following in exponent form with 3 as a base:

- **a** 3
- **b** 27
- **c** 81
- **d** 729

6 Write the following in exponent form with 10 as a base:

- **a** 100
- **b** 1000
- **c** 100 000
- **d** 1 000 000

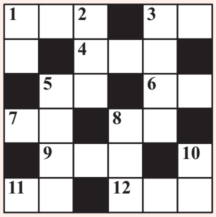
7 Write the following in exponent form:

- **a** 25
- **b** 36
- **c** 125
- **d** 343

PUZZLE OPERATIONS WITH WHOLE NUMBERS

Click on the icon to obtain a printable version of this puzzle.

PUZZLE



Across

1 19^2
3 2^4
4 4^4
5 5^2
6 3^4
7 3^3
8 9^2
9 22^2
11 4^3
12 13^2

Down

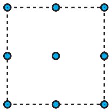
1 6^2
2 5^3
3 41^2
5 14^3
8 29^2
10 7^2

F SQUARES AND CUBES

SQUARE NUMBERS

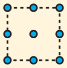
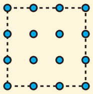
If a number can be represented by a square arrangement of dots it is called a **square number**.

For example, 9 is a square number as it can be represented by the 3×3 square shown:



We say 'three squared is equal to nine' and we write $3^2 = 9$.

The table alongside shows the first four square numbers:

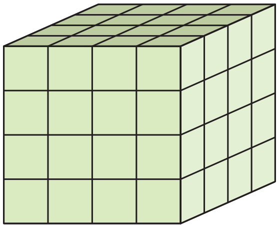
Square number	*Geometric form*	*Symbolic form*	*Factor form*	*Value*
1		1^2	1×1	1
2		2^2	2×2	4
3		3^2	3×3	9
4		4^2	4×4	16

A calculator can help you to work out the value of square numbers.

For example, 15^2 can be found by pressing 15 $\boxed{\times}$ 15 $\boxed{=}$

or 15 $\boxed{x^2}$ $\boxed{=}$. The answer is 225.

CUBIC NUMBERS



Consider a cube that is made of smaller cubic blocks. It is 4 blocks high, 4 blocks wide, and 4 blocks deep. If we count the blocks there are a total of 64.

We notice that $4^3 = 4 \times 4 \times 4 = 64$.

This is why a number to the power 3 is called a **cube**.

EXERCISE 1F

1 For each of the 5th and 6th square numbers:

a draw a diagram to represent it **b** state its value.

2 **a** Manually calculate the 7th, 8th, 9th and 10th square numbers.

b Use your calculator to find the 17th, 20th and 50th square numbers.

3 **a** Write down two numbers between 7 and 41 that are both odd and square.

b Write down two numbers between 45 and 105 that are both even and square.

4 **a** Use a calculator to complete the following:

$$\begin{aligned} 1^2 &= \\ 11^2 &= \\ 111^2 &= \\ 1111^2 &= \end{aligned}$$

b Have you noticed a pattern? Complete the following *without* using your calculator:

i $11\,111^2 =$ **ii** $111\,111^2 =$

c Investigate other such patterns with square numbers. If you find any, share them with your class!

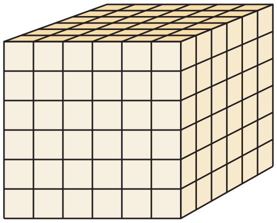
5 **a** Copy and complete the following pattern:

$$\begin{aligned} 1 &= 1 = 1^2 \\ 1+3 &= 4 = 2^2 \\ 1+3+5 &= 9 = 3^2 \\ 1+3+5+7 &= \\ 1+3+5+7+9 &= \end{aligned}$$

b Use the pattern to find the sum of the first:

i 6 odd numbers **ii** 10 odd numbers **iii** 'n' odd numbers

6



How many tiny cubes are in this block?

7 A cube like the one above contains 512 blocks. How many blocks high, wide and deep is the cube?

8 **a** Copy and complete the following pattern:

$$\begin{aligned} 1^2 - 0^2 &= \\ 2^2 - 1^2 &= \\ 3^2 - 2^2 &= \\ 4^2 - 3^2 &= \end{aligned}$$

b Predict the value of: **i** $17^2 - 16^2$ **ii** $89^2 - 88^2$

Check your answers using a calculator.

9 **a** Copy and complete the following pattern:

$$\begin{array}{rlll} 1^3 = & 1 & = 1 & = 1^2 \\ 1^3 + 2^3 = & 1 + 8 & = 9 & = 3^2 \\ 1^3 + 2^3 + 3^3 = & & = & = \\ 1^3 + 2^3 + 3^3 + 4^3 = & & = & = \end{array}$$

b Predict the value of:

i $1^3 + 2^3 + 3^3 + 4^3 + 5^3$ **ii** $1^3 + 2^3 + 3^3 + \ldots\ldots + 10^3$

Check your answers using a calculator.

G ORDER OF OPERATIONS

When two or more operations are carried out, different answers can result depending on the **order** in which the operations are performed.

For example, consider the expression $16 - 10 \div 2$.

Norio decided to subtract first then divide:

$$\begin{aligned} & 16 - 10 \div 2 \\ =\ & 6 \div 2 \\ =\ & 3 \end{aligned}$$

Chika divided first then subtracted:

$$\begin{aligned} & 16 - 10 \div 2 \\ =\ & 16 - 5 \\ =\ & 11 \end{aligned}$$

Which answer is correct, 3 or 11?

To avoid this problem, a set of rules for the **order of operations** has been agreed upon by all mathematicians.

RULES FOR ORDER OF OPERATIONS

- Perform operations within **B**rackets first.
- Then, calculate any part involving **E**xponents.
- Then, starting from the left, perform all **D**ivisions and **M**ultiplications as you come to them.
- Finally, working from the left, perform all **A**dditions and **S**ubtractions.

The word **BEDMAS** may help you remember this order.

Note:
- If an expression contains more than one set of brackets, evaluate the innermost brackets first.
- The division line of fractions behaves like a set of brackets. This means that the numerator and denominator must each be found before doing the division.

Using these rules, Chika's method is correct, and $16 - 10 \div 2 = 11$.

Example 13 Self Tutor

Evaluate: $35 - 10 \div 2 \times 5 + 3$

$$\begin{aligned} & 35 - 10 \div 2 \times 5 + 3 \\ &= 35 - 5 \times 5 + 3 && \{\text{division and multiplication working from left}\} \\ &= 35 - 25 + 3 \\ &= 10 + 3 && \{\text{subtraction and addition working from left}\} \\ &= 13 \end{aligned}$$

EXERCISE 1G

1 Evaluate the following:

- **a** $5 + 6 - 6$
- **b** $7 + 8 \div 2$
- **c** $8 \div 2 + 7$
- **d** $9 \div 3 + 4$
- **e** $100 + 6 - 7$
- **f** $7 \times 9 \div 3$
- **g** $30 \div 3 \div 5$
- **h** $18 \div 3 + 11 \times 2$
- **i** $6 + 3 \times 5 \times 2$
- **j** $7 \times 4 - 3 \times 5$
- **k** $8 + 6 \div 3 \times 4$
- **l** $4 + 5 - 3 \times 2$

Example 14 Self Tutor

Evaluate: $2 \times (3 \times 6 - 4) + 7$

$$\begin{aligned} & 2 \times (3 \times 6 - 4) + 7 \\ &= 2 \times (18 - 4) + 7 && \{\text{inside brackets, multiply}\} \\ &= 2 \times 14 + 7 && \{\text{complete brackets}\} \\ &= 28 + 7 && \{\text{multiplication next}\} \\ &= 35 && \{\text{addition last}\} \end{aligned}$$

If you do not follow the order rules, you are likely to get the wrong answer.

2 Evaluate the following, remembering to complete the brackets first:

- **a** $(12 + 3) \times 2$
- **b** $(17 - 8) \times 2$
- **c** $(3 + 7) \div 10$
- **d** $5 \times (7 + 3)$
- **e** $36 - (8 - 6) \times 5$
- **f** $18 \div 6 + 5 \times 3$
- **g** $5 + 4 \times 7 + 27 \div 9$
- **h** $(14 - 8) \div 2$
- **i** $6 \times (7 - 2)$
- **j** $17 - (5 + 3) \div 8$
- **k** $(12 + 6) \div (8 - 5)$
- **l** $5 \times (4 - 2) + 3$
- **m** $36 - (12 - 4)$
- **n** $52 - (10 + 2)$
- **o** $25 - (10 - 3)$

3 Evaluate:

- **a** $6 \times 8 - 18 \div (2 + 4)$
- **b** $10 \div 5 + 20 \div (4 + 1)$
- **c** $5 + (2 \times 10 - 5) - 6$
- **d** $18 - (15 \div 3 + 4) + 1$
- **e** $(2 \times 3 - 4) + (33 \div 11 + 5)$
- **f** $(18 \div 3 + 3) \div (4 \times 4 - 7)$
- **g** $(50 \div 5 + 6) - (8 \times 2 - 4)$
- **h** $(10 \times 3 - 20) + 3 \times (9 \div 3 + 2)$
- **i** $(7 - 3 \times 2) \div (8 \div 4 - 1)$
- **j** $(5 + 3) \times 2 + 10 \div (8 - 3)$

Example 15 **Self Tutor**

Evaluate: $5+[13-(8\div 4)]$

$$\begin{aligned}
&5+[13-(8\div 4)] && \\
&=5+[13-2] && \{\text{innermost brackets first}\}\\
&=5+11 && \{\text{remaining bracket next}\}\\
&=16 && \{\text{addition last}\}
\end{aligned}$$

Evaluate the innermost brackets first.

4 Evaluate:

a $[3\times(4+2)]\times 5$

b $[(3\times 4)-5]\times 4$

c $[4\times(16-1)]-6$

d $[(3+4)\times 6]-11$

e $5+[6+(7\times 2)]\div 5$

f $4\times[(4\times 3)\div 2]\times 7$

g $[(2\times 3)+(11-5)]\div 3$

h $19-[\{3\times 7\}-\{9\div 3\}]+14$

Example 16 **Self Tutor**

Evaluate: $\dfrac{16-(4-2)}{14\div(3+4)}$

$$\begin{aligned}
&\frac{16-(4-2)}{14\div(3+4)} && \\
&=\frac{16-2}{14\div 7} && \{\text{brackets first}\}\\
&=\frac{14}{2} && \{\text{evaluate numerator, denominator}\}\\
&=7 && \{\text{do the division}\}
\end{aligned}$$

In a fraction the **numerator** is the top line and the **denominator** is the bottom line.

5 Simplify:

a $\dfrac{21}{16-9}$

b $\dfrac{18\div 3}{14-11}$

c $\dfrac{(8\times 7)-5}{17}$

d $\dfrac{12+3\times 4}{5+7}$

e $\dfrac{3\times 7-5}{2}$

f $\dfrac{3\times(7-5)}{2}$

g $\dfrac{2\times 8-1}{8-6\div 2}$

h $\dfrac{56\div 8-7}{56\div(8-7)}$

i $\dfrac{25-(16-11)}{12\div 4+2}$

Example 17 **Self Tutor**

Simplify: $3\times(6-2)^2$

$$\begin{aligned}
&3\times(6-2)^2 && \\
&=3\times 4^2 && \{\text{brackets first}\}\\
&=3\times 16 && \{\text{exponent next}\}\\
&=48 && \{\text{multiplication last}\}
\end{aligned}$$

6 Simplify:

- **a** 3×4^2
- **b** 2×3^3
- **c** $3^2 + 2^3$
- **d** $(5 - 2)^2 - 6$
- **e** $3 \times 4 + 5^2$
- **f** $4 \times 3^2 - (3 + 2)^2$
- **g** $3 - 2^2 \div 2 + 1$
- **h** $(5 - 2)^2 - 2^2$
- **i** $(15 - 3^2) \div 3$
- **j** $5 \times 2^2 + 2 \times 3^2$
- **k** $5 \times (2^2 + 2) + 6$
- **l** $35 - 3 \times 2^3 + 7$

7 Replace $\ast$ with either +, −, × or ÷ to make a true statement:

- **a** $3 + 15 \ast 3 = 8$
- **b** $10 \ast 7 + 15 = 18$
- **c** $8 \ast 4 - 10 = 22$
- **d** $(18 \ast 2) \div 10 = 2$
- **e** $(10 \ast 3) \ast 7 = 1$
- **f** $15 \ast 3 + 2 \ast 5 = 15$

8 Insert brackets into the following to make them true:

- **a** $18 - 6 \times 3 + 2 = 38$
- **b** $48 - 6 \times 3 + 4 = 6$
- **c** $32 \div 8 \div 2 = 8$
- **d** $8 + 4 \div 2 + 2 = 3$
- **e** $5 + 3 \times 6 - 10 = 38$
- **f** $13 + 5 \div 5 + 4 = 2$

ACTIVITY 2 CONSECUTIVE SUMS

We can write the number 10 as a sum of 4 **consecutive** natural numbers: $10 = 1 + 2 + 3 + 4$. Some naturals can be written as a sum of *consecutive* natural numbers in more than one way.

For example, 9 can be written as $9 = 4 + 5$ or $9 = 2 + 3 + 4$.

What to do:

1 Write 15 as the sum of *consecutive* natural numbers in as many ways as you can. Be systematic with your approach. For example, start with $1 + 2 + 3 + ...$, then $2 + 3 + 4 + ...$, then $3 + 4 + 5 + ...$, and so on.

2 Write the following as the sum of *consecutive* natural numbers in as many ways as possible: **a** 27 **b** 28 **c** 37

3 Consider *all* the natural numbers from 2 to 40. Where possible, write each as a sum of two or more *consecutive* natural numbers.

4 Which natural numbers greater than 1 do you suspect cannot be written as a sum of consecutive integers?

ACTIVITY 3 NUMBER PRODUCT SEQUENCES

$73 \rightarrow 21 \rightarrow 2$ is a number sequence. It has length 3 since there are 3 terms. The sequence has been constructed from the following rule:

Start with a two-digit number. The next number is the product of the digits of the previous number. Stop when you get to a one digit number.

What to do:

1 Use the rule above to find the sequences beginning with: **a** 67 **b** 96

2 Give an example of a sequence of length: **a** 2 **b** 4

3 Try to find the *one and only sequence* of length 5.

PUZZLE

1 In each of the following puzzles, each letter stands for a different digit. Work out which digit each letter represents.

a $\begin{array}{r} 2\,a\,4 \\ -\ 1\,3\,c \\ \hline 1\,3\,8 \end{array}$ **b** $\begin{array}{r} d\,3 \\ \times\quad e \\ \hline 3\,1\,8 \end{array}$ **c** $\begin{array}{r} 2\,f\,4\,g \\ \times\qquad 9 \\ \hline 1\,9\,2\,h\,7 \end{array}$ **d** $\begin{array}{r} r\,r \\ +\,p\,p \\ \hline q\,p\,q \end{array}$

2 In the multiplication alongside, each letter stands for a different one of the digits 0, 1, 2, 3, ..., 9. There are several solutions which make the multiplication true. Can you find one of them? Can you find all of them?

$$\begin{array}{r} \mathrm{Y\,O\,U} \\ \times\quad \mathrm{M\,E} \\ \hline \mathrm{L\,O\,V\,E} \end{array}$$

KEY WORDS USED IN THIS CHAPTER

- approximation
- BEDMAS
- counting number
- denominator
- difference
- dividend
- divisor
- estimate
- expanded form
- exponent
- Hindu-Arabic system
- index
- infinite
- natural number
- numeral
- numerator
- place value
- power
- product
- quotient
- round down
- round up
- significant figures
- square number
- sum
- whole number

REVIEW SET 1A

1 Write 4738 in word form.

2 What number is represented by the digit 4 in 86 482?

3 What is the difference between 895 and 2718?

4 Round off 48 526 to the nearest 100.

5 Find an approximate value for 106×295.

6 Calculate 123×36.

7 How many buses would be required to transport 329 students if each bus holds a maximum of 47 students?

8 Simplify the following: **a** $20 \times 33 \times 5$ **b** $125 \times 7 \times 8$

9 A class contains 14 boys and 15 girls. If each student is given 13 pencils, how many pencils are given out altogether?

10 If a baker sells 24 dozen rolls at 15 cents each, how much money does he receive?

11 Find $1834 - 712 + 78$.

12 Find the 9th square number.

13 A woman has €255 in her purse. She gives €35 to each of her five children. How much money does she have left?

14 Find the sum of the first five square numbers.

15 Find: **a** 3^6 **b** 8^3

16 Simplify:

a $7 \times 4 - 18 \div 2$ **b** $20 - (8 - 4) \times 3$ **c** $\dfrac{31 - 11}{12 \div 6}$ **d** $\dfrac{8 + 14 \div 2}{12 \div 4}$

17 Insert brackets into the following to make a true statement:

a $4 \times 2 + 3 - 5 = 15$ **b** $20 \div 4 + 1 + 5 = 2$

REVIEW SET 1B

1 Simplify 14×0.

2 What number is represented by the digit 3 in 43 209?

3 Round off 46 804 to two significant figures.

4 Determine $2763 + 427$.

5 By how much does 738 exceed 572?

6 Write $4000 + 20$ in simplest form.

7 Use the digits 7, 5, 9, 0, and 2 once only to make the largest number you can.

8 Find the sum of 8, 15 and 9.

9 Find the difference between 108 and 19.

10 What number must be increased by 293 to get 648?

11 Determine $119 + 19 + 81$.

12 Determine $5 \times 39 \times 20$.

13 Find the product of 14 and 38.

14 Find the quotient of 437 and 19.

15 Use a one figure approximation to estimate 5032×295.

16 The Year 8 students at a school are split into 4 equal classes of 27 students. The school decides to increase the number of classes to 6. How many students will there be in each of the new classes if the students are divided equally between them?

17 My bank account contains £3621 and I make monthly withdrawals of £78 for 12 months. What is my new bank balance?

18 How many square numbers are there between 60 and 130?

19 Simplify the following:

a $5 \times 6 - 4 \times 3$ **b** $3 + 18 \times 2^2$ **c** $5 \times 8 - 18 \div (2 + 4)$

d $(5 + 4) \times 2 + 20 \div (8 - 3)$ **e** $\dfrac{8 \div 2^2}{1 + 7}$

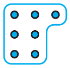
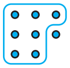
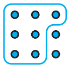
20 Insert brackets into the following to make a true statement:

a $9 - 8 \div 1 + 3 = 7$ **b** $2 + 8 \div 4 - 2 = 5$

EXTENSION A DIFFERENT BASE SYSTEM

Our number system has a base of 10, probably because we have 10 fingers on our hands.

Suppose we had only 7 fingers on our hands. We would probably work in a base 7 number system which only includes the digits 0 to 6. This would work as shown in the table below:

number of objects	•	• •	• • •	• • • •	• • • • •	• • • • • •			
numeral	1	2	3	4	5	6	10_7	11_7	12_7
how it is said	one	two	three	four	five	six	one zero in base 7	one one in base 7	one two in base 7

Try these:

1 **a** Write down the first 20 numbers in base 7.

b What would be the next number after 66_7?

2 Try converting these base 10 numerals to base 7:

a 11_{10} **b** 26_{10} **c** 35_{10} **d** 43_{10}

e 58_{10} **f** 63_{10} **g** 120_{10} **h** 145_{10}

3 Convert these base 7 numbers to base 10:

a 6_7 **b** 13_7 **c** 36_7 **d** 42_7

e 125_7 **f** 150_7 **g** 210_7 **h** 455_7

4 Complete these sums:

a
$$\begin{array}{r} 34_7 \\ 46_7 \\ +\ 51_7 \\ \hline \end{array}$$

b
$$\begin{array}{r} 66_7 \\ -\ 23_7 \\ \hline \end{array}$$

c
$$\begin{array}{r} 6_7 \\ \times\ 4_7 \\ \hline \end{array}$$

5 Make up a base 6 number system that might be used if we had only 6 fingers.

a Write the first 20 numbers in this new system.

b Write some conversions like in **2** and **3** above where you convert between base 6 and base 10. Ask your friend to answer them.

Chapter 2

Angles, lines and parallelism

Contents:

If we look carefully, we can see **angles** in many objects and situations. We see them in the framework of buildings, the pitches of roof structures, the steepness of ramps, and the positions of boats from a harbour and aeroplanes from an airport.

The measurement of angles dates back more than 2500 years and is still very important today in architecture, building, surveying, engineering, navigation, space research, and many other industries.

HISTORICAL NOTE DEGREE MEASURE

The Babylonian Empire was founded in the 18th century BC by Hammurabi in lower Mesopotamia, which is today in southern Iraq. It lasted over a thousand years, being finally absorbed into the Persian Empire of Darius in the 6th century BC.

The Babylonians were great astronomers, and so were very interested in angles. They invented the **astrolabe** to help them. The Babylonians decided that there should be 360 degrees in one full turn or rotation. This was probably because their calendar has 360 days, being 12 months of 30 days each. It was a convenient choice for them because the Babylonian number system uses base 60.

360^o is also divisible by 2, 3, 4, 5, 6, 8, 9, 10, and 12, which makes it very easy for us to divide a full turn into different fractions or parts.

OPENING PROBLEM

Without using a protractor, how can we construct angles which measure 60^o, 30^o and 15^o?

Can we use a compass to construct angles of 90^o, 45^o and $22\frac{1}{2}^o$?

A POINTS AND LINES

POINTS

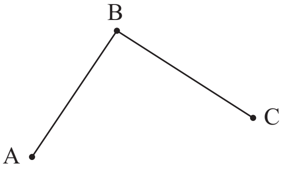
We use a **point** to mark a location or position.

Examples of points are:

- the corner of your desk
- the tip of your compass needle.

Points do not have size. We say they are **infinitely small**. In geometry, however, a point is represented by a small dot so we can see it. To help identify the point we label it with a capital letter.

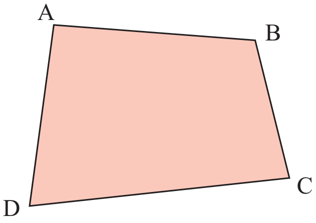
For example:



The letters A, B and C identify the points.

We can make statements like:
“the distance from A to B is .....” or
“the angle at B measures .....”.

FIGURES AND VERTICES

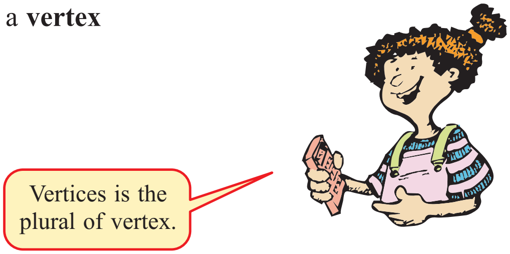


A **figure** is a drawing which shows points we are interested in.

The figure alongside contains four points which have been labelled A, B, C and D.

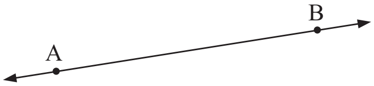
These corner points are also known as **vertices**.

Vertices is the plural of vertex, so point B is a **vertex** of the figure.



STRAIGHT LINES

A **straight line**, usually just called a **line**, is a continuous infinite collection of points with no beginning or end which lie in a particular direction.



(AB) is the **line** which passes through points A and B. We can call it “**line** AB” or “**line** BA”. There is only one straight line which passes through both A and B.

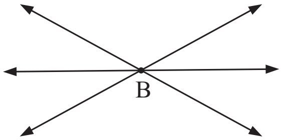
[AB] is the **line segment** which joins points A and B. We call it “**line segment** AB” or “**line segment** BA”. It is only part of the line (AB).

[AB) is the **ray** which starts at A, passes through B, then continues on forever in that direction.

If three or more *points* lie on a single straight line, we say that the points are **collinear**.

The points A, B, C and D shown are collinear.

If three or more *lines* meet or intersect at the same point, we say that the lines are **concurrent**.

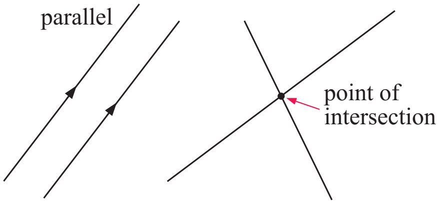


The lines shown are concurrent at point B.

PARALLEL AND INTERSECTING LINES

In mathematics, a **plane** is a flat surface like a table top or a sheet of paper. It goes on indefinitely in all directions, so it has no boundaries.

Two straight lines in the same plane may either be **parallel** or **intersecting**. Arrow heads are used to show parallel lines.



Parallel lines are lines which are always a fixed distance apart and never meet.

EXERCISE 2A

1 Give *two* examples in the classroom of:

a a point **b** a line **c** a flat surface

2 Describe with a sketch the meaning of:

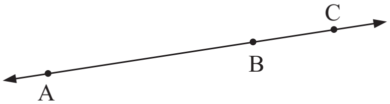
a a vertex **b** an angle **c** a point of intersection

d parallel lines **e** collinear points **f** concurrent lines

3 Give *all* ways of naming the following lines:

a

b

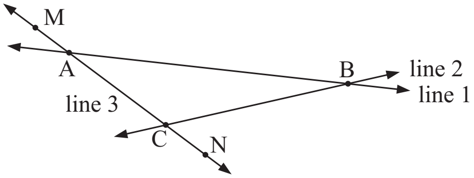


4 Name the point of intersection of:

a line 1 and line 2

b line 2 and line 3

c (AB) and [MN].



5 ABCD is a quadrilateral. The line segment [BD] is called a **diagonal**.

a Name the four sides of the quadrilateral.

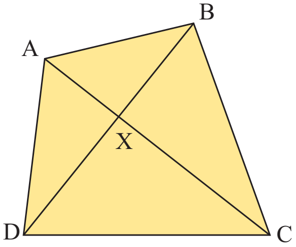
b Name the two diagonals of the quadrilateral.

c At what point do the diagonals meet?

d How many line segments meet at A?

e What can be said about points A, X and C?

f What can be said about the line segments [AB], [DB] and [CB]?



6 How many different lines can you draw through:

a two points A and B **b** all three collinear points A, B and C

c one point A **d** all three non-collinear points A, B, C?

7 Draw a different diagram to fit each statement:

a C is a point on (AB). **b** (AB) and (CD) meet at point X.

c Point A does not lie on (BC). **d** X, Y and Z are collinear.

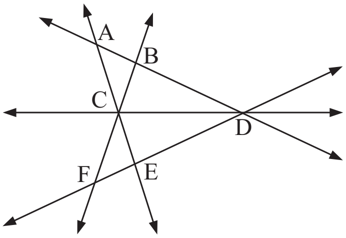
e Line segments [AB], [CD] and [EF] are concurrent at G.

8 **a** Name line (AC) in two other ways.

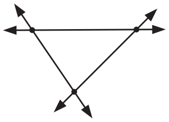
b Name two different lines containing point B.

c What can be said about:

- **i** points A, B and D
- **ii** lines (BF) and (AD)
- **iii** lines (FE), (CD) and (AB)?



9 When drawing lines through three different points, there are *two* possible cases:



3 different lines

1 line, as the points are collinear

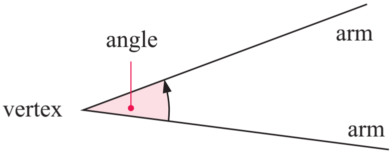
a How many different cases can we have for four different points? Illustrate each case.

b Draw the cases for five different points.

B MEASURING AND CLASSIFYING ANGLES

Whenever two lines or edges meet, an **angle** is formed between them. In mathematics, an angle is made up of two arms which meet at a point called the **vertex**.

The **size** of the angle is the amount of turning or rotation from one arm to the other.

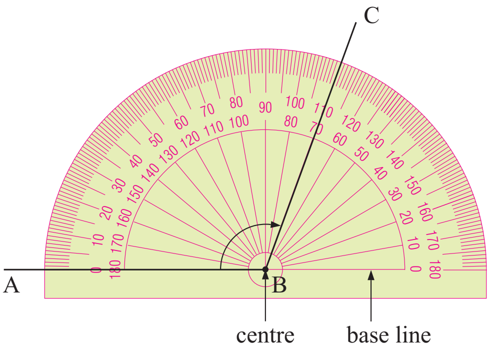


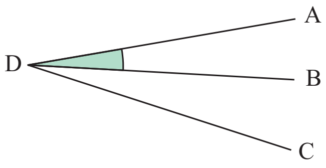
THE PROTRACTOR

Alongside is a **protractor** placed with its centre at B and its base line on [AB]. The amount of turning from [AB] to [BC] is 110 degrees.

We write $A\widehat{B}C = 110^o$ which reads 'the angle ABC measures 110 degrees'.

$A\widehat{B}C$ is called **three point notation**. We use it to make it clear which angle we are referring to.



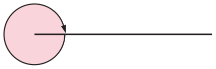
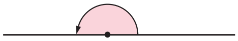
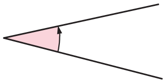
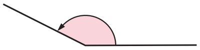
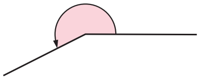


For example, if we want to talk about the shaded angle in this figure, we cannot say the angle at D, as there are three such angles.

The shaded angle is $A\widehat{D}B$ or $B\widehat{D}A$.

CLASSIFYING ANGLES

Angles are **classified** according to their size.

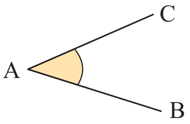
Revolution	**Straight Angle**	**Right Angle**
 One complete turn. One revolution $= 360^o$.	 $\frac{1}{2}$ turn 1 straight angle $= 180^o$.	 $\frac{1}{4}$ turn 1 right angle $= 90^o$.
Acute Angle	**Obtuse Angle**	**Reflex Angle**
 Less than a $\frac{1}{4}$ turn. An acute angle has size between 0^o and 90^o.	 Between $\frac{1}{4}$ turn and $\frac{1}{2}$ turn. An obtuse angle has size between 90^o and 180^o.	 Between $\frac{1}{2}$ turn and 1 turn. A reflex angle has size between 180^o and 360^o.

EXERCISE 2B

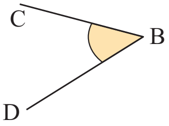
1 Match the names to the correct angles:

a $A\widehat{B}C$ **b** $C\widehat{A}B$ **c** $B\widehat{C}A$ **d** $C\widehat{B}D$

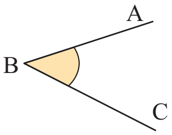
A



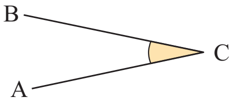
B



C



D



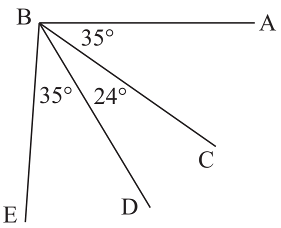
2 Draw and label each of the following angles:

a $D\widehat{E}F$ **b** $Z\widehat{X}Y$ **c** $X\widehat{Y}Z$ **d** $P\widehat{Q}R$ **e** reflex $R\widehat{P}Q$

3 Find the size of these angles without your protractor:

a $A\widehat{B}C$ **b** $D\widehat{B}C$

c $A\widehat{B}D$ **d** $A\widehat{B}E$



4 Use your ruler and protractor to draw angles with the following sizes:

a 35^o **b** 131^o **c** 258^o

Get someone else to check the accuracy of your angles.

5 Draw a free-hand sketch of:

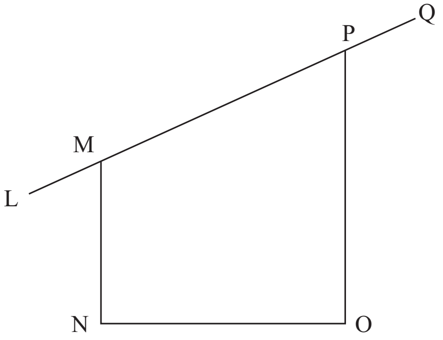
a acute angle BPQ **b** right angle NXZ **c** straight angle QDT

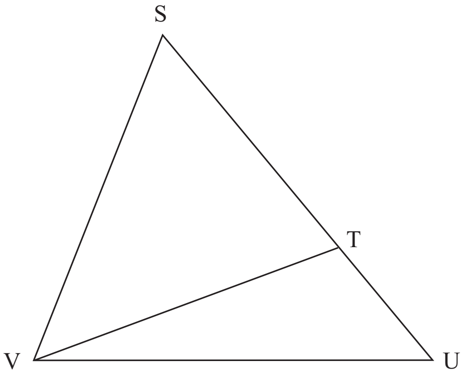
d obtuse angle CPT **e** reflex angle DSM **f** revolution $\widehat{E}$.

6 Use a protractor to measure the named angles:

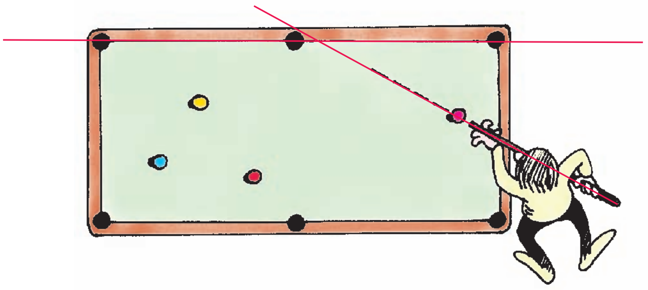
a **i** $P\widehat{M}N$ **ii** $O\widehat{P}L$ **iii** $P\widehat{O}N$

b **i** $V\widehat{T}U$ **ii** reflex $V\widehat{S}T$ **iii** reflex $T\widehat{V}U$





7 Kim hits the billiard ball so that it follows the path shown. What acute angle will it make with the edge of the table?



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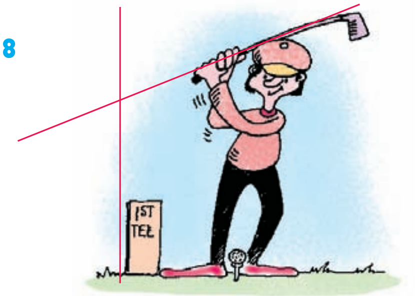
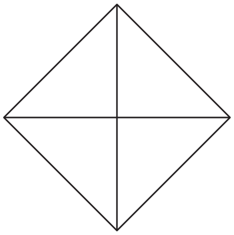


A golfer completing his back swing holds the golf club behind his body. What is the size of the reflex angle between his body and the club?

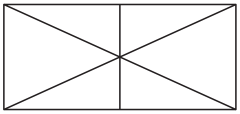
9 For each figure find the total number of:

i right angles **ii** acute angles

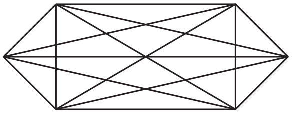
a



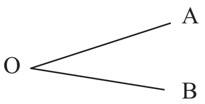
b



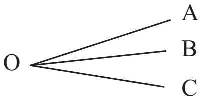
c



10

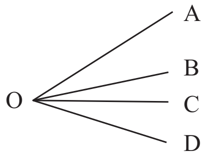


has one acute angle at O.



has 3 acute angles at O. These are $A\widehat{O}B$, $A\widehat{O}C$, and $B\widehat{O}C$.

a



How many acute angles can we identify at O given the 4 vertices A, B, C and D?

b Draw a diagram with 5 vertices A, B, C, D and E like the one in **a**. How many acute angles are at O in this case?

c Copy and complete:

Number of vertices	*Number of acute angles*
2	1
3	3
4	
5	

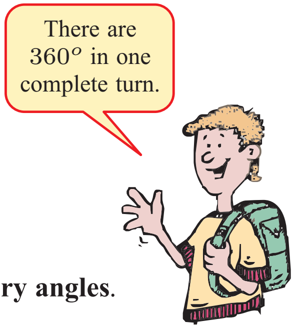
d Without drawing them, predict the number of acute angles for:

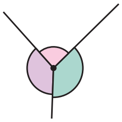
i 6 vertices **ii** 10 vertices.

DISCUSSION

For any angle ABC, what is the difference in size between $A\widehat{B}C$ and reflex $A\widehat{B}C$?

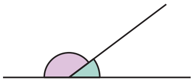
C ANGLE PROPERTIES





These angles are **angles at a point**.

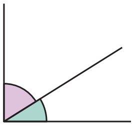
Angles at a point add to 360^o.



These angles are **angles on a line**.

Angles on a line add to 180^o.

Angles which add to 180^o are called **supplementary angles**.

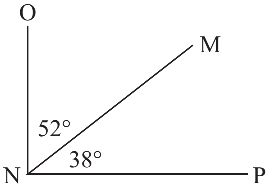


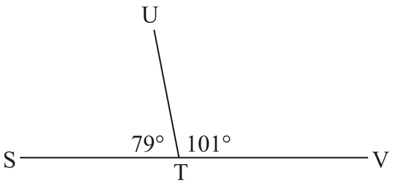
These angles are **angles in a right angle**.

Angles in a right angle add to 90^o.

Angles which add to 90^o are called **complementary angles**.
Lines which meet at 90^o are said to be **perpendicular**.

For example:





$M\widehat{N}O$ and $M\widehat{N}P$ are complementary because $52^o + 38^o = 90^o$.

$S\widehat{T}U$ and $U\widehat{T}V$ are supplementary because $79^o + 101^o = 180^o$.

[ON] and [NP] meet in a right angle, so they are perpendicular.

Two angles are **equal** if they have the same size or degree measure.

Example 1 **Self Tutor**

a Are angles with sizes 37^o and 53^o complementary?

b What angle size is the supplement of 48^o?

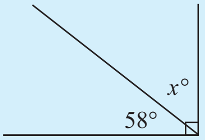
a $37^o + 53^o = 90^o$. So, the angles are complementary.

b The angle size is $180^o - 48^o = 132^o$.

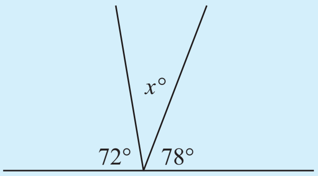
Example 2 **Self Tutor**

Find the value of the unknown in:

a



b



a The angles 58^o and x^o are complementary.

$\therefore \quad x = 90 - 58$

$\therefore \quad x = 32$

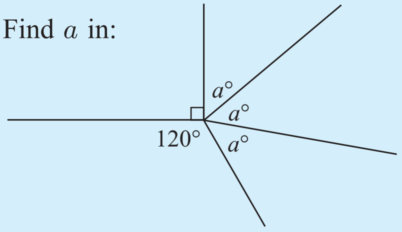
b The three angles add to 180^o

$\therefore \quad x = 180 - 72 - 78$

$\therefore \quad x = 30$

Example 3 **Self Tutor**

Find a in:



The sum of the five angles is 360^o

$\therefore$ the three equal angles add to $360^o - 90^o - 120^o = 150^o$

So, each must be $150^o \div 3 = 50^o$

$\therefore \quad a = 50.$

EXERCISE 2C

1 Add the following pairs of angles and state whether they are complementary, supplementary, or neither:

a 20^o, 70^o **b** 30^o, 150^o **c** 110^o, 40^o

d 47^o, 43^o **e** 107^o, 63^o **f** 35^o, 55^o

2 Find the size of the angle complementary to: **a** 30^o **b** 5^o **c** 85^o

3 Find the size of the angle supplementary to: **a** 100^o **b** 5^o **c** 90^o

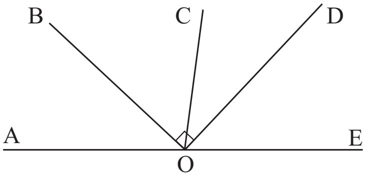
4 Classify the following angle pairs as complementary, supplementary or neither:

a $\text{B}\widehat{\text{O}}\text{C}$ and $\text{C}\widehat{\text{O}}\text{D}$

b $\text{A}\widehat{\text{O}}\text{C}$ and $\text{C}\widehat{\text{O}}\text{E}$

c $\text{C}\widehat{\text{O}}\text{D}$ and $\text{D}\widehat{\text{O}}\text{E}$

d $\text{A}\widehat{\text{O}}\text{B}$ and $\text{B}\widehat{\text{O}}\text{E}$



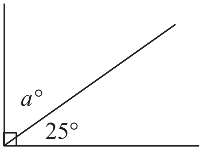
5 Copy and complete:

a the size of the complement of x^o is ....

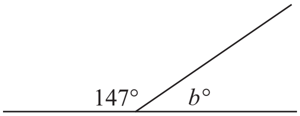
b the size of the supplement of y^o is ....

6 Find the value of the unknown in:

a



b



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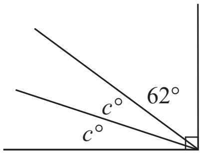

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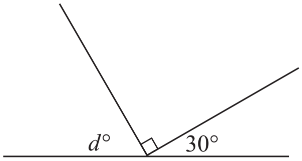

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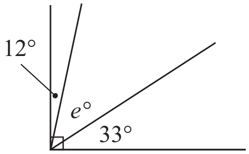

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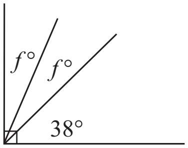
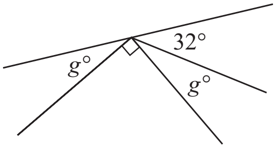


g



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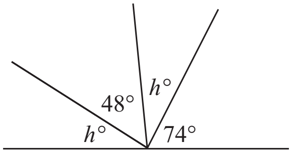
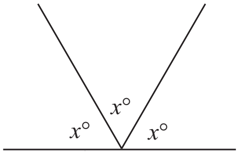


i



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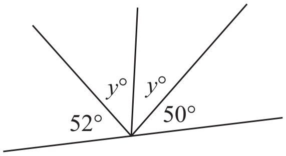

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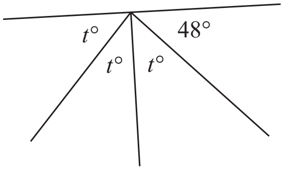
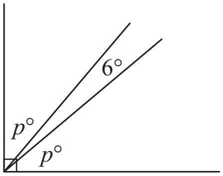
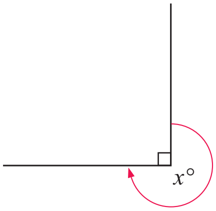


l

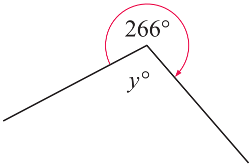


7 Find the sizes of the unknown angles:

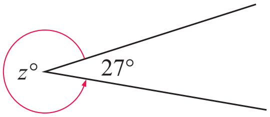
a



b

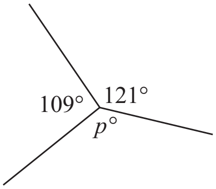


c



8 Find the values of the unknowns in:

a



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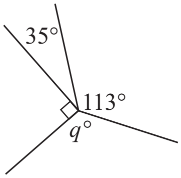

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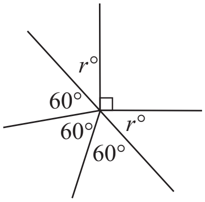
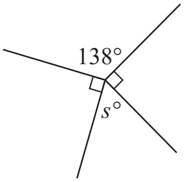
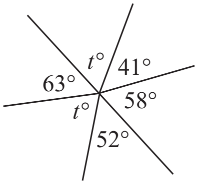


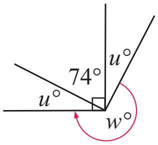
d



e



f

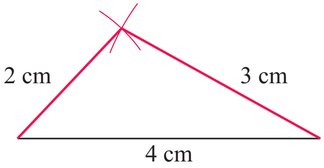


D GEOMETRIC CONSTRUCTION

In **geometric constructions** we use a ruler and compass to accurately draw diagrams. When you perform geometric constructions, **do not erase** the construction lines.

In previous courses you should have learnt how to construct:

- **triangles with known side lengths**
- **angle bisectors**



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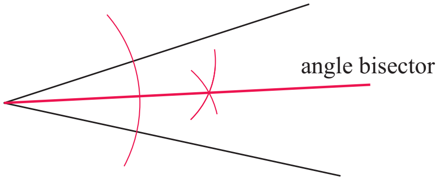


DEMO

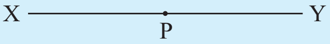
However, if necessary you can click on the icons to review these constructions.

CONSTRUCTING A 90° ANGLE TO A LINE

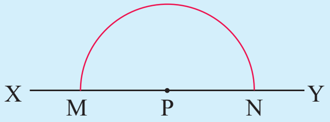
A **right angle** or **90° angle** can be constructed without a protractor or set square. Consider the following example:

Example 4 **Self Tutor**

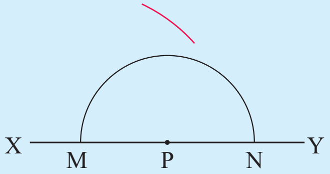
Construct an angle of 90^o at P on the line segment [XY].



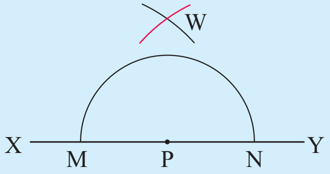
Step 1: On a line segment [XY], draw a semi-circle with centre P and convenient radius which cuts [XY] at M and N.



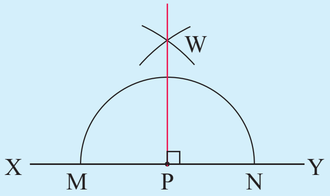
Step 2: With centre M and convenient radius larger than MP, draw an arc above P.

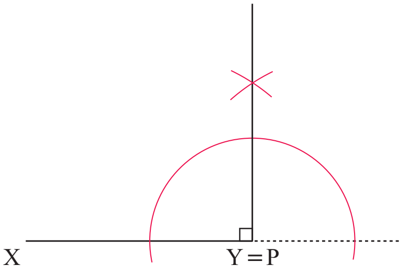


Step 3: With centre N and the same radius draw an arc to cut the first one at W.



Step 4: Draw the line from P through W.
$W\widehat{P}Y$ and $W\widehat{P}X$ are both 90^o.

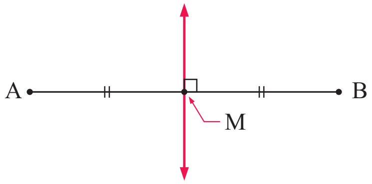




In the example above, in some cases the point P may be close to one end of the line segment, or be the end of the line segment. In these cases you may need to extend the line segment.

For example, in the construction alongside, Y and P are the same point. We say they **coincide**. We can still construct a right angle at P, but we need to extend the line segment first.

CONSTRUCTING A PERPENDICULAR BISECTOR



The red line on this figure is the **perpendicular bisector** of line segment [AB].

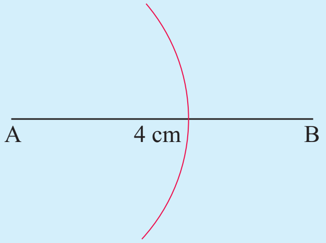
It is at right angles to [AB] so it is **perpendicular**, and since M is midway between A and B we say M **bisects** [AB].

We can use a perpendicular bisector to locate the midpoint of a line segment. We will see this in the following example.

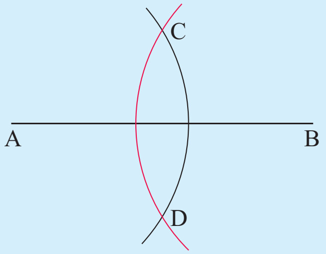
Example 5 **Self Tutor**

[AB] has length 4 cm. Locate the midpoint of [AB] by construction using a perpendicular bisector.

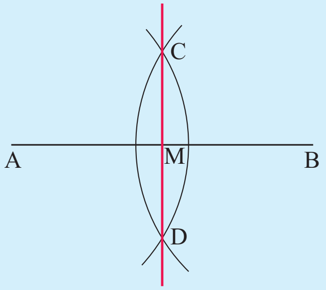
Step 1: With centre A and radius more than 2 cm, draw an arc of a circle to cut [AB] as shown.



Step 2: Repeat *Step 1*, but with centre B. Make sure that the first arc is crossed twice at C and D.

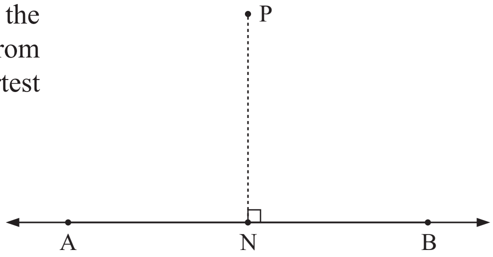


Step 3: With pencil and ruler, join C and D.
The point where (CD) and [AB] meet is the midpoint M.
(CD) and [AB] are perpendicular.



CONSTRUCTING A PERPENDICULAR TO AN EXTERNAL POINT

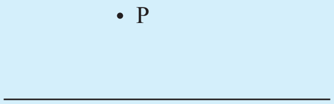
If we are given a line (AB) and a point P not on the line, we can construct the perpendicular to P from the line. This is useful because PN is the shortest distance from P to the line (AB).



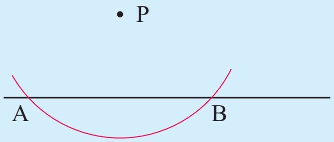
Example 6

Self Tutor

Using a compass, ruler and pencil only, construct a perpendicular from the line to an external point P.

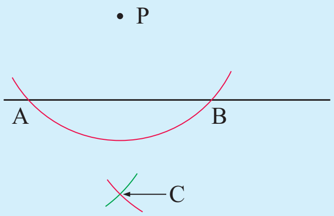


Step 1: With centre P, draw an arc to cut the line in two places A and B.

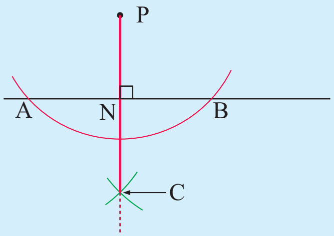


Step 2: With centre A, draw the green arc shown.

With centre B draw the red arc to cut the other one at C.



Step 3: Join P to C. We let N be the point of intersection with the original line. [PC] is perpendicular to the original line and PN is the shortest distance from the line to P.



EXERCISE 2D

Do not erase any construction lines.

1 **a** Use your protractor to draw accurately $A\hat{B}C$ of size 50^o.

b Use a compass and ruler only to bisect $A\hat{B}C$.

c Use a protractor to check the accuracy of your construction.

2 **a** Use your protractor to draw accurately $P\hat{Q}R$ of size 122^o.

b Use a compass and ruler only to bisect $P\hat{Q}R$.

c Use a protractor to check the accuracy of your construction.

3 **a** Draw any triangle ABC and carefully bisect its three angles.

b Repeat with another triangle DEF of different shape.

c Check with other students in your class for any observations about the three angle bisectors.

d Copy and complete: "The three angle bisectors of a triangle ......".

4 **a** Use a set square or a protractor to draw an angle of 90^o. Let this angle be $P\widehat{Q}R$.

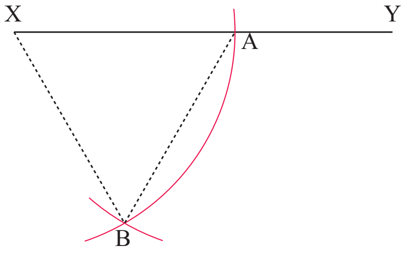
b Bisect $P\widehat{Q}R$ using a compass and ruler only. Check that each angle measures 45^o.

c Using a compass and ruler only, construct an angle measuring $22\frac{1}{2}^o$.

5 **a** [XY] is a line segment 5 cm long. At X an arc is drawn to cut [XY] at A. Keeping the same radius another arc is drawn with centre A to cut the first arc at B.

[AB] and [BX] are then joined by straight line segments.

Explain why triangle ABX is equilateral.

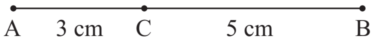


b What is the size of angle $A\widehat{X}B$?

c Perform the construction in **a**. Use it to construct angles of size:

i 30^o **ii** 15^o.

6 **a** Accurately draw the line segment [AB] as shown. Mark on it the points A, B and C.



b Use a compass and ruler construction to draw right angles at B above the line segment [AB], and at C below [AB].

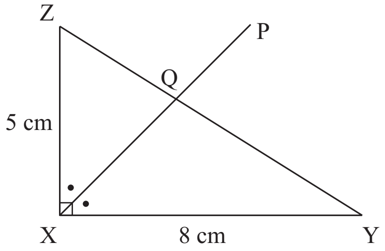
7 **a** Draw a line segment [XY] of length 8 cm.

b At X use a compass and ruler to construct a 90^o angle.

c Draw [XZ] of length 5 cm and join [ZY] as shown.

d Measure [ZY] to the nearest mm.

e Bisect $Z\widehat{X}Y$ using a compass and ruler only.



f Find by measuring:

i the length of [QY] **ii** the size of $X\widehat{Q}Y$.

8 **a** Draw line segment [AB] of length 4 cm.

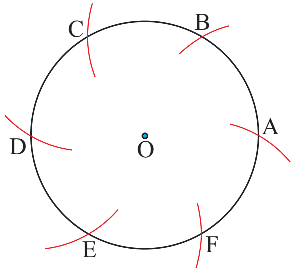
b At A use a compass and ruler to construct a 90^o angle.

c Locate the point C so that $C\widehat{A}B = 90^o$ and the length of [AC] is 3 cm.

d Join [BC] and measure its length.

e Use a protractor to measure angle $C\widehat{B}A$.

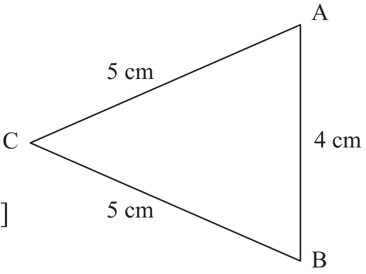
9



Draw a circle of radius 3 cm. Select a point A on it. Keeping the same radius, with centre A draw an arc meeting the circle at B. With centre B and the same radius, draw an arc meeting the circle at C. Continue this around the circle to obtain D, E and F.

If you have done this very accurately, a final arc will pass through A, the starting point.

Explain why ABCDEF is a regular hexagon.



10 **a** Accurately construct the isosceles triangle illustrated. Use a compass and ruler to do this.

b Bisect $A\widehat{C}B$ using a compass.

c If the bisector in **b** meets [AB] at point X, measure:

i the length of [AX] **ii** the length of [BX]

iii the size of $A\widehat{C}B$ **iv** the size of $A\widehat{X}C$

11 **a** Draw a line segment [PQ] where $PQ = 5$ cm.

b Use a compass and ruler to construct the perpendicular bisector of [PQ].

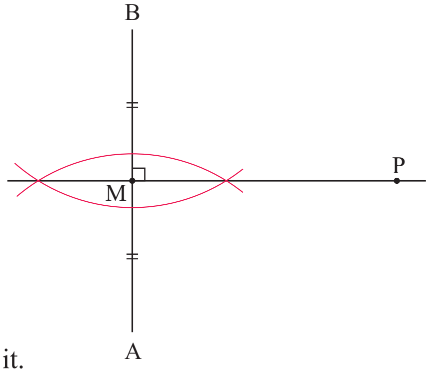
c If the perpendicular bisector of [PQ] meets [PQ] at Y, measure the lengths of [PY] and [QY].

12 **a** Draw any triangle ABC and construct the perpendicular bisectors of its three sides.

b Repeat with another triangle PQR of different shape.

c What do you observe from **a** and **b**?

d Copy and complete: “The three perpendicular bisectors of the sides of a triangle ......”.



13 **a** In the figure alongside, the line segment [AB] has been perpendicularly bisected. Repeat this construction with $AB = 4$ cm.

b Choose P on the perpendicular bisector such that the length of [MP] is 3.5 cm.

c Measure the lengths of [AP] and [BP]. What do you notice?

d Measure $A\widehat{P}M$ and $B\widehat{P}M$ with a protractor. What do you notice?

14 [AB] is a line segment. The point P is 3 cm from it.

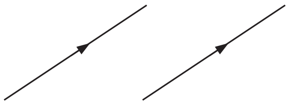
a Construct the perpendicular from [AB] to P using a compass and ruler only.

b Explain why the length of the perpendicular must be the shortest distance from [AB] to P.

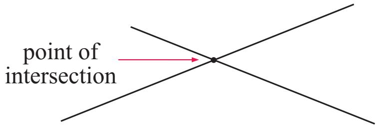
E ANGLE PAIRS

We have already seen how lines drawn in a plane are either **parallel** or **intersecting**.

For example,



parallel



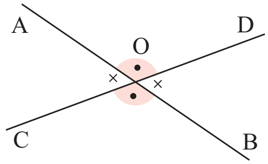
intersecting

When we are dealing with several lines in a plane, we can identify a number of **angle pairs**.

VERTICALLY OPPOSITE ANGLES

Vertically opposite angles are formed when two straight lines intersect. The two angles are directly opposite each other through the vertex.

For example:

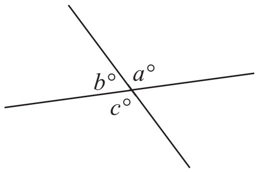


$A\widehat{O}C$ and $D\widehat{O}B$ are vertically opposite.
$A\widehat{O}D$ and $C\widehat{O}B$ are vertically opposite.

Measure the pairs of vertically opposite angles carefully. You should conclude that vertically opposite angles have the same size.

GEOMETRY PACKAGE

Consider the following diagram:



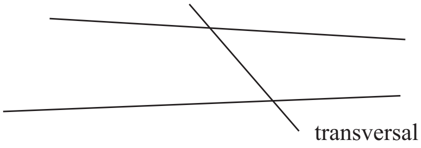
$a + b = 180$ {angles on a line}
and $c + b = 180$ {angles on a line}
so $a = c$ and these are vertically opposite.

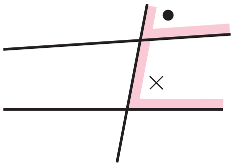
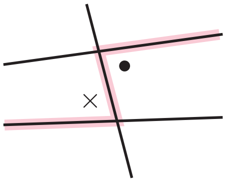
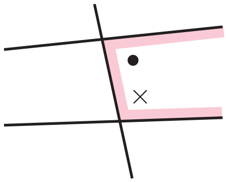
When two straight lines intersect, pairs of **vertically opposite** angles are *equal* in size.

CORRESPONDING, ALTERNATE AND CO-INTERIOR ANGLES

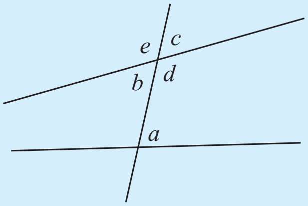
If a third line crosses two other straight lines we call it a **transversal**.

When two or more straight lines are cut by a transversal, three different angle pairs are formed:



Corresponding angle pairs	Alternate angle pairs	Co-interior angle pairs
 The angles marked • and × are corresponding angles because they are both in the *same position*. They are on the *same side* of the transversal and the *same side* of the two straight lines, in this case *both above* the line.	 The angles marked • and × are alternate angles. They are on *opposite sides* of the transversal and *between* the two straight lines.	 The angles marked • and × are co-interior angles. They are on the *same side* of the transversal and *between* the two straight lines.

Example 7 **Self Tutor**



Name the following angle pairs:

a a and d **b** a and b

c d and e **d** a and c

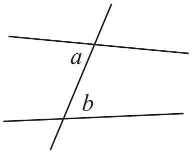
a a and d are co-interior angles **b** a and b are alternate angles

c d and e are vertically opposite angles **d** a and c are corresponding angles

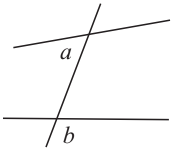
EXERCISE 2E

1 In which of the following diagrams are a and b alternate angles?

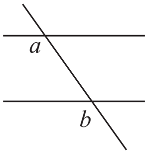
a



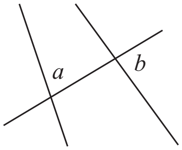
b



c

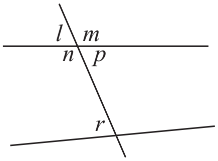


d

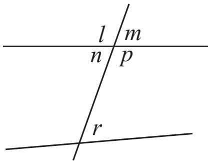


2 Which angle is alternate to angle r?

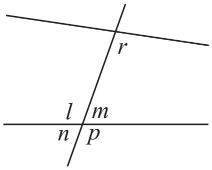
a



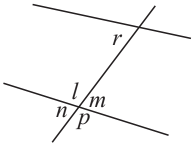
b



c

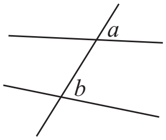


d

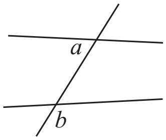


3 In which of the following diagrams are a and b corresponding angles?

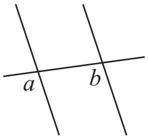
a



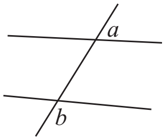
b



c

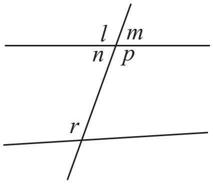


d

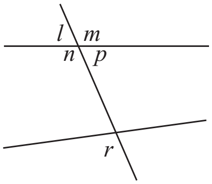


4 Which angle is corresponding to angle r?

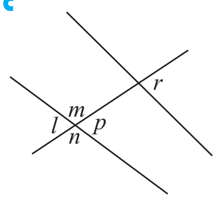
a



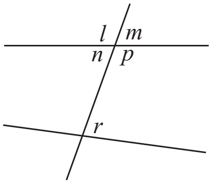
b



c

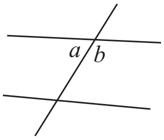


d

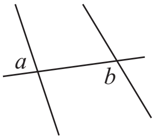


5 In which of the following diagrams are a and b co-interior angles?

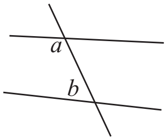
a



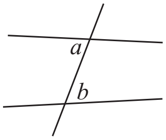
b



c

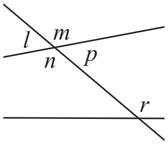


d

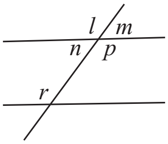


6 Which angle is co-interior with angle r?

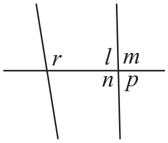
a



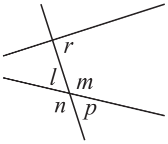
b



c

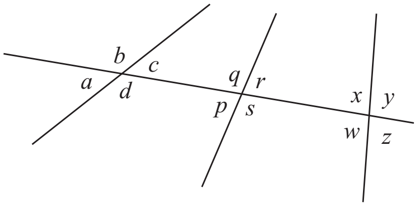


d



7 Classify the following angle pairs as either corresponding, alternate, co-interior, or vertically opposite:

a a and p **b** r and w **c** r and x
d z and s **e** b and q **f** a and c
g x and z **h** w and s **i** c and p



F PARALLEL LINES

INVESTIGATION ANGLE PAIRS ON PARALLEL LINES

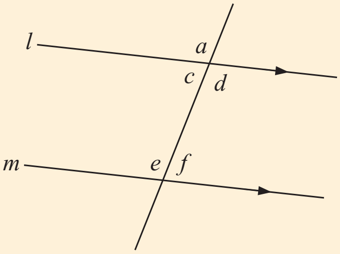
What to do:

1. Print this worksheet from the CD so that you can write directly onto it.

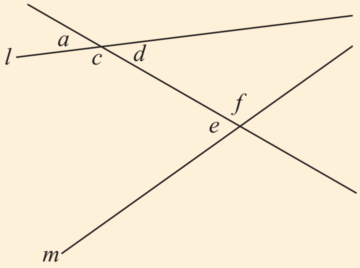
WORKSHEET

2. In the following five diagrams, l and m are two lines. The other labels refer to angles. In each diagram measure the angles marked and answer the related questions in the table below:

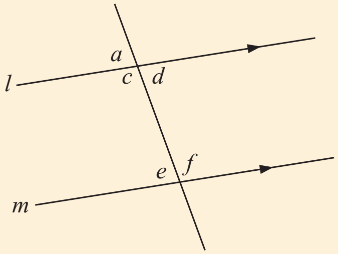
a



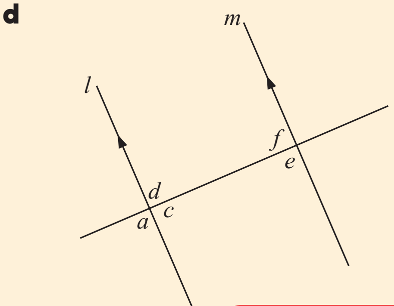
b



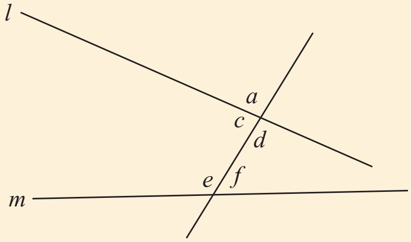
c



d



e



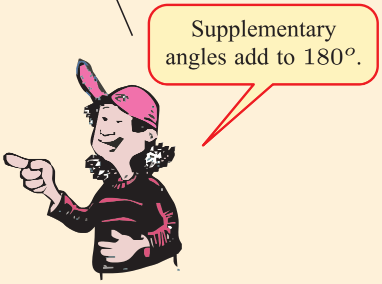


Diagram	Are the lines l and m parallel?	Are the corresponding angles a and e equal?	Are the alternate angles c and f equal?	Are the co-interior angles d and f equal? If not, are they supplementary?	
				equal	supplementary
a					
b					
c					
d					
e					

3 Discuss the results of the table and write your conclusions.

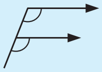
4 Click on the icon to further investigate angle pairs on parallel lines.

GEOMETRY PACKAGE

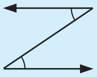
From the **Investigation** you should have discovered some important facts about special angle pairs.

When **parallel lines** are cut by a **transversal**:

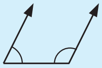
- corresponding angles are equal in size



- alternate angles are equal in size



- co-interior angles are supplementary, or add up to 180°.

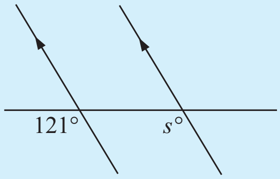


With these geometrical facts we can find unknown values for angles on parallel lines.

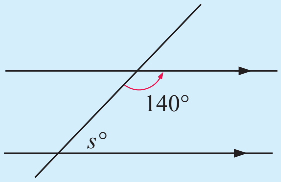
Example 8 **Self Tutor**

Find the value of the unknown, giving a brief reason for your answer:

a



b



a $s = 121$
{equal corresponding angles}

b Using co-interior angles
$s + 140 = 180$
$\therefore \quad s = 40$

TESTS FOR PARALLELISM

GEOMETRY PACKAGE

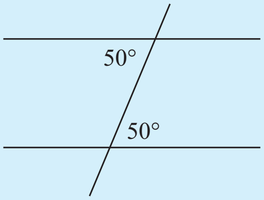
The facts about parallel lines and special angle pairs give us tests for finding whether lines cut by a transversal are parallel.

- If pairs of corresponding angles are equal in size then the lines must be parallel.
- If pairs of alternate angles are equal in size then the lines must be parallel.
- If pairs of co-interior angles are supplementary then the lines must be parallel.

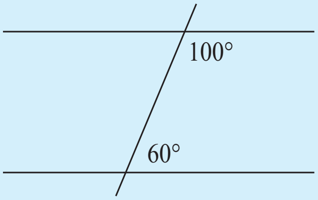
Example 9 **Self Tutor**

Decide if the figure contains parallel lines, giving a brief reason for your answer:

a



b



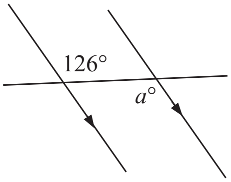
a These alternate angles are equal so the lines are parallel.

b These co-interior angles add to 160^o, so they are not supplementary. The lines are *not* parallel.

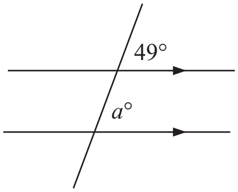
EXERCISE 2F

1 Find, giving brief reasons, the values of the unknowns in alphabetical order:

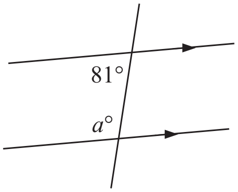
a



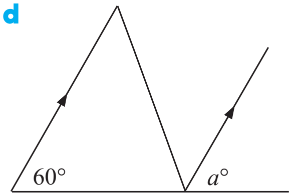
b



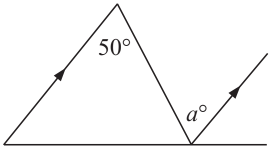
c



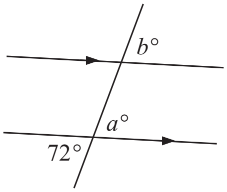
d



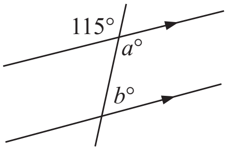
e



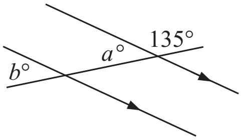
f



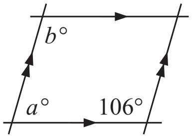
g



h

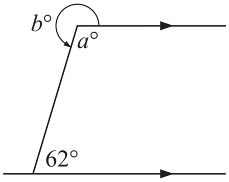


i

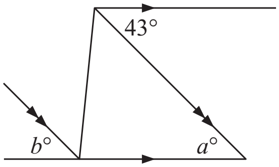


2 Find, giving brief reasons, the values of the unknowns in alphabetical order:

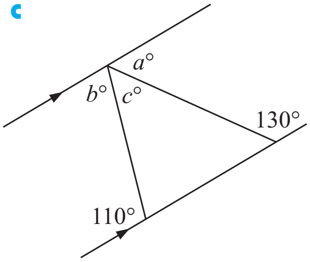
a



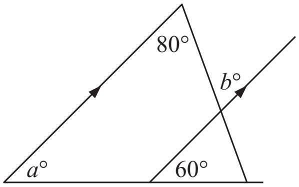
b



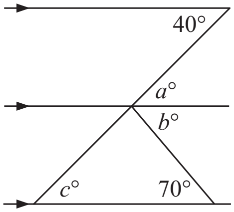
c



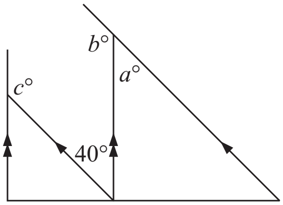
d



e

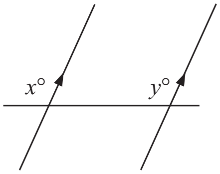


f

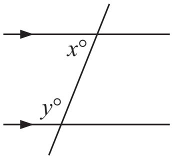


3 Write a statement connecting the unknowns, giving a brief reason:

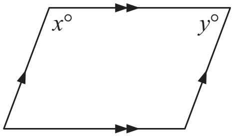
a



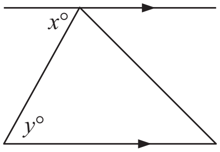
b



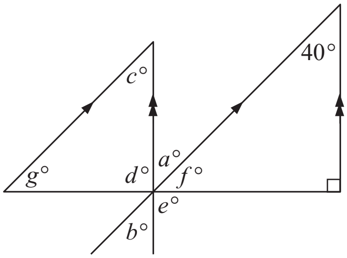
c



d

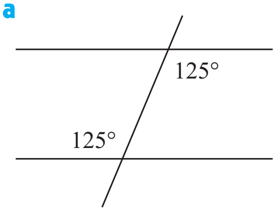


4 Working in alphabetical order, find the values of the unknowns.
Give a reason for each answer.



5 The following figures are not drawn to scale. Does each figure contain a pair of parallel lines? Give a brief reason for each answer.

a



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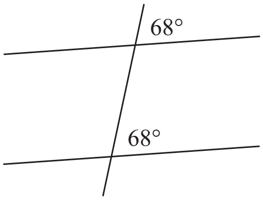

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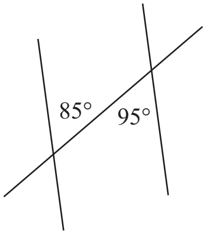

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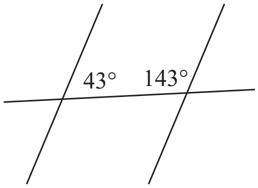

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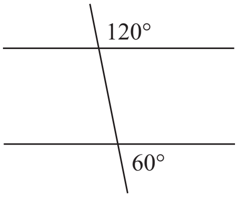

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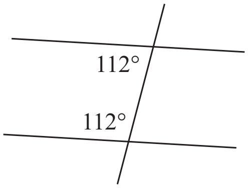


KEY WORDS USED IN THIS CHAPTER

- acute angle
- co-interior angles
- concurrent
- figure
- line segment
- perpendicular
- protractor
- revolution
- supplementary angles
- vertex
- alternate angles
- collinear
- corresponding angles
- intersecting lines
- obtuse angle
- perpendicular bisector
- ray
- right angle
- three point notation
- vertically opposite
- angle
- complementary angles
- diagonal
- line
- parallel lines
- point
- reflex angle
- straight angle
- transversal

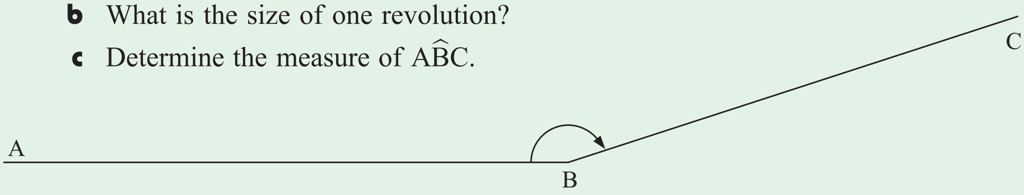
LINKS
click here

STAINED GLASS WINDOWS

Areas of interaction:
Human ingenuity, Approaches to learning

REVIEW SET 2A

1 **a** State the complement of 41^o.
b What is the size of one revolution?
c Determine the measure of $A\hat{B}C$.



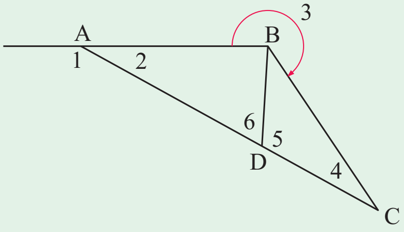
2 Consider the given figure alongside.

a Find the angle number corresponding to:

i $B\widehat{D}A$ **ii** $D\widehat{C}B$ **iii** $B\widehat{A}C$

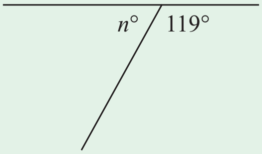
b Classify the following angles as acute, obtuse or reflex:

i 3 **ii** 1 **iii** 4



3 Find the value of the unknown in:

a



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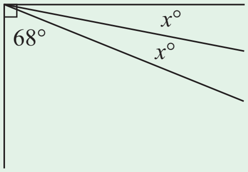

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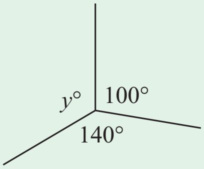


4 How many points are needed to determine the position of a line?

5 Draw a diagram to illustrate the following statement:
"Line segments [AB] and [CD] intersect at P."

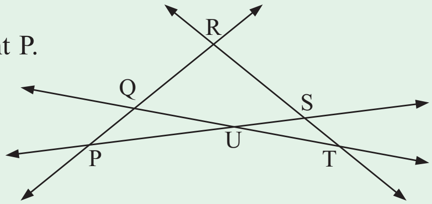
6 **a** Name line (RS) in two other ways.

b Name two different lines containing point P.

c What can be said about:

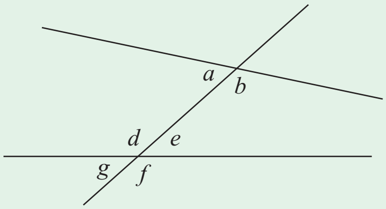
i points P, Q and R

ii lines (PQ) and (RS)?



7 Use your protractor to draw an angle of 56^o. Bisect this angle using your compass. Check that the two angles produced are each 28^o.

8



Name the angle:

a corresponding to a

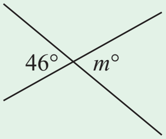
b alternate to a

c co-interior to e

d vertically opposite f

9 Find, giving a reason, the value of m in the following:

a



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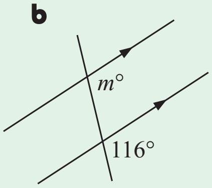
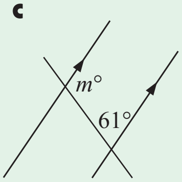


c



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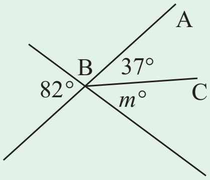
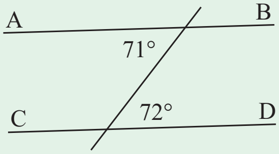
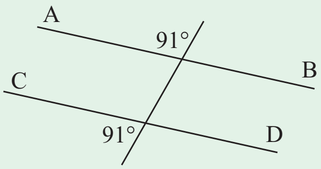


10 In each of the following, state whether (AB) is parallel to (CD). Give reasons for your answers.

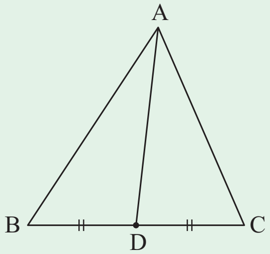
a



b



11 In any triangle a line from a vertex to the midpoint of the side opposite is called a *median*.

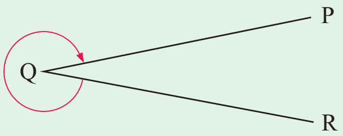


- **a** Draw any triangle ABC. Use a compass and ruler construction to find the midpoints of all three sides.
- **b** What do you notice about the three medians?

12 Use a compass and ruler to construct an angle of 15^o.

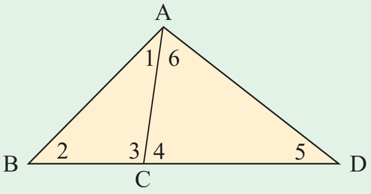
REVIEW SET 2B

1
- **a** Draw a diagram to illustrate an obtuse angle.
- **b** If $\widehat{ABC}$ and $\widehat{DEF}$ are equal angles and the measure of $\widehat{ABC}$ is 72^o, find the measure of $\widehat{DEF}$.
- **c** Determine the measure of reflex $\widehat{PQR}$.



2 Match the angle name to the angle number:

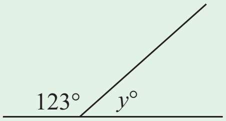
a $\widehat{ADC}$ **b** $\widehat{BAC}$ **c** $\widehat{ABD}$



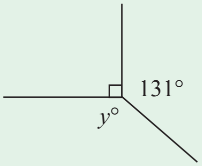
3 **a** What is the complement of 63^o? **b** What is the supplement of 70^o?

4 Find, giving a reason, the value of y in each of the following:

a



b



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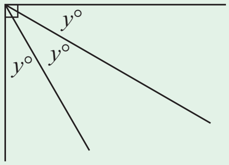
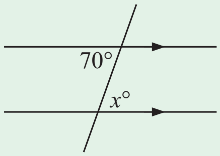


5 Copy and complete: "Points are collinear if ......".

6 Draw [AB] of length 4 cm. Construct an angle of 90^o at B using a compass and ruler only. Draw [BC] of length 3 cm and perpendicular to [AB]. Draw in and measure the length of [AC].

7 Find, giving reasons, the value of x:

a



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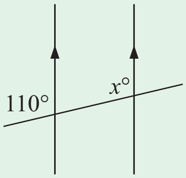

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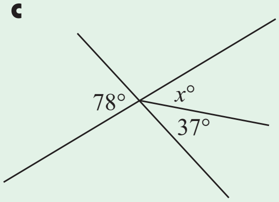
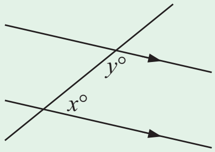


8 Write down a statement connecting the unknowns. Give reasons for your answers.

a



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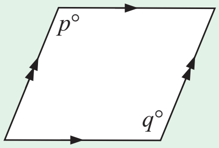
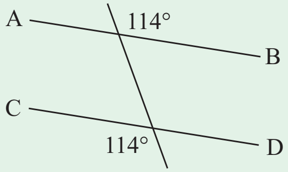


9 In each of the following, state whether [AB] is parallel to [CD]. Give reasons for your answers.

a



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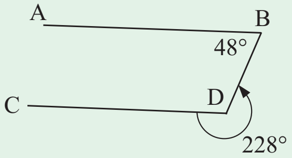
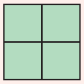


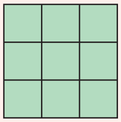
10 Using only a compass and ruler, construct a rectangle with sides 4 cm and 2.5 cm. Show all construction lines.

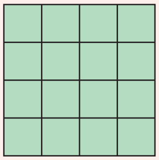
PUZZLE COUNTING SQUARES

consists of 1 square.



consists of $4+1=5$ squares, 4 with sides one unit and 1 with sides 2 units.





What to do:

a Investigate the total number of squares in a:

  i 3 by 3 square
  ii 4 by 4 square
  iii 5 by 5 square.

b Can you determine, without drawing figures, the total number of squares in a:

  i 6 by 6 square
  ii 20 by 20 square?

Chapter 3

Properties of numbers

Contents:

OPENING PROBLEM ONE THOUSAND LOCKERS

Imagine your school has 1000 students and 1000 lockers. The lockers are lined up in a row, all closed. Consider the following scenario:

- The first student goes down the row and opens every locker.
- The second student then goes down the row and closes every *second* locker, starting with locker number 2.
- The third student then *changes the state* of every third locker, starting with locker number 3. If it is open the student closes it, and if it is closed the student opens it.
- The fourth student changes the state of every fourth locker, starting with locker number 4.

Suppose this process continues for each of the 1000 students. At the end of the process, how many lockers are left open? Which lockers are they?

At first glance, the questions in the **Opening Problem** may appear very difficult and time consuming. However, some knowledge of the properties of numbers can make solving them relatively straight forward.

A DIVISIBILITY TESTS

One number is **divisible** by another if, when we divide, the quotient is a whole number.

For example, 16 is divisible by 2 because $16 \div 2 = 8$, but
16 is not divisible by 3 as $16 \div 3 = 5$ remainder 1.

We sometimes need to quickly decide whether one number is divisible by another. Obviously this can be done using a calculator provided the number is not too big. However, there are also some simple rules we can follow to test for divisibility without actually doing the division!

For example, we know that any even number is divisible by 2 and so its last digit must be 0 or one of the even digits 2, 4, 6, or 8.

INVESTIGATION 1 DIVISIBILITY BY 4 AND 9

One of the joys of mathematics comes from investigating and discovering things for yourself. In this investigation you should discover rules for divisibility by 4 and by 9.

What to do:

1 Copy and complete the following table. Start with the third column by writing down the last two digits of each number. Then use your calculator to check the numbers for divisibility by 4.

Number	*Divisibility by 4 (Yes/No)*	*Last 2 digits*	*Divisibility of last 2 digits by 4 (Yes/No)*
81			
154			
252			
3624			
8185			
9908			

2 Copy and complete: "A natural number is divisible by 4 if ......".

3 Copy and complete the following table:

Number	*Divisibility by 9 (Yes/No)*	*Sum of its digits*
81		$8 + 1 = 9$
154		
252		
3624		
8185		
9908		

4 Copy and complete: "A natural number is divisible by 9 if ......".

DIVISIBILITY TESTS FOR NATURAL NUMBERS

Number	**Divisibility Test**
2	If the last digit is 0 or even, then the original number is divisible by 2.
3	If the sum of the digits is divisible by 3, then the original number is divisible by 3.
4	If the number formed by the last *two* digits is divisible by 4, then the original number is divisible by 4.
5	If the last digit is 0 or 5 then the number is divisible by 5.
6	If a number is divisible by both 2 and 3 then it is divisible by 6.
8	If the number formed by the last three digits is divisible by 8, then the original number is divisible by 8.
9	If the sum of the digits is divisible by 9, then the original number is divisible by 9.
10	If the last digit of a number is 0, then the number is divisible by 10.
11	Add the digits in odd positions. Add the digits in the even positions. Find the difference between your two answers. If the difference is 0 or divisible by 11, the original number is divisible by 11.

There is also a rule for testing divisibility by 7, but it is too complicated to be of use. You could probably do the actual division in the time that it takes to recall and follow the testing procedure.

Example 1 **Self Tutor**

Test for divisibility by 3 and 11: **a** 846 **b** 2618

a The sum of the digits of 846 is $8 + 4 + 6 = 18$.
Since 18 is divisible by 3, so is 846.
For the number 846, the sum of the digits in the odd positions is $6 + 8 = 14$ and the sum of the digits in the even positions is 4.
The difference is $14 - 4 = 10$. Since 10 is not divisible by 11, 846 is also not divisible by 11.

b The sum of the digits of 2618 is $2 + 6 + 1 + 8 = 17$.
Since 17 is not divisible by 3, 2618 is also not divisible by 3.
For the number 2618, the sum of the digits in the odd positions is $8 + 6 = 14$ and the sum of the digits in the even positions is $1 + 2 = 3$.
The difference is $14 - 3 = 11$, which is divisible by 11.
So, 2618 is also divisible by 11.

EXERCISE 3A

1 Answer *true* or *false* for the following:

- **a** 45 is divisible by 5
- **b** 70 is divisible by 5
- **c** 402 is divisible by 5
- **d** 75 is divisible by 2
- **e** 96 is divisible by 2
- **f** 2338 is divisible by 2
- **g** 92 is divisible by 3
- **h** 126 is divisible by 3
- **i** 56 235 is divisible by 3

2 Which of the following are divisible by 3?

- **a** 87
- **b** 153
- **c** 512
- **d** 861
- **e** 977
- **f** 1002
- **g** 111 111
- **h** 56 947
- **i** 12 321
- **j** 778 899
- **k** 123 456 789
- **l** 124 124 124

3 Determine whether the following are divisible by 4:

- **a** 1250
- **b** 4234
- **c** 30 420
- **d** 315 422

4 Determine whether the following are divisible by 9:

- **a** 801
- **b** 2979
- **c** 35 533
- **d** 59 283

5 Determine which of the numbers 2, 3, 4, 5, 9, 10 the following are divisible by:

- **a** 120
- **b** 616
- **c** 960
- **d** 1443

6 Find all possible values of the missing digit if the following are divisible by 3:

- **a** 1□3
- **b** □36
- **c** 6□34
- **d** 89□12

7 Discuss and then write down concise tests for divisibility by:

- **a** 12
- **b** 15
- **c** 24

8 A four digit number has digit form '$a2b4$'. If it is divisible by 3, what are the possible values of $a + b$?

9 Find the smallest positive integer which is divisible by the numbers 2, 3, 7, 10, 15, and 21.

B FACTORS OF NATURAL NUMBERS

The **factors** of a natural number are the natural numbers which divide exactly into it.

For example, the factors of 10 are $1, 2, 5$ and 10 since $10 \div 1 = 10$, $10 \div 2 = 5$, $10 \div 5 = 2$, and $10 \div 10 = 1$.

3 is not a factor of 10 since $10 \div 3 = 3$ with remainder 1. 10 is not *divisible* by 3.

All natural numbers can be split into **factor pairs**.

For example, $10 = 1 \times 10$ or 2×5.
$1701 = 63 \times 27$.

When we write a number as a product of factors, we say it is **factorised**.

10 may be factorised as a product of two factors in two ways: 1×10 or 2×5.

12 has factors $1, 2, 3, 4, 6, 12,$ and can be factorised as a product of two factors in three ways: 1×12, 2×6 and 3×4.

EVEN AND ODD NUMBERS

A natural number is **even** if it has 2 as a factor and thus is divisible by 2.
A natural number is **odd** if it is not divisible by 2.

EXERCISE 3B.1

1 **a** List all the factors of 9.
b List all the factors of 12.
c Copy and complete: $12 = 2 \times ....$
d Write another pair of factors which multiply to give 12.

2 List **all** the factors of:

a 10 **b** 18 **c** 30 **d** 35 **e** 44 **f** 56
g 50 **h** 84 **i** 39 **j** 42 **k** 66 **l** 75

3 Complete the factorisations below:

a $24 = 6 \times ....$ **b** $25 = 5 \times ....$ **c** $28 = 4 \times ....$
d $100 = 5 \times ....$ **e** $88 = 11 \times ....$ **f** $88 = 2 \times ....$
g $36 = 2 \times ....$ **h** $36 = 3 \times ....$ **i** $36 = 9 \times ....$
j $49 = 7 \times ....$ **k** $121 = 11 \times ....$ **l** $72 = 6 \times ....$
m $60 = 12 \times ....$ **n** $48 = 12 \times ....$ **o** $96 = 8 \times ....$

4 Write the largest factor other than itself, for each of the following numbers:

a 12 **b** 18 **c** 27 **d** 48
e 44 **f** 75 **g** 90 **h** 39

5 What is the smallest whole number which:

a has factors of 2, 3 and 5
b has factors of 3, 5 and 7
c has factors of 2, 3, 5 and 7?

6 What can be said about:

  a the sum of three even numbers
  b the sum of three odd numbers
  c the product of an even and two odd numbers
  d the sum of four consecutive odd numbers?

7 $(\text{even})^3$ means 'an even number is raised to the power 3'. Which of these are odd and which are even?

  a $(\text{even})^2$
  b $(\text{odd})^2$
  c $(\text{even})^3$
  d $(\text{odd})^3$
  e $(\text{even})^4$
  f $(\text{odd})^4$

8 What is the smallest positive whole number which when increased by 1, is divisible by 3, 4 and 5?

PRIME AND COMPOSITE NUMBERS

Some numbers have only two factors, one and the number itself.

For example, the only two factors of 7 are 7 and 1, and of 31 are 31 and 1.

Numbers of this type are called **prime numbers**.

> A **prime** number is a natural number which has exactly two different factors, 1 and itself.
>
> A **composite** number is a natural number which has more than two factors.

From the definitions of prime and composite numbers we can see that

> the number 1 is neither prime nor composite.

PRIME FACTORS

The number 6 is **composite** since it has 4 factors: 1, 6, 2, 3.

Notice that $6 = 2 \times 3$, and this factor pair are both prime numbers.

The number 12 can be written as the product of prime factors $2 \times 2 \times 3$.

> All composite numbers can be written as the product of prime number factors in exactly one way.

To find the prime factors of a composite number we systematically divide the number by the prime numbers which are its factors, starting with the smallest.

When we write a number as the product of prime factors, it is usual to express it in index form.

For example, we would write $12 = 2^2 \times 3$.

Example 2

Self Tutor

a Express 252 as the product of prime factors in index form.

b What are the prime factors of 252?

a

$$\begin{array}{r|r} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

So, $252 = 2 \times 2 \times 3 \times 3 \times 7$
$= 2^2 \times 3^2 \times 7$

b The prime factors of 252 are 2, 3 and 7.

EXERCISE 3B.2

1 **a** List all the prime numbers less than 50.
   b Is 1 a prime number? Give a reason for your answer.
   c Are there any prime numbers which are even?

2 Show that the following numbers are composites:

a 5485 **b** 8230 **c** 7882 **d** 999

3 Express each of the following numbers as a product of prime factors in index form:

a 24 **b** 28 **c** 63 **d** 72 **e** 136
f 84 **g** 216 **h** 528 **i** 405 **j** 784
k 138 **l** 250 **m** 189 **n** 726 **o** 2310

4 Use a list of prime numbers to help you find:
   a the smallest one-digit odd prime
   b the only odd two-digit composite number less than 20
   c a prime number which is a factor of 105, 20 and 30.

5 The two digits of a number are the same. Their product is not a composite number. What is the original number?

6 **a** How many different factors do the following numbers have:
      i 4 **ii** 9 **iii** 25 **iv** 100
   b What numbers have an odd number of factors?
   c Consider the **Opening Problem** on page **58**.
      i Click on the icon for a demonstration of the process.
      ii How does the number of factors a number has determine whether a locker ends up open or closed?
      iii Answer the questions in the **Opening Problem**.

DEMO

HIGHEST COMMON FACTOR

A number which is a factor of two or more other numbers is called a **common factor** of these numbers.

For example, 7 is a common factor of 28 and 35.

We can use the method of finding prime factors to find the **highest common factor** (**HCF**) of two or more natural numbers.

Example 3 **Self Tutor**

Find the highest common factor (HCF) of 18 and 24.

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$\therefore \quad 18 = 2 \times 3 \times 3$

and $\quad 24 = 2 \times 2 \times 2 \times 3$

2×3 is common to the factorisations of both 18 and 24.

So, the highest common factor of 18 and 24 is $2 \times 3 = 6$.

EXERCISE 3B.3

1 Find the highest common factor of:

a 9, 12 **b** 8, 16 **c** 18, 42 **d** 14, 42
e 18, 30 **f** 24, 32 **g** 12, 36 **h** 15, 33
i 72, 96 **j** 108, 144 **k** 243, 246 **l** 78, 130

2 Find the HCF of:

a 25, 50, 75 **b** 22, 33, 44 **c** 21, 42, 84 **d** 39, 13, 26

3 Find the HCF of:

a 25, 35, 50, 60 **b** 36, 44, 52, 56 **c** 10, 18, 20, 36 **d** 32, 56, 72, 88

INVESTIGATION 2 THE SIEVE OF ERATOSTHENES

Eratosthenes (pronounced Er-ra-toss-tha-nees) was a Greek mathematician and geographer who lived between 275 BC and 194 BC. He is credited with many useful mathematical discoveries and calculations.

Eratosthenes was probably the first person to calculate the **circumference** of the earth, which is the distance around the equator. He did this using the lengths of shadows. His calculation was in terms of '*stadia*', which were the units of length in his era. When converted to metres, his calculation was found to be very close to modern day calculations.

Eratosthenes also found a method for 'sieving' out composite numbers from the set of natural numbers from 1 to 100 to leave only the primes.

His method was:

- cross out 1
- cross out all evens, except 2
- cross out all multiples of 3, except 3
- cross out all multiples of 5, except 5
- cross out all multiples of 7, except 7
- continue this process using the smallest uncrossed number not already used.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What to do:

1 Print the table of natural numbers from 1 to 100. Use Eratosthenes' method to discover the primes between 1 and 100.

PRINTABLE TABLE OF NUMBERS

2 Are there patterns in the way prime numbers occur? Copy the table into your book and count the number of primes in each set of numbers. Is there a pattern?

Set of numbers	*Total number of prime numbers*
0 to 9	
10 to 19	
20 to 29	
30 to 39	
40 to 49	
50 to 59	
60 to 69	

DEMO

C MULTIPLES OF NATURAL NUMBERS

The multiples of any whole number have that number as a factor. They are obtained by multiplying it by 1, then 2, then 3, then 4, and so on.

The multiples of 10 are: 10×1, 10×2, 10×3, 10×4, 10×5, ...
10, 20, 30, 40, 50, ...

The multiples of 15 are 15, 30, 45, 60, 75, ...

The number 30 is a multiple of both 10 and 15, so we say 30 is a **common multiple** of 10 and 15.

Example 4 **Self Tutor**

Find the common multiples of 4 and 6 between 20 and 40.

The multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, **36**, 40, ...

The multiples of 6 are 6, **12**, 18, **24**, 30, **36**, 42, ...

$\therefore$ the common multiples between 20 and 40 are 24 and 36.

EXERCISE 3C.1

1 List the first six multiples of:

a 3 **b** 8 **c** 12 **d** 17 **e** 25 **f** 34

2 Find the:

a fifth multiple of 6 **b** eighth multiple of 5

c tenth multiple of 15 **d** hundredth multiple of 49

3 List the numbers from 1 to 30.

a Put a circle around each multiple of 3.

b Put a square around each multiple of 4.

c List the common multiples of 3 and 4 which are less than 30.

4 Use the following list of multiples of 15 to answer the following questions:

15 30 45 60 75 90 105 120 135 150

State the numbers which are common multiples of both:

a 15 and 10 **b** 15 and 9 **c** 20 and 30 **d** 4 and 30

5 Use lists of multiples to help you find the following unknown numbers:

a I am an odd multiple of 9. The product of my two digits is also a multiple of 9. What two numbers could I be?

b I am a multiple of 7 and a factor of 210. The product of my two digits is odd. What number am I?

LOWEST COMMON MULTIPLE

The **lowest common multiple (LCM)** of two or more numbers is the smallest number which is a multiple of *each* of these numbers.

Example 5 **Self Tutor**

Find the lowest common multiple of 9 and 12.

The multiples of 9 are: 9, 18, 27, **36**, 45, 54, 63, **72**, 81, ...

The multiples of 12 are: 12, 24, **36**, 48, 60, **72**, 84, ...

$\therefore$ the common multiples are 36, 72, ... and 36 is the smallest of these

$\therefore$ the LCM is 36.

Another method for finding lowest common multiples is to write each number as the product of its prime factors. By writing these products one above the other, we can include in the LCM only those factors that are necessary. This method is particularly useful if we are seeking the LCM of several numbers.

Example 6 **Self Tutor**

Find the LCM of:

a 9 and 12 **b** 2, 3, 4 and 10

a

$$\begin{array}{r|l} 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Prime factors of 9:					3	×	3
Prime factors of 12:	2	×	2	×	3		
Prime factors of LCM:	2	×	2	×	3	×	3

∴ LCM = 36.

b

$$\begin{array}{r|l} 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Prime factors of 2:	2						
Prime factors of 3:					3		
Prime factors of 4:	2	×	2				
Prime factors of 10:	2					×	5
Prime factors of LCM:	2	×	2	×	3	×	5

∴ LCM = 60.

EXERCISE 3C.2

1 Find the lowest common multiples of the following sets of numbers:

a 3, 6 **b** 4, 6 **c** 5, 8 **d** 12, 15

e 6, 8 **f** 3, 8 **g** 5, 9 **h** 7, 10

2 Find the lowest common multiples of the following sets of numbers:

a 2, 4, 6 **b** 5, 9, 12 **c** 3, 4, 9 **d** 4, 6, 7

3 Find the lowest common multiple of 2, 3, 4 and 5.

4 Chris has a piece of rope. It can be cut exactly into either 12 metre or 18 metre lengths. Find the shortest length that Chris' rope could be.

5 Find the smallest positive integer which is exactly divisible by 2, 3, 10, 14 and 15.

6 Three bells toll at intervals of 4, 6 and 9 seconds respectively. If they start to ring at the same instant, how long will it take before they will again ring together?

7 Find the:

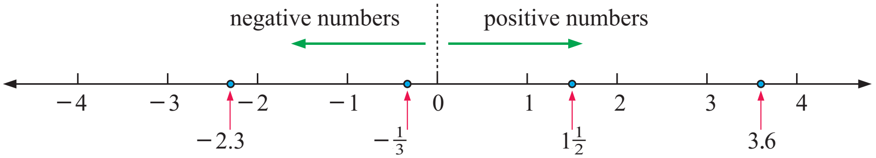
a smallest multiple of 6 that is greater than 200

b greatest multiple of 11 that is less than 500.

D DIRECTED NUMBERS

In previous years you have seen that numbers can be either **positive** or **negative**.

We can show positive and negative numbers on a **number line**. It shows not just whole numbers, but fractions and decimals also.



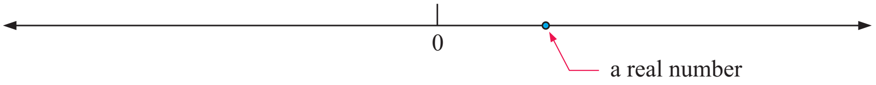
Notice that:

- zero or 0 is neither positive nor negative
- numbers to the right of 0 are positive numbers
- numbers to the left of 0 are negative numbers
- 2 or $+2$ is the **opposite** of -2 and vice versa
- $1\frac{1}{2} > -\frac{1}{3}$ as $1\frac{1}{2}$ is to the right of $-\frac{1}{3}$
- numbers like $\frac{2}{0}$ and $\sqrt{-4}$ are meaningless and do not appear on the number line.

REAL NUMBERS

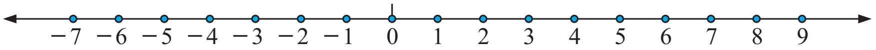
The set of all **real numbers** are the numbers which can be placed on a number line. They include all whole numbers and decimals.

We use a dot on the number line to represent a real number.



INTEGERS

The set of all **integers** consists of all the positive whole numbers, zero and the set of all negative whole numbers.



Example 7 **Self Tutor**

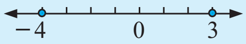
a What are the opposites of -3 and 7?

b Explain why $3 > -4$ is a true statement.

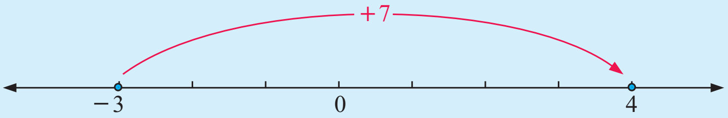
c Increase -3 by 7.

a The opposite of -3 is $+3$ or simply 3. The opposite of 7 is -7.

b $3 > -4$ reads "3 is greater than -4".
This is true as 3 is to the right of -4 on the number line.



c



$-3 + 7 = 4$

EXERCISE 3D.1

If necessary, draw a number line to help answer these questions:

1 Find the opposite of:
a 7 **b** -3 **c** 0 **d** 2.8 **e** -3.17 **f** -0.6731

2 Increase:
a 3 by 5 **b** 0 by 4 **c** -2 by 3 **d** -4 by 4 **e** -7 by 3

3 Decrease:
a 7 by 5 **b** 2 by 6 **c** 0 by 4 **d** -2 by 5 **e** -6 by $\frac{1}{2}$

4 By how much is:
a 7 larger than 2 **b** 6 larger than -3 **c** -2 larger than -11
d 3 smaller than 12 **e** 0 smaller than 7 **f** -5 smaller than 5?

5 True or false?
a $6 > -10$ **b** $-6 < -11$ **c** $-8 > 5$ **d** $-15 < -6$

6 Draw a number line to show:
a $\{-5, -2.5, -0.5, 0, 1.5, 3\}$ **b** $\{-4, -\frac{5}{2}, -1\frac{1}{2}, 1, 2, 2\frac{3}{4}\}$

7 What number is halfway between:
a 1 and 5 **b** -3 and 7 **c** -5 and -1 **d** -6 and 8
e $-\frac{3}{4}$ and $\frac{3}{4}$ **f** -6 and -7 **g** 2.4 and 4.2 **h** -2.4 and 4.2?

8 Find:
a $6 + 2$ **b** $-6 + 2$ **c** $7 + 3$ **d** $-7 + 3$
e $11 + 7$ **f** $-11 + 7$ **g** $16 + 10$ **h** $-16 + 10$
i $23 + 8$ **j** $-23 + 8$ **k** $31 + 15$ **l** $-31 + 15$

9 Find:
a $11 - 3$ **b** $-11 - 3$ **c** $8 - 13$ **d** $-8 - 13$
e $18 - 7$ **f** $-18 - 7$ **g** $23 - 12$ **h** $-23 - 12$

10 Find:
a $5 + 3$ **b** $5 - 3$ **c** $-5 + 3$ **d** $-5 - 3$
e $7 - 8$ **f** $-7 + 8$ **g** $-7 - 8$ **h** $1 + 2.5$
i $1 - 2.5$ **j** $-1 + 2.5$ **k** $-1 - 2.5$ **l** $6.9 - 11.4$

11 **a** The temperature was -7^oC and it has fallen a further 18^oC. What is the temperature now?
b A submarine is 23 m below sea level and dives a further 36 m. How deep is it now?
c A bird flying 9 m above a lake suddenly drops 10.5 m to catch a fish. How far under the surface was the fish?

12 If we add or subtract two integers, is the answer always an integer?

ADDING AND SUBTRACTING DIRECTED NUMBERS

Consider these patterns:

$$\begin{array}{rcl} 5+2=7 & & 5-2=3 \\ 5+1=6 & & 5-1=4 \\ 5+0=5 & & 5-0=5 \\ 5+-1=4 & \text{and} & 5--1=6 \\ 5+-2=3 & & 5--2=7 \\ 5+-3=2 & & 5--3=8 \end{array}$$

Adding a **negative** number is the same as **subtracting** a **positive** number.

Subtracting a **negative** number is the same as **adding** a **positive** number.

Example 8 Self Tutor

Find:

a $6+-3$ **b** $6--3$ **c** $-6+-3$ **d** $-6--3$

a $6+-3 = 6-3 = 3$

b $6--3 = 6+3 = 9$

c $-6+-3 = -6-3 = -9$

d $-6--3 = -6+3 = -3$

EXERCISE 3D.2

1 Find:

a $8+-5$ **b** $8--5$ **c** $-8+-5$ **d** $-8--5$
e $4+-9$ **f** $4--9$ **g** $-4+-9$ **h** $-4--9$
i $17-12$ **j** $12-17$ **k** $12--17$ **l** $17--12$
m $-6+-1$ **n** $-6--1$ **o** $6--5$ **p** $-6--5$
q $11-19$ **r** $-11-19$ **s** $11--19$ **t** $-11--19$

2 Find:

a $3--2+4$ **b** $13+-5--2$ **c** $-6--4+-5$
d $-6+-1--8$ **e** $5-12--4$ **f** $8-15+-11$
g $3--1--5$ **h** $-6--8+2$ **i** $-2-8--6+-1$
j $-3+-4--6$ **k** $11-17--5$ **l** $-12-8+17-9$
m $-63-258$ **n** $174-317$ **o** $-28-39-47$

MULTIPLYING AND DIVIDING DIRECTED NUMBERS

By considering patterns like those for addition and subtraction above, we can obtain the following rules for multiplication and division.

Multiplication

- (positive) × (positive) = (positive)
- (positive) × (negative) = (negative)
- (negative) × (positive) = (negative)
- (negative) × (negative) = (positive)

Division

- (positive) ÷ (positive) = (positive)
- (positive) ÷ (negative) = (negative)
- (negative) ÷ (positive) = (negative)
- (negative) ÷ (negative) = (positive)

Example 9 Self Tutor

Find:

a 3×7 **b** 3×-7 **c** -3×7 **d** -3×-7

a 3×7
$= 21$

b 3×-7
$= -21$

c -3×7
$= -21$

d -3×-7
$= 21$

Example 10 Self Tutor

Find:

a $12 \div -3$ **b** $-12 \div -4$ **c** $18 \div -3$ **d** $-24 \div 8$

a $12 \div -3$
$= -4$

b $-12 \div -4$
$= 3$

c $18 \div -3$
$= -6$

d $-24 \div 8$
$= -3$

EXERCISE 3D.3

1 Find:

a 3×4 **b** -3×4 **c** 3×-4 **d** -3×-4
e 5×11 **f** 5×-11 **g** -5×-11 **h** -5×11
i 6×9 **j** 6×-9 **k** -6×9 **l** -6×-9
m $(-3)^2$ **n** -3^2 **o** $-(-3^2)$ **p** $-(-3)^2$
q $(-1)^4$ **r** $(-1)^5$ **s** $-(-1)^6$ **t** $-2 \times (-3)^2$

2 Find:

a $5 \times -2 \times 4$ **b** $-3 \times 2 \times -1$ **c** $(-4)^2 \times -3$ **d** $-3^2 \times -5$
e $8 \times (-2)^3$ **f** $-5 \times (-2)^4$ **g** $(-1)^5 \times (-3)^3$ **h** $-5 \times -2 \times -5$

3 Find:

a $15 \div 3$ **b** $-15 \div 3$ **c** $15 \div -3$ **d** $-15 \div -3$
e $24 \div 6$ **f** $24 \div -6$ **g** $-24 \div -6$ **h** $-24 \div 6$
i $7 \div 7$ **j** $-7 \div -7$ **k** $7 \div -7$ **l** $-7 \div 7$

COMBINED OPERATIONS

The order of operations rules also apply to negative numbers.

- **B**rackets are evaluated first.
- **E**xponents are calculated next.
- **D**ivisions and **M**ultiplications are done next, in the order that they appear.
- **A**ddition and **S**ubtractions are then done, in the order that they appear.

Example 11 **Self Tutor**

Find: **a** $3 - -6 \times 2$ **b** $13 + 25 \div -5$

a $3 - -6 \times 2$

$= 3 - -12$ {multiplication first}

$= 3 + 12$ {simplify}

$= 15$

b $13 + 25 \div -5$

$= 13 + -5$ {division first}

$= 13 - 5$ {simplify}

$= 8$

EXERCISE 3D.4

1 Find:

a $8 - 2 \times (-1)^3$

b $15 - 6 \times -3$

c $-2 \times (3 - 5)^2$

d $5 - 14 \div -2 + 1$

e $6 - 7 - (-2)^2$

f $(-2)^2 - -2^2$

g $-5 - 15 \div -3$

h $-12 + 3 \times -2$

i $3 \times -2 - -1 \times 5$

j $(-2)^3 - (-2)^2$

k $15 - (-4)^2 \times -1$

l $-16 \div (23 - -9)$

E ROOTS OF WHOLE NUMBERS

SQUARE ROOTS

The **square root** of some number a is the positive number which when squared gives a.

We write the square root of a as $\sqrt{a}$. $\boldsymbol{\sqrt{a} \times \sqrt{a} = a}$

For example, since $4 = 2^2$, $9 = 3^2$ and $16 = 4^2$,
$\sqrt{4} = 2$, $\sqrt{9} = 3$ and $\sqrt{16} = 4$.

The square roots of 4, 9 and 16 are whole numbers, so 4, 9 and 16 are **perfect squares**.

The square roots of most numbers are not whole numbers. For example, $\sqrt{2} \approx 1.414\,213$ ......

We can find square roots like this using our calculator. You need to look for the $\sqrt{}$ symbol. You may need to press a key such as $\boxed{\text{2nd F}}$ or $\boxed{\text{SHIFT}}$ to access this function.

CUBE ROOTS

The **cube root** of a is the number which when cubed gives a.

We write the cube root of a as $\sqrt[3]{a}$. $\boldsymbol{\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a}$

For example, since $8 = 2^3$ and $27 = 3^3$,

$\sqrt[3]{8} = 2$ and $\sqrt[3]{27} = 3$.

Notice that $(-2)^3 = -8$ and so $\sqrt[3]{-8} = -2$

In contrast, we cannot find the *square* root of a negative number.

HIGHER ROOTS

The nth root of a is the number which when raised to the power n gives a.

We write the nth root of a as $\sqrt[n]{a}$.

$$\boldsymbol{\underbrace{\sqrt[n]{a} \times \sqrt[n]{a} \times \ldots\ldots \times \sqrt[n]{a}}_{n \text{ times}} = (\sqrt[n]{a})^n = a}$$

A TEST FOR PRIME NUMBERS

The usual **test for a prime number** is:

- Find the square root of the number.
- Divide the original number by all known prime numbers less than the square root.
- If an integer results, the number is not a prime.
- If the division is never exact, the integer is a prime.

For example, to test if 391 is a prime we notice that $\sqrt{391} \approx 19.773$

So, we divide by all primes which are < 19.773

These are 2, 3, 5, 7, 11, 13, 17, and 19.

We divide first by 2, then by 3, then by 5, and so on.

We notice that $391 \div 17 = 23$,
so 391 is not a prime number.

EXERCISE 3E

1 Find:

a $\sqrt{25}$ **b** $\sqrt{36}$ **c** $\sqrt{81}$ **d** $\sqrt{144}$ **e** $\sqrt{289}$ **f** $\sqrt{441}$

g $\sqrt{625}$ **h** $\sqrt{1024}$ **i** $\sqrt{0}$ **j** $\sqrt{1369}$ **k** $\sqrt{6889}$ **l** $\sqrt{10\,000}$

2 Find correct to 2 decimal places:

a $\sqrt{3}$ **b** $\sqrt{7}$ **c** $\sqrt{10}$ **d** $\sqrt{200}$ **e** $\sqrt{1764}$

3 Find:

a $\sqrt[3]{1}$ **b** $\sqrt[3]{64}$ **c** $\sqrt[3]{125}$ **d** $\sqrt[3]{343}$ **e** $\sqrt[4]{1}$
f $\sqrt[4]{0}$ **g** $\sqrt[4]{16}$ **h** $\sqrt[3]{-1}$ **i** $\sqrt[3]{-27}$ **j** $\sqrt[4]{-1}$
k $\sqrt[4]{625}$ **l** $\sqrt[5]{1}$ **m** $\sqrt[5]{-1}$ **n** $\sqrt[5]{-32}$ **o** $\sqrt[6]{64}$

4 **a** Show that 223 is a prime number.
b Show that 527 is not a prime number.
c Is 5273 a prime number?

KEY WORDS USED IN THIS CHAPTER

- common multiple
- divisible
- factor pairs
- integer
- odd number
- prime number
- composite number
- even number
- factorised
- negative number
- perfect square
- real number
- cube root
- factor
- highest common factor
- number line
- positive number
- square root

LINKS
click here

MATCHSTICK MATHEMATICS

Areas of interaction:
Approaches to learning

REVIEW SET 3A

1 Find □ if 43□ is divisible by 5.

2 Find □ if 26□ is divisible by 6.

3 List all factors of 63.

4 List the multiples of 6 which lie between 30 and 50.

5 If an even number is multiplied by an odd number, and then an odd number is subtracted from the result, is the final answer odd or even?

6 List the prime numbers between 40 and 50.

7 Write $2^2 \times 3 \times 5$ as a natural number.

8 Determine the LCM of 6 and 8.

9 Write 420 as the product of its prime factors in index form.

10 Find the largest number which divides exactly into both 63 and 84.

11 I am a two digit number. I am a multiple of 8. Both my digits are prime numbers. Which number am I?

12 Write down the 21st odd number.

13 Find the smallest multiple of 11 which is greater than 300.

14 What is the minimum number of sweets needed if they are to be shared exactly between either 5, 6 or 9 children?

15 Find: **a** $\sqrt{169}$ **b** $\sqrt[3]{-125}$

16 Is 36 201 a prime number?

17 Find: **a** $-11 - -8$ **b** $6 + -5 - -11$ **c** $-2 \times (-1)^3$

REVIEW SET 3B

1 Find ☐ if 8☐4 is divisible by 3.

2 List the first 6 powers of 2, starting with 2.

3 List the factors of 162.

4 Write down the 28th even number.

5 Write down the seventh multiple of 12.

6 Is 4536 divisible by: **a** 3 **b** 4 **c** 5?

7 **a** List all the factors of 72.
  b List the prime numbers between 20 and 30.
  c Express 124 as the product of prime factors in index form.
  d Find the highest common factor of 14 and 49.

8 **a** Write in index form: $3 \times 3 \times 5 \times 5 \times 5$
  b Convert into a natural number: $2^4 \times 3^2$

9 Find the lowest common multiple of 15 and 18.

10 What is the largest prime number which will divide exactly into 91?

11 I am a two digit prime number. The number 5 less than me is a multiple of 11. Which number am I?

12 Write the following in index form with a prime number as a base:
  a 16 **b** 121

13 Suppose the garbage man visits your house once every 14 days, and the gardener visits your house once every 10 days. If they both visit your house today, how long will it be before they are both at your house on the same day again?

14 When I woke up this morning it was -5^oC. Since then the temperature has risen 17^oC. What is the temperature now?

15 Find: **a** $\sqrt{-16}$ **b** $\sqrt[4]{16}$

16 Is 2743 a prime number?

17 Find: **a** $17 + -5 - -2$ **b** $(-1)^3 \times 5^2$ **c** $-(-3)^2$
  d $\dfrac{6 \times -2}{-4}$ **e** $\dfrac{-3}{9}$ **f** $\dfrac{7-16}{-2-7}$

ACTIVITY GOLDBACH'S "GOLDEN RULES"?

In 1742, **Christian Goldbach** stated two "golden rules":

- Every even number greater than 4 can be written as the sum of **two odd primes**.
- Every odd number greater than 8 is the sum of **three odd primes**.

What to do:

1 Complete the following table to test Goldbach's first "rule". The first 15 primes are given to help you: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

$6 = 3 + 3$	$8 = 3 + 5$	$10 =$	$12 =$	$14 =$
$16 =$	$18 =$	$20 =$	$22 =$	$24 =$
$26 =$	$28 =$	$30 =$	$32 =$	$34 =$
$36 =$	$38 =$	$40 =$	$42 =$	$44 =$
$46 =$	$48 =$	$50 =$	$52 =$	$54 =$

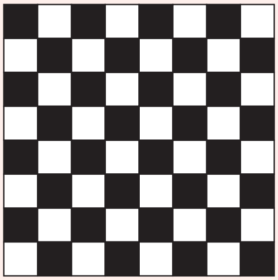
2 Complete the following table to test Goldbach's second "rule".

$9 = 3 + 3 + 3$	$11 = 3 + 3 + 5$	$13 =$	$15 =$	$17 =$
$19 =$	$21 =$	$23 =$	$25 =$	$27 =$
$29 =$	$31 =$	$33 =$	$35 =$	$37 =$
$39 =$	$41 =$	$43 =$	$45 =$	$47 =$
$49 =$	$51 =$	$53 =$	$55 =$	$57 =$

3 Can you find any numbers that do not follow Goldbach's "golden rules"?

PUZZLE TILES ON A CHESS BOARD

If you had sufficient of them, which of the following shapes could be used to completely cover a chess board?



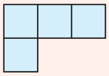
You may not cut, fold or break the shapes.

If it is possible to cover the chess board, show how it can be done. If it is not possible, explain why not.

a

b

c



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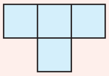

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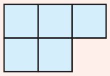

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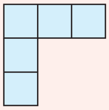


Chapter 4

Fractions

Contents:

In previous years we have seen how **fractions** are obtained when we divide a whole into equal portions.

In general, the division $a \div b$ can be written as the **fraction** $\frac{a}{b}$.

$\frac{a}{b}$ means we divide a whole into b equal portions, and then consider a of them.

$\frac{a}{b}$

the **numerator** is the number of portions considered

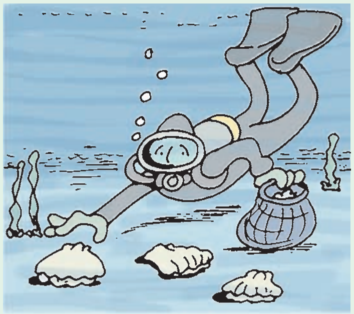
the **bar** indicates division

the **denominator** is the number of portions we divide a whole into.

The denominator cannot be zero, as we cannot divide a whole into zero pieces.

OPENING PROBLEM

A scallop fisherman has a daily catch limit. One day in the first hour he catches $\frac{1}{5}$ of his limit, in the second hour $\frac{1}{4}$, and in the third hour $\frac{1}{3}$.



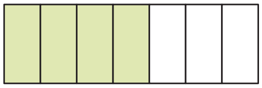
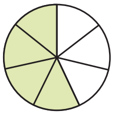
1 What fraction of his limit has he caught so far?

2 What fraction of his limit is he yet to catch?

3 If he can catch a further 40 kg without exceeding his limit, what is his limit?

A MANIPULATING FRACTIONS

The fraction four sevenths can be represented in a number of different ways:

Words	four sevenths
Diagram	as a shaded region *or* as pieces of a pie  
Number line	 four sevenths
Symbol	$\frac{4}{7}$ numerator bar denominator

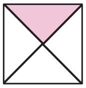
A fraction written in symbolic form with a bar is called a **common fraction**.

PROPER AND IMPROPER FRACTIONS

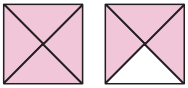
A fraction which has numerator **less** than its denominator is called a **proper fraction**.

A fraction which has numerator **greater** than its denominator is called an **improper fraction**.

For example, $\frac{1}{4}$ is a proper fraction.



$\frac{7}{4}$ is an improper fraction.



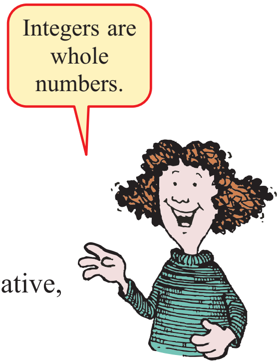
When an improper fraction is written as a whole number and a fraction, it is called a **mixed number**.

For example, $\frac{7}{4}$ can be written as the mixed number $1\frac{3}{4}$. We can see this in the diagram above as there is one whole square shaded plus three quarters of another square.

RATIONAL NUMBERS

A **rational number** is a number which can be written in the form $\dfrac{a}{b}$ where a and b are both **integers** and $b \neq 0$.

We can see that rational numbers are another special type of fraction. Most of the fractions we deal with in this course are rational numbers.



NEGATIVE FRACTIONS

In **Chapter 3** we saw that whenever we divided a positive by a negative, or a negative by a positive, the result is a negative.

Since the bar of a fraction indicates division, the fraction

$$\frac{-1}{2} \quad \text{means} \quad \underset{\substack{\uparrow \\ \text{negative}}}{(-1)} \div \underset{\substack{\uparrow \\ \text{positive}}}{2} = \underset{\substack{\uparrow \\ \text{negative}}}{-\frac{1}{2}}$$

Also,

$$\frac{1}{-2} \quad \text{means} \quad \underset{\substack{\uparrow \\ \text{positive}}}{1} \div \underset{\substack{\uparrow \\ \text{negative}}}{(-2)} = \underset{\substack{\uparrow \\ \text{negative}}}{-\frac{1}{2}}$$

So, $\frac{-1}{2} = \frac{1}{-2} = -\frac{1}{2}$, and in general $\boldsymbol{\dfrac{-a}{b} = \dfrac{a}{-b} = -\dfrac{a}{b}}$

SIMPLIFYING FRACTIONS

We can **simplify** a fraction by cancelling **common factors** in the numerator and denominator.

When a fraction is written as a rational number with the smallest possible denominator, we say it is in **lowest terms**.

Example 1 Self Tutor

Simplify: **a** $\frac{7}{21}$ **b** $\frac{-2}{4}$

a
$$\begin{aligned}&\frac{7}{21}\\&=\frac{1\times\cancel{7}^{1}}{3\times\cancel{7}_{1}}\\&=\frac{1}{3}\end{aligned}$$

b
$$\begin{aligned}&\frac{-2}{4}\\&=-\frac{2}{4}\\&=-\frac{1\times\cancel{2}^{1}}{2\times\cancel{2}_{1}}\\&=-\frac{1}{2}\end{aligned}$$

The division line of fractions behaves like a set of brackets. This means that using the BEDMAS rule, the numerator and denominator must be found before doing the division.

Example 2 Self Tutor

Simplify: **a** $\dfrac{3-9}{2^2+4}$ **b** $\dfrac{4\times 5}{7-18\div 2}$

a
$$\begin{aligned}&\frac{3-9}{2^2+4}\\&=\frac{-6}{4+4} && \{\text{simplify numerator and denominator first}\}\\&=-\frac{6}{8}\\&=-\frac{3\times\cancel{2}^{1}}{4\times\cancel{2}_{1}} && \{\text{cancel common factor}\}\\&=-\frac{3}{4}\end{aligned}$$

b
$$\begin{aligned}&\frac{4\times 5}{7-18\div 2}\\&=\frac{20}{7-9}\\&=\frac{20}{-2}\\&=-\frac{10\times\cancel{2}^{1}}{\cancel{2}_{1}}\\&=-10\end{aligned}$$

Two fractions are **equal** or **equivalent** if they can be written in the same lowest terms.

We can convert a fraction to an equivalent fraction by multiplying or dividing both the numerator and denominator by the same non-zero number.

Example 3 Self Tutor

Express: **a** $\frac{3}{4}$ with denominator 32 **b** $\frac{25}{45}$ with numerator 15

a To convert the denominator to 32 we need to multiply by 8. We must therefore multiply the numerator by 8 also.

$$\begin{array}{c} \times 8 \\ \dfrac{3}{4} = \dfrac{24}{32} \\ \times 8 \end{array}$$

b We first notice there is a common factor of 5 in the numerator and denominator.

$$\frac{25}{45} = \frac{5\times\cancel{5}^{1}}{9\times\cancel{5}_{1}} = \frac{5}{9}$$

To convert the numerator to 15 we need to multiply by 3. We must also multiply the denominator by 3.

$$\begin{array}{c} \times 3 \\ \dfrac{5}{9} = \dfrac{15}{27} \\ \times 3 \end{array}$$

COMPARING FRACTIONS

To compare fractions we first convert them to equal fractions with a common denominator which is the lowest common multiple of the original denominators. This denominator is called the **lowest common denominator** or **LCD**.

For example, consider the fractions $\frac{4}{5}$ and $\frac{7}{9}$.

The lowest common denominator is 45.

$$\frac{4}{5} = \frac{4\times 9}{5\times 9} = \frac{36}{45} \qquad \frac{7}{9} = \frac{7\times 5}{9\times 5} = \frac{35}{45}$$

$$\frac{36}{45} > \frac{35}{45}, \quad \text{so} \quad \frac{4}{5} > \frac{7}{9}.$$

EXERCISE 4A

1 Represent the fraction three fifths using:

a a diagram **b** a number line **c** symbol notation.

2 Express with denominator 12:

a $\frac{2}{3}$ **b** $\frac{3}{4}$ **c** $\frac{5}{6}$ **d** $\frac{6}{18}$ **e** $\frac{15}{45}$

3 Express with numerator 12:

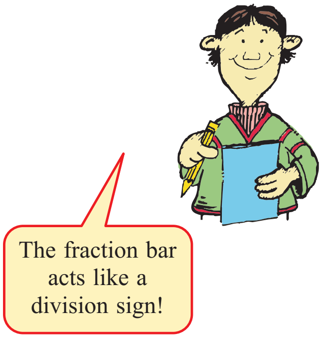
a $\frac{3}{7}$ **b** $\frac{6}{5}$ **c** $\frac{4}{9}$ **d** $\frac{24}{28}$ **e** $\frac{18}{42}$

4 Express in lowest terms:

a $\frac{6}{10}$ **b** $\frac{6}{18}$ **c** $\frac{25}{10}$ **d** $\frac{14}{35}$
e $\frac{33}{77}$ **f** $\frac{48}{72}$ **g** $\frac{78}{117}$ **h** $\frac{125}{1000}$

5 Simplify:

a $\frac{15}{3}$ **b** $\frac{-15}{5}$ **c** $\frac{20}{-4}$ **d** $\frac{22}{-2}$
e $\frac{18}{6}$ **f** $\frac{-18}{6}$ **g** $\frac{-12}{-4}$ **h** $\frac{3}{-6}$
i $\frac{-2}{-8}$ **j** $\frac{-5}{15}$ **k** $\frac{-7}{-14}$ **l** $\frac{4}{-8}$



6 Simplify:

a $\dfrac{3^2}{4-7}$ **b** $\dfrac{11-3}{16\div 4}$ **c** $\dfrac{3-7}{5+3}$ **d** $\dfrac{7+2}{4\times 3}$

e $\dfrac{8-2\times 5}{4\times 3}$ **f** $\dfrac{7\times 3-5}{2^2}$ **g** $\dfrac{-1+8\div 2}{8-5}$ **h** $\dfrac{3^2-2\times 5}{8-3^2}$

7 Simplify:

a $\dfrac{4\times -3}{6}$ **b** $\dfrac{-5\times -4}{-10}$ **c** $\dfrac{5\times -8}{4}$ **d** $\dfrac{24}{-3\times -4}$

e $\dfrac{6--12}{-3}$ **f** $\dfrac{3+-9}{-6}$ **g** $\dfrac{5--15}{6-8}$ **h** $\dfrac{-5\times -6}{-11-4}$

8 Plot each set of fractions on a number line:

a $-\frac{3}{5}, \frac{1}{5}, \frac{7}{5}, \frac{18}{5}$ **b** $-\frac{5}{2}, -\frac{4}{3}, \frac{1}{6}, \frac{2}{3}, \frac{7}{2}$

9 What fraction is greatest?

a $\frac{3}{5}$ or $\frac{4}{7}$ **b** $\frac{2}{3}$ or $\frac{5}{7}$ **c** $\frac{1}{6}$ or $\frac{2}{11}$ **d** $\frac{1}{4}$ or $\frac{3}{10}$ or $\frac{2}{7}$

10 Place these fractions in ascending order:

a $\frac{1}{8}, -\frac{2}{3}, \frac{3}{11}, -\frac{1}{6}, -\frac{3}{4}$

b $\frac{4}{3}, \frac{7}{5}, \frac{5}{7}, -\frac{3}{4}, -\frac{6}{11}$

11 Place these fractions in descending order:

a $\frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{13}, \frac{6}{10}$

b $-\frac{5}{8}, -\frac{1}{2}, -\frac{4}{7}, -\frac{7}{11}, -\frac{6}{13}$

Ascending means smallest to largest. Descending means largest to smallest.

B OPERATIONS WITH FRACTIONS

In this section we revise rules for operations with fractions that you should have seen in previous years.

ADDITION AND SUBTRACTION

To **add** or **subtract** fractions:

- If necessary, convert the fractions so they have the lowest common denominator.
- Add or subtract the new numerators. The denominator stays the same.

Example 4 **Self Tutor**

Find: **a** $\frac{3}{8}+\frac{1}{2}$ **b** $\frac{3}{4}-\frac{2}{3}+\frac{1}{2}$

a $\frac{3}{8}+\frac{1}{2}$

$=\frac{3}{8}+\frac{1\times 4}{2\times 4}$

$=\frac{3}{8}+\frac{4}{8}$ {LCD = 8}

$=\frac{7}{8}$ {adding numerators}

b $\frac{3}{4}-\frac{2}{3}+\frac{1}{2}$

$=\frac{3\times 3}{4\times 3}-\frac{2\times 4}{3\times 4}+\frac{1\times 6}{2\times 6}$

$=\frac{9}{12}-\frac{8}{12}+\frac{6}{12}$ {LCD = 12}

$=\frac{9-8+6}{12}$

$=\frac{7}{12}$

When adding or subtracting mixed numbers, you can first convert them to improper fractions and then perform the operation. However you can also add the whole numbers and fractions separately, then combine the result.

Example 5 **Self Tutor**

Find: $2\frac{1}{3}-3\frac{1}{2}+1\frac{1}{4}$

$2\frac{1}{3}-3\frac{1}{2}+1\frac{1}{4}$

$=\frac{7}{3}-\frac{7}{2}+\frac{5}{4}$ {converting to improper fractions}

$=\frac{7\times 4}{3\times 4}-\frac{7\times 6}{2\times 6}+\frac{5\times 3}{4\times 3}$

$=\frac{28}{12}-\frac{42}{12}+\frac{15}{12}$ {LCD = 12}

$=\frac{28-42+15}{12}$

$=\frac{1}{12}$

EXERCISE 4B.1

1 Find:

a $\frac{2}{3}+\frac{1}{3}$
b $\frac{3}{4}-\frac{1}{4}$
c $\frac{2}{5}+\frac{4}{5}$
d $\frac{3}{7}-\frac{5}{7}$
e $-\frac{1}{2}+\frac{5}{2}$
f $\frac{4}{3}-\frac{7}{3}$
g $\frac{1}{5}-\frac{3}{5}+1$
h $-\frac{1}{4}+\frac{3}{4}-2$

2 Find:

a $\frac{2}{5}+\frac{1}{2}$
b $\frac{3}{5}-\frac{1}{4}$
c $\frac{1}{3}-\frac{1}{2}$
d $\frac{2}{3}+\frac{4}{5}$
e $\frac{3}{7}-\frac{1}{2}$
f $-\frac{1}{2}+\frac{3}{4}$
g $-\frac{2}{3}-\frac{5}{6}$
h $\frac{1}{6}+\frac{3}{2}$
i $\frac{1}{10}-\frac{4}{5}$
j $\frac{7}{9}+\frac{2}{3}$
k $\frac{5}{8}-\frac{7}{4}$
l $-\frac{5}{7}+\frac{11}{14}$

3 Find:

a $1\frac{2}{3}-2$
b $3\frac{3}{4}-1\frac{1}{2}$
c $\frac{3}{4}-2\frac{1}{2}$
d $1\frac{2}{3}+3\frac{1}{4}$
e $4\frac{1}{3}+2\frac{1}{6}$
f $2\frac{2}{3}-5\frac{5}{6}$
g $-2\frac{1}{4}+3\frac{1}{8}$
h $4\frac{1}{5}-2\frac{1}{6}$

4 Find:

a the sum of $\frac{1}{3}$ and $\frac{2}{5}$
b the difference between $\frac{1}{4}$ and $\frac{2}{3}$
c the number 3 less than $\frac{2}{3}$
d the number $\frac{2}{3}$ more than $1\frac{1}{4}$.

5 **a** What must $\frac{1}{5}$ be increased by to get $\frac{2}{3}$?

b What number is $\frac{3}{4}$ less than $-1\frac{1}{2}$?

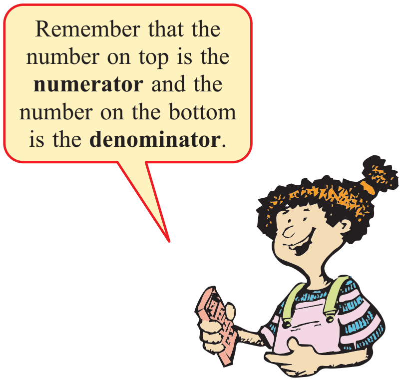
6 Find:

a $\frac{5}{6}+\frac{3}{5}+\frac{1}{3}$ **b** $\frac{2}{5}+\frac{3}{8}+1$ **c** $\frac{3}{4}+\frac{1}{6}-\frac{1}{2}$ **d** $\frac{1}{3}-\frac{2}{5}+\frac{1}{4}$

MULTIPLICATION OF FRACTIONS

To **multiply** two fractions, we multiply the two numerators to get the new numerator and multiply the two denominators to get the new denominator.

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$



Remember that the number on top is the **numerator** and the number on the bottom is the **denominator**.

Example 6 Self Tutor

Find: **a** $\frac{2}{3}\times\left(-\frac{4}{5}\right)$ **b** $\frac{1}{3}\times\left(\frac{2}{5}\right)^2$

a $\frac{2}{3}\times\left(-\frac{4}{5}\right)$

$=-\frac{2\times4}{3\times5}$ $\{(+)\times(-)=(-)\}$

$=-\frac{8}{15}$

b $\frac{1}{3}\times\left(\frac{2}{5}\right)^2$

$=\frac{1}{3}\times\frac{2}{5}\times\frac{2}{5}$

$=\frac{1\times2\times2}{3\times5\times5}$

$=\frac{4}{75}$

To help make multiplication easier, we can **cancel** any **common factors** in the numerator and denominator *before* we multiply.

Example 7 Self Tutor

Find: **a** $\frac{4}{9}\times\frac{3}{5}$ **b** $\frac{4}{9}\times1\frac{7}{8}$

a $\frac{4}{9}\times\frac{3}{5}$

$=\frac{4}{{}_3\cancel{9}}\times\frac{\cancel{3}^1}{5}$

$=\frac{4}{15}$

b $\frac{4}{9}\times1\frac{7}{8}$

$=\frac{{}^1\cancel{4}}{{}_3\cancel{9}}\times\frac{\cancel{15}^5}{\cancel{8}_2}$

$=\frac{5}{6}$

EXERCISE 4B.2

1 Find:

a $\frac{1}{2} \times \frac{3}{5}$
b $\frac{1}{4}$ of $\frac{3}{4}$
c $\left(\frac{3}{5}\right)^2$
d $\left(-\frac{1}{3}\right) \times \frac{2}{5}$

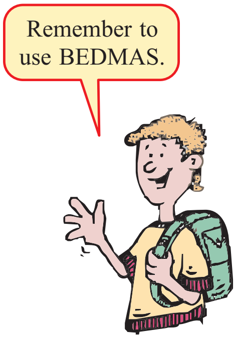
2 Evaluate, giving your answer in simplest form:

a $\frac{3}{4} \times \frac{1}{3}$
b $\frac{2}{5} \times \frac{3}{4}$
c $\left(-\frac{1}{2}\right) \times \frac{2}{3}$
d $\frac{4}{7}$ of 28
e $\frac{5}{4} \times \left(-\frac{2}{3}\right)$
f $\frac{4}{7} \times \frac{21}{16}$
g $1\frac{1}{2} \times \left(-\frac{1}{3}\right)$
h $\left(-\frac{3}{4}\right)^2$
i $\frac{3}{4}$ of 124
j $\frac{3}{8} \times \frac{4}{9}$
k $\left(-\frac{2}{3}\right) \times \left(-\frac{9}{8}\right)$
l $\frac{2}{5}$ of -65

3 Find the product of $\frac{2}{7}$ and $1\frac{2}{5}$.

4 Find:

a $\frac{2}{3} \times \frac{1}{4} \times \frac{3}{5}$
b $\frac{3}{8} \times \left(-\frac{4}{3}\right) \times \left(-\frac{2}{5}\right)$
c $\frac{2}{3} + \frac{3}{4} \times \frac{2}{3}$
d $\frac{3}{5} - \frac{5}{2} \times \frac{4}{3}$
e $\frac{3}{5} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{4}$
f $\frac{4}{3} \times \frac{1}{2} - \frac{1}{6} \times \frac{2}{3}$
g $\left(\frac{2}{3}\right)^2 - \frac{3}{4} \times 1\frac{2}{3}$
h $4 \times 1\frac{1}{3} - 5 \times \frac{2}{7}$



RECIPROCALS

Two numbers are **reciprocals** of each other if their product is one.

For any fraction $\frac{a}{b}$, we notice that $\frac{a}{b} \times \frac{b}{a} = 1$.

So, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

DIVIDING FRACTIONS

To **divide** by a number, we multiply by its reciprocal.

Example 8 **Self Tutor**

Find:

a $\frac{5}{4} \div \frac{2}{3}$
b $1\frac{1}{3} \div 3\frac{1}{2}$

a $\frac{5}{4} \div \frac{2}{3}$

$= \frac{5}{4} \times \frac{3}{2}$ {multiplying by reciprocal}

$= \frac{15}{8}$

b $1\frac{1}{3} \div 3\frac{1}{2}$

$= \frac{4}{3} \div \frac{7}{2}$ {converting to improper fractions}

$= \frac{4}{3} \times \frac{2}{7}$

$= \frac{8}{21}$

EXERCISE 4B.3

1 State the reciprocal of:

a $\frac{3}{4}$ **b** $\frac{2}{7}$ **c** 4 **d** $-\frac{1}{2}$

e -2 **f** $\frac{5}{8}$ **g** $-\frac{5}{2}$ **h** -1

2 Find:

a $\frac{2}{3}\div\frac{1}{6}$ **b** $\frac{5}{7}\div\frac{1}{3}$ **c** $\frac{3}{4}\div\left(-\frac{1}{2}\right)$ **d** $\frac{4}{5}\div 3$

e $\frac{1}{4}\div 1\frac{2}{3}$ **f** $2\frac{3}{4}\div\frac{2}{3}$ **g** $1\frac{1}{2}\div\left(-\frac{3}{4}\right)$ **h** $3\frac{1}{5}\div 1\frac{1}{3}$

3 Find:

a $\frac{2}{3}-\frac{3}{2}\div\frac{4}{5}$ **b** $\frac{5}{3}\div\frac{1}{2}+\frac{4}{3}$ **c** $\frac{1}{2}\times\frac{2}{5}-\frac{3}{4}\div\frac{6}{5}$ **d** $\frac{2}{5}\div\left(-\frac{1}{2}\right)+\frac{3}{4}\times\frac{2}{5}$

4 Find:

a the average of $\frac{1}{4}$ and $\frac{3}{4}$

b the number midway between $-\frac{1}{2}$ and $\frac{2}{3}$

c the average of $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$

d the quotient of $\frac{1}{3}$ and $\frac{3}{4}$

EVALUATING FRACTIONS USING A CALCULATOR

When we enter operations into a calculator, it automatically uses the BEDMAS rules. However, we need to be careful with more complicated fractions because we need to divide *the whole of the numerator* by *the whole of the denominator*. To make sure the calculator knows what we mean, we insert brackets around the numerator and the denominator.

For example, consider the expression $\dfrac{5+6}{3-1}$.

If we type in $5\ \boxed{+}\ 6\ \boxed{\div}\ 3\ \boxed{-}\ 1$, the calculator will think we want $5+\frac{6}{3}-1$, and so it will give us the wrong answer.

We need to insert brackets around both the numerator and denominator, giving $\dfrac{(5+6)}{(3-1)}$.

We type in $\boxed{(}\ 5\ \boxed{+}\ 6\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ 3\ \boxed{-}\ 1\ \boxed{)}$.

Example 9 **Self Tutor**

Find the value of: **a** $\dfrac{15-33}{17-7\times 3}$ **b** $\dfrac{15+3\times 5^2}{11-25\div 2}$

a $\dfrac{15-33}{17-7\times 3}=\dfrac{(15-33)}{(17-7\times 3)}=4\frac{1}{2}$

Calculator: $\boxed{(}\ 15\ \boxed{-}\ 33\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ 17\ \boxed{-}\ 7\ \boxed{\times}\ 3\ \boxed{)}\ \boxed{=}$

b $\dfrac{15+3\times 5^2}{11-25\div 2}=\dfrac{(15+3\times 5^2)}{(11-25\div 2)}=-60$

Calculator: $\boxed{(}\ 15\ \boxed{+}\ 3\ \boxed{\times}\ 5\ \boxed{x^2}\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ 11\ \boxed{-}\ 25\ \boxed{\div}\ 2\ \boxed{)}\ \boxed{=}$

EXERCISE 4B.4

1 Use a calculator to find the value of:

a $5+\frac{10}{5}$ **b** $\dfrac{5+10}{5}$ **c** $3-\frac{9}{6}$

d $\dfrac{3-9}{6}$ **e** $15-8\div4+10$ **f** $\dfrac{15-8}{4+10}$

g $\dfrac{4+8^2}{11-35}$ **h** $\dfrac{(4+8)^2}{11-35}$ **i** $\dfrac{4+\dfrac{8^2}{2}}{4-13}$

j $\dfrac{18-2^2}{18-8\times2}$ **k** $\dfrac{-4-11}{12-9\div2}$ **l** $\dfrac{22+11\div2}{23-3\times4}$

FRACTIONS WITHIN FRACTIONS

When faced with fractions such as $\dfrac{2+\frac{1}{3}}{2-\frac{1}{3}}$, it may be very tempting to reach for a calculator.

However, this fraction can actually be simplified easily by hand.

We multiply the fraction top and bottom by the lowest common denominator (LCD) of the little fractions within it. We are really just multiplying by 1, so the value of the fraction is not changed.

Example 10 **Self Tutor**

Simplify: **a** $\dfrac{2+\frac{1}{3}}{2-\frac{1}{3}}$ **b** $\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{2}{3}-\frac{3}{4}}$ **c** $\dfrac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{5}}$

a
$$\begin{aligned}&\frac{2+\frac{1}{3}}{2-\frac{1}{3}}\\ &=\left(\frac{2+\frac{1}{3}}{2-\frac{1}{3}}\right)\frac{3}{3} \quad \{\text{LCD}=3\}\\ &=\frac{6+1}{6-1}\\ &=\frac{7}{5}\end{aligned}$$

b
$$\begin{aligned}&\frac{\frac{2}{3}+\frac{3}{4}}{\frac{2}{3}-\frac{3}{4}}\\ &=\left(\frac{\frac{2}{3}+\frac{3}{4}}{\frac{2}{3}-\frac{3}{4}}\right)\frac{12}{12} \quad \{\text{LCD}=12\}\\ &=\frac{8+9}{8-9}\\ &=\frac{17}{-1}\\ &=-17\end{aligned}$$

c
$$\begin{aligned}&\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{5}}\\ &=\left(\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{5}}\right)\frac{60}{60} \quad \{\text{LCD}=60\}\\ &=\frac{20+15}{60-12}\\ &=\frac{35}{48}\end{aligned}$$

EXERCISE 4B.5

1 Simplify:

a $\dfrac{1+\frac{1}{2}}{1-\frac{1}{2}}$ **b** $\dfrac{2-\frac{1}{4}}{2+\frac{1}{4}}$ **c** $\dfrac{1+\frac{2}{3}}{2-\frac{1}{3}}$

d $\dfrac{\frac{1}{2}+\frac{3}{4}}{\frac{3}{4}-\frac{1}{2}}$ **e** $\dfrac{\frac{1}{2}+\frac{1}{5}}{\frac{1}{2}-\frac{1}{5}}$ **f** $\dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}-\frac{1}{4}}$

g $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{3}-\frac{1}{4}}$ **h** $\dfrac{1-\frac{1}{3}}{\frac{1}{2}-\frac{2}{5}}$ **i** $\dfrac{1+\frac{1}{4}-\frac{1}{3}}{1-\frac{1}{2}+\frac{1}{5}}$

INVESTIGATION 1 DIVISION BY ZERO

We have already indicated that division by zero (0) is not permitted. In fact, numbers like $\frac{2}{0}$ are excluded from being rational numbers because $\frac{2}{0}$ is not real and cannot be placed on a number line.

What to do:

1 **a** Copy and complete: **i** Since $\frac{6}{2}=3,\ \ 2\times 3=......$

ii Since $\frac{20}{5}=4,\ 5\times 4=......$

iii Since $\frac{2}{0}=a,\ \ 0\times a=......$

b In **iii** above, we are saying that if $\frac{2}{0}$ is equal to some number a, then $0=2$. Do you agree with this deduction?

c What can we conclude from **b**?

2 **a** Evaluate the following:

i $1\div\frac{1}{2}$ **ii** $1\div\frac{1}{5}$ **iii** $1\div\frac{1}{20}$ **iv** $1\div\frac{1}{1000}$ **v** $1\div\frac{1}{1\,000\,000}$

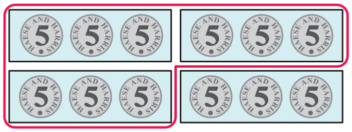
b Copy and complete: As the number we are dividing 1 by gets smaller and smaller, the answer gets ....

C PROBLEM SOLVING

In this section we see how fractions are applied to the real world. They can describe a part of a quantity or a group of objects.

For example, $\frac{3}{4}$ of 12 coins is 9 coins

and $\frac{3}{4}\times 12=\frac{3}{{}_1\cancel{4}}\times\frac{\cancel{12}^{\,3}}{1}=9$



Examples like this one tell us that 'of' is replaced by ×.

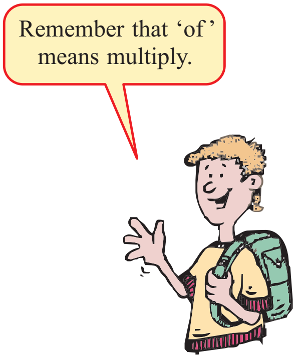
Example 11 **Self Tutor**

Find $\frac{3}{5}$ of €85.

$\frac{3}{5}$ of €85

$= \frac{3}{\cancel{5}_{1}} \times \text{€}\frac{\cancel{85}^{17}}{1}$

$= \text{€}51$



EXERCISE 4C

1 Find $\frac{2}{3}$ of \$213.

2 Find $\frac{3}{7}$ of £434.

3 Julie owes Gigi $\frac{3}{5}$ of €1336.25. How much does she owe Gigi?

4 Millie calculated that her bicycle cost $\frac{1}{83}$ of the cost of her father's car. If the car cost \$38 014, what did her bicycle cost?

5 The price of a shirt is $\frac{2}{13}$ of the cost of a suit. If the suit costs €292.50, find the cost of the shirt.

Example 12 **Self Tutor**

Rob eats $\frac{1}{3}$ of a watermelon one day and $\frac{3}{8}$ of it the next day.

What fraction of the watermelon remains?

The fraction remaining

$= 1 - \frac{1}{3} - \frac{3}{8}$ {from the whole we subtract the fractions eaten}

$= \frac{24}{24} - \frac{1\times 8}{3\times 8} - \frac{3\times 3}{8\times 3}$ {LCD = 24}

$= \frac{24-8-9}{24}$

$= \frac{7}{24}$

6 Pam uses $\frac{5}{8}$ of a cabbage for the evening meal. What fraction remains?

7 Phong eats $\frac{1}{3}$ of a chocolate bar in the morning and $\frac{5}{8}$ of it in the afternoon. What fraction remains?

8 Over three successive days Colin builds $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{4}$ of the brickwork of his new garage. What fraction must he complete on the fourth and final day?

9 200 kg of sugar must be poured into packets so there is $\frac{2}{5}$ kg of sugar per packet. How many packets will be filled?

10 2400 kg of icecream is put into plastic containers which hold $\frac{3}{4}$ kg each. How many plastic containers are needed?

11 Jon says that his income is now $3\frac{1}{2}$ times what it was 20 years ago. If his current annual income is €63 000, what was his income 20 years ago?

12 Silvia owns $\frac{1}{3}$ of a bakery, $\frac{2}{5}$ of a grocers shop and $\frac{1}{6}$ of a clothing store. The profits from these stores last year were \$215 610, \$436 215, and \$313 200 respectively. How much profit did Silvia make from the three businesses last year?

D THE UNITARY METHOD WITH FRACTIONS

If $\frac{4}{7}$ of an amount of money is \$480, then

$\frac{1}{7}$ of the amount $= \$480 \div 4 = \$120$.

Thus $\frac{6}{7}$ of the amount $= 6 \times \$120 = \$720$.

and $\frac{7}{7}$ is the whole amount which is $7 \times \$120 = \$840$.

So, given the value of a number of parts of a quantity we can find one part of the quantity and then the whole quantity or another fraction of the quantity. This is called the **unitary method.**

Example 13 **Self Tutor**

If $\frac{3}{8}$ of a shipping container holds 2100 identical cartons, how many cartons will fit into:

a $\frac{5}{8}$ of the container

b the whole container?

$\frac{3}{8}$ of the container holds 2100 cartons.

$\therefore$ $\frac{1}{8}$ of the container holds $2100 \div 3 = 700$ cartons.

a So, $\frac{5}{8}$ holds 700×5
$= 3500$ cartons

b The whole is $\frac{8}{8}$ which is 700×8
$= 5600$ cartons

EXERCISE 4D

1 $\frac{2}{3}$ of an amount of money is \$519. Find:

a $\frac{1}{3}$ of the money

b the whole amount.

2 $\frac{4}{5}$ of a soy bean crop is 2104 tonnes. Find the weight of:

a $\frac{1}{5}$ of the crop

b the whole crop.

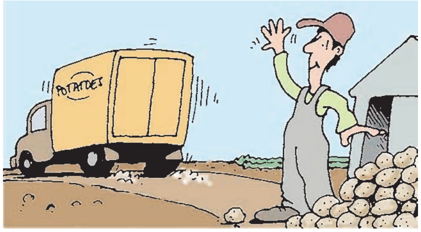
3 $\frac{2}{11}$ of Jo's weekly earnings are paid as income tax. She has \$666 remaining after tax. What is her total weekly pay?

4 $\frac{3}{13}$ of a field was searched for truffles and 39 were found. How many truffles would we expect to find in the remainder of the field?

5 Last week we picked $\frac{1}{3}$ of our grapes and this week we picked $\frac{1}{4}$ of them. So far we have picked 3682 kg of grapes. What is the total weight of grapes we expect to pick?

6 Alfredo sent $\frac{2}{5}$ of his potato crop to market last week. This week he sent $\frac{2}{3}$ of the remainder.

- **a** What fraction of his crop has now gone to market?
- **b** If he has 860 kg remaining, what was the original weight of the crop?



7 Annika pays $\frac{2}{25}$ of her weekly income into a retirement fund. If she pays £42 into the retirement fund, what is her:

a weekly income **b** annual income?

8 Jamil spent $\frac{1}{4}$ of his weekly salary on rent, $\frac{1}{5}$ on food, and $\frac{1}{6}$ on clothing and entertainment. The remaining money was banked.

- **a** What fraction of Jamil's money was banked?
- **b** If he banked \$138.00, what is his weekly salary?
- **c** How much did Jamil spend on food?

9 In autumn a tree starts to shed its leaves. $\frac{2}{5}$ of the leaves fall off in the first week, $\frac{1}{2}$ of those remaining fall off in the second week, and $\frac{2}{3}$ of those remaining fall off in the third week. 85 leaves now remain.

- **a** What fraction of leaves have fallen off at the end of:
  - **i** the second week **ii** the third week?
- **b** How many leaves did the tree have to start with?

E SQUARE ROOTS OF FRACTIONS

We have seen previously how $3^2 = 9$ indicates that $\sqrt{9} = 3$.

In the same way, $\left(\frac{2}{5}\right)^2 = \frac{4}{25}$ indicates that $\sqrt{\frac{4}{25}} = \frac{2}{5}$.

However, $\frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$ also.

So, we observe that $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ for positive numbers a and b.

Example 14 **Self Tutor**

Find **a** $\sqrt{\frac{4}{9}}$ **b** $\sqrt{2\frac{1}{4}}$

a $\sqrt{\frac{4}{9}}$

$= \frac{\sqrt{4}}{\sqrt{9}}$

$= \frac{2}{3}$

b $\sqrt{2\frac{1}{4}}$

$= \sqrt{\frac{9}{4}}$

$= \frac{\sqrt{9}}{\sqrt{4}}$

$= \frac{3}{2}$

$= 1\frac{1}{2}$

Before finding the square root, convert mixed numbers to improper fractions.

EXERCISE 4E

1 Copy and complete:

a Since $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, $\sqrt{\frac{1}{4}} = \ldots\ldots$

b Since $\left(\frac{2}{3}\right)^2 = \ldots\ldots = \ldots\ldots$, $\sqrt{\ldots\ldots} = \frac{2}{3}$

c Since $\left(\frac{3}{7}\right)^2 = \ldots\ldots = \ldots\ldots$, $\sqrt{\ldots\ldots} = \ldots\ldots$

2 Find:

a $\sqrt{\frac{1}{9}}$ **b** $\sqrt{\frac{1}{16}}$ **c** $\sqrt{\frac{1}{121}}$ **d** $\sqrt{\frac{4}{25}}$ **e** $\sqrt{\frac{9}{16}}$ **f** $\sqrt{\frac{16}{49}}$

g $\sqrt{\frac{25}{9}}$ **h** $\sqrt{\frac{25}{4}}$ **i** $\sqrt{\frac{81}{100}}$ **j** $\sqrt{\frac{100}{49}}$ **k** $\sqrt{\frac{9}{121}}$ **l** $\sqrt{\frac{36}{169}}$

3 Find:

a $\sqrt{6\frac{1}{4}}$ **b** $\sqrt{1\frac{7}{9}}$ **c** $\sqrt{1\frac{9}{16}}$ **d** $\sqrt{5\frac{4}{9}}$ **e** $\sqrt{3\frac{1}{16}}$ **f** $\sqrt{11\frac{1}{9}}$

INVESTIGATION 2 CONTINUED FRACTIONS

In this investigation you will need to find the reciprocals of fractions using your calculator. To do this you can use the function marked $\boxed{\frac{1}{x}}$ or $\boxed{x^{-1}}$.

What to do:

1 Use your calculator to find the decimal values of the following fractions. Give all answers using the full display of your calculator.

a $1 + 2$ $1\ \boxed{+}\ 2\ \boxed{=}$ and keep this answer in display.

b $1 + \dfrac{2}{1 + 2}$ then $\boxed{\frac{1}{x}}\ \boxed{\times}\ 2\ \boxed{+}\ 1\ \boxed{=}$ and keep this answer in display.

c $1 + \cfrac{2}{1 + \cfrac{2}{1 + 2}}$ then $\boxed{\frac{1}{x}}$ $\boxed{\times}$ 2 $\boxed{+}$ 1 $\boxed{=}$

d $1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + 2}}}$

e Continue this process until it is obvious not to proceed any further.

2 Make up other **continued fractions** of your own choosing. For example,

$2 + 3$, $2 + \cfrac{3}{2 + 3}$, $2 + \cfrac{3}{2 + \cfrac{3}{2 + 3}}$, ...

Evaluate each fraction and record your observations.

3 Use your skills in adding and dividing fractions to explain the results above.

KEY WORDS USED IN THIS CHAPTER

- common fraction
- improper fraction
- lowest terms
- numerator
- reciprocal
- denominator
- integer
- mixed number
- proper fraction
- fraction
- lowest common denominator
- number line
- rational number

REVIEW SET 4A

1 Simplify:

a $\dfrac{-24}{8}$ **b** $\dfrac{-3}{-9}$ **c** $\dfrac{4-7}{11+2^2}$ **d** $\dfrac{6-3\div 3}{2+10\div 2}$

2 Plot the fractions $-\frac{1}{3}$, $\frac{2}{3}$, $1\frac{1}{3}$ and $2\frac{2}{3}$ on a number line.

3 Write in ascending order: $-\frac{3}{4}$, $1\frac{1}{4}$, $\frac{2}{3}$, $-1\frac{1}{2}$ and $\frac{4}{5}$.

4 Find:

a $\frac{3}{7} + \frac{5}{14}$ **b** $\frac{2}{3} - \frac{4}{5}$ **c** $-1\frac{1}{4} + -\frac{2}{3}$ **d** $\frac{1}{4} - \frac{3}{5} - \frac{1}{2}$

5 What number is $\frac{3}{4}$ more than $\frac{2}{3}$?

6 Find:

a $\frac{2}{3} \times 1\frac{1}{2}$ **b** $-\frac{2}{3} \div \frac{1}{2}$ **c** $-3 \times \left(-\frac{2}{3}\right)^2$ **d** $\frac{4}{7}$ of \$630

7 Find the number which is midway between $\frac{3}{4}$ and -1.

8 Simplify: **a** $\dfrac{2+\frac{1}{3}}{1+\frac{2}{3}}$ **b** $\dfrac{\frac{3}{4}-\frac{2}{5}}{1+\frac{3}{5}}$

9 Ken spent $\frac{1}{4}$ of his money on Monday and $\frac{2}{5}$ of it on Tuesday. What fraction of his money remains?

10 200 kg of brass is melted down and cast into ornamental frogs each weighing $\frac{3}{20}$ kg. How many frogs are made?

11 $\frac{2}{9}$ of Freda's income is used to pay rent. If her rent is €115 per week, what is her weekly income?

12 Fong's family bought a large sack of rice. They consumed $\frac{7}{20}$ last month and $\frac{8}{11}$ of the remainder this month. What fraction of rice:

a has been consumed **b** remains?

13 Find: **a** $\left(\frac{2}{3}\right)^2$ **b** $\left(2\frac{1}{2}\right)^2$ **c** $\sqrt{6\frac{1}{4}}$

REVIEW SET 4B

1 Plot $\frac{3}{4}$, $-1\frac{1}{2}$, $1\frac{1}{4}$ and 3 on a number line.

2 Simplify:

a $\dfrac{-12}{-4}$ **b** $\dfrac{5}{-25}$ **c** $\dfrac{6+2^2}{6-2^3}$ **d** $\dfrac{12+8\div 2}{12-8\times 2}$

3 What number is $\frac{2}{3}$ less than $1\frac{1}{2}$?

4 Find: **a** $\left(1\frac{1}{2}\right)^2$ **b** $\left(\frac{2}{3}\right)^3$ **c** $\sqrt{2\frac{7}{9}}$

5 Write in descending order: $-\frac{4}{5}$, $-1\frac{1}{3}$, $\frac{1}{2}$, $-\frac{1}{10}$, $\frac{5}{6}$.

6 Find:

a $\frac{1}{3}-\frac{2}{5}$ **b** $2\frac{1}{3}-1\frac{1}{2}$ **c** $\frac{3}{5}\div(-2)$ **d** $\frac{1}{10}-\frac{2}{3}+\frac{1}{2}$

7 Find:

a $-\frac{3}{4}\times 2$ **b** $\frac{3}{5}\div -\frac{1}{2}$ **c** $\frac{3}{4}$ of \$84 **d** $12\times\left(-\frac{1}{2}\right)^3$

8 Find the average of 2, $\frac{3}{4}$ and $\frac{1}{2}$.

9 Simplify: **a** $\dfrac{1-\frac{3}{4}}{2+\frac{1}{4}}$ **b** $\dfrac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\frac{1}{12}-\frac{1}{4}}$

10 What must $-\frac{1}{3}$ be increased by to get $\frac{4}{5}$?

11 What fraction of material is left if $\frac{1}{5}$, $\frac{1}{4}$ and $\frac{1}{6}$ are used to make dresses?

12 Jacob's business investments have been bad this year. He has lost a $\frac{1}{3}$ share in \$45 000 and a $\frac{2}{5}$ share in \$65 000. How much has he lost from these two investments?

13 $\frac{3}{25}$ of Jim's income is used to pay for health insurance and superannuation. If this amounts to £105 per week, find Jim's:

a weekly income **b** annual income.

Chapter 5

Algebra: Patterns and models

Contents:

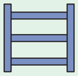
Algebra is a branch of mathematics which uses symbols in place of numbers. It provides a simple shorthand way of writing expressions, finding rules, and solving problems that deal with unknown quantities.

HISTORICAL NOTE

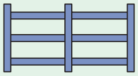
Algebra dates back 4000 years to Ancient Egyptian and Babylonian times. It was extended by the Alexandrian mathematicians **Hero** and **Diophantus**. The work *Arithmetica* by Diophantus contains much high powered algebra, including solutions to some very difficult equations.

OPENING PROBLEM

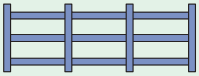
Wiktor builds standard steel fences which are put together in sections. In each section the vertical and horizontal steel pieces are all the same length.



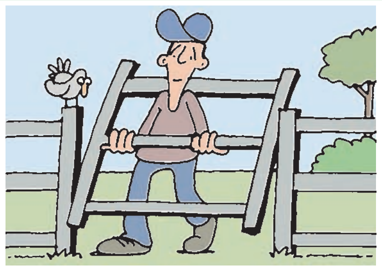
1-section
5 pieces



2-sections
9 pieces



3-sections
13 pieces

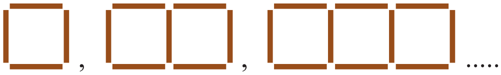


Things to think about:

- How many steel lengths does Wiktor require to make a fence of:
  a 4 sections **b** 5 sections **c** 20 sections?
- Is there a rule which allows Wiktor to find the number of steel lengths required to make a fence of any given number of sections?

In algebra we use letters or symbols to represent unknown quantities or **variables**.

A GEOMETRIC PATTERNS

 and 

are examples of **geometric patterns** which continue indefinitely.

We often refer to them as **matchstick patterns** because we can construct them from matchsticks.

For the pattern  we may wish to know how many matchsticks are necessary to create a future diagram, for example the 10th figure.

We may also wish to establish a **rule** or **formula** which will tell us how many matchsticks there are in a figure further along the pattern.

Suppose the *figure number* is n and the *number of matches* is M.

From the first four figures in the pattern, we can create a **summary table**:

Figure number (n)	1	2	3	4
Number of matches (M)	4	7	10	13

+3 +3 +3

Each time the *figure number* n is increased by 1, the *number of matches* M is increased by 3. This suggests we compare $n \times 3$ with M.

$n \times 3$	3	6	9	12
M	4	7	10	13

We can see that M is always one more than $n \times 3$, so the rule connecting the variables is $M = n \times 3 + 1$ or "*the number of matches* is three times the *figure number*, plus 1".

In Mathematics we use **product notation** to make expressions simpler. We agree to:
- omit the × sign whenever possible
- write the numbers in products before the variables.

So, the formula can be written more simply as $M = 3n + 1$.

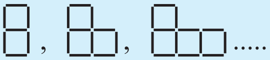
To help understand how the formula relates to the pattern, we can construct a more detailed table:

Figure number	*Figure*	*Matches*	*Pattern*	*Explanation*
1		4	$1 \times 3 + 1$	
2		7	$2 \times 3 + 1$	
3		10	$3 \times 3 + 1$	
4		13	$4 \times 3 + 1$	

We can use the pattern to predict that:

the 10th figure has $10 \times 3 + 1 = 31$ matches,
and the 100th figure has $100 \times 3 + 1 = 301$ matches.

Example 1 Self Tutor



a Complete a table for the first four figures in the pattern:

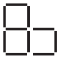
b Find how many matches are required to make the:

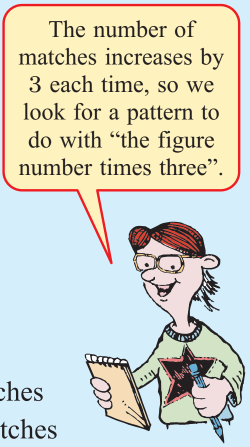
  i 10th figure **ii** 50th figure

c Write a description of the pattern in words.

d Predict a general rule for finding the number of matches M in the nth figure.

a

Figure number	*Figure*	*Matches*	*Pattern*	*Explanation*
1		7	$1 \times 3 + 4$	
2		10	$2 \times 3 + 4$	
3		13	$3 \times 3 + 4$	
4		16	$4 \times 3 + 4$	



b **i** 10th figure: Matches required $= 10 \times 3 + 4 = 34$ matches

ii 50th figure: Matches required $= 50 \times 3 + 4 = 154$ matches

c The number of matches is the figure number times three, plus four.

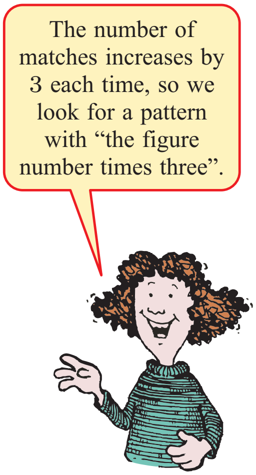
d For the nth figure the rule is $M = n \times 3 + 4$ or $M = 3n + 4$.

EXERCISE 5A

1 **a** Consider the pattern:  ..... .

Draw the next two figures and hence complete:

Figure number	*Figure*	*Matches*	*Pattern*	*Explanation*
1		3	1×3	
2		6	2×3	
3				
4				
5			5×3	



b Find how many matches would be required to make the:

i 6th figure **ii** 20th figure.

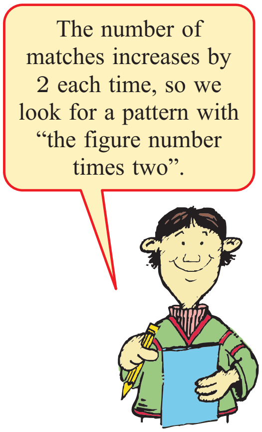
c Copy and complete: "The number of matches is the figure number ......".

d Write a general rule for determining the number of matches M in the nth figure.

2 **a** Consider the pattern:  ..... .

Draw the next two figures and hence complete:

Figure number	*Figure*	*Matches*	*Pattern*	*Explanation*
1		4	$1 \times 2 + 2$	
2		6	$2 \times 2 + 2$	
3				
4			$4 \times 2 + 2$	
5				



b Find how many matches would be required to make the:
  i 6th figure   **ii** 15th figure.

c Copy and complete this description of the pattern:
"The number of matches is the figure number .... plus .... ."

d Write a general rule for determining the number of matches M in the nth figure.

3 **a** Consider the pattern: 

Draw the next two figures and hence complete:

Figure number	*Figure*	*Matches*	*Pattern*	*Explanation*
1		5		
2		8		
3				
4				
5				

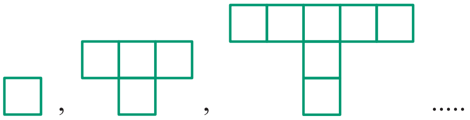
Look for a pattern with "the figure number times 3".

b Find how many matches are required to make the:
  i 10th figure   **ii** 15th figure.

c Write a description of the pattern in words.

d Write a general rule for determining the number of matches M in the nth figure.

4 Consider the pattern:



a Continue the pattern for 2 more figures.

b How many matchsticks are required to make each of the first five figures?

c Copy and complete:

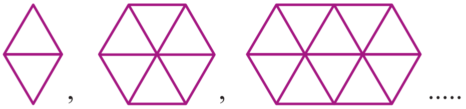
Number of figure (n)	1	2	3	4	5	8
Number of matchsticks (M)						

d Write a description of the pattern.

e Write a general rule for the connection between M and n.

f Predict the number of matchsticks needed for the 30th figure.

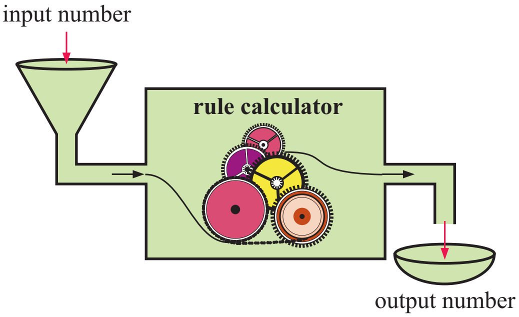
5 Repeat **4** for the pattern:



B NUMBER CRUNCHING MACHINES

Rules or **formulae** which give the total number of matchsticks in patterns can be likened to *number crunching machines*.

For any **input number** put into the machine, for example, 1, 2, 3, 4, 5, ..., the machine calculates an **output number** according to a rule.



For example, suppose the output number M is '*three times the input number* n*, plus seven*'
or $M = 3 \times n + 7$.
We have:

Input number (n)	*Calculation*	*Output number* (M)
1	$3 \times 1 + 7$	10
2	$3 \times 2 + 7$	13
3	$3 \times 3 + 7$	16
⋮	⋮	⋮

Example 2 **Self Tutor**

Consider the rule: *three times the input number, less one*. Calculate the output numbers when the input numbers are 1, 2, 5 and 10.

We let n represent the input number and M represent the output number.

In symbols, the rule is: $M = 3 \times n - 1$ or $M = 3n - 1$.

Input number (n)	*Output number* (M)
1	$3 \times 1 - 1 = 2$
2	$3 \times 2 - 1 = 5$
5	$3 \times 5 - 1 = 14$
10	$3 \times 10 - 1 = 29$

So, 1 → **2**, 2 → **5**, 5 → **14**, 10 → **29**.

EXERCISE 5B.1

1 For the following input numbers, calculate output numbers under the given rules:

a *Four times the input number.*

Input number	*Output number*
1	
2	
3	
4	

b *The input number plus three.*

Input number	*Output number*
4	
6	
12	
26	

c *Add two then multiply by* 3.

Input number	*Output number*
0	
1	
2	
5	

d *Multiply the input number by itself.*

Input number	*Output number*
1	
2	
3	
5	

e *Double the input number then add one.*

Input number	*Output number*
1	
2	
3	
8	

f *Subtract* 3 *then multiply by* 2.

Input number	*Output number*
3	
4	
10	
15	

g *Halve the input number then add four.*

Input number	*Output number*
2	
6	
10	
23	

h *Add* 10 *then divide by* 3.

Input number	*Output number*
2	
8	
23	
98	

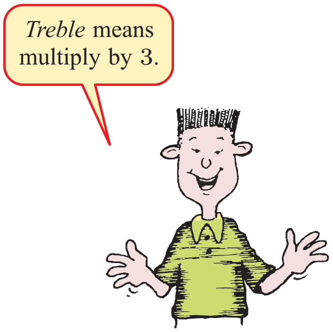
INPUTS FROM OUTPUTS

Now let us look at the **reverse process**. Given an output number and the rule, can we find the input number?

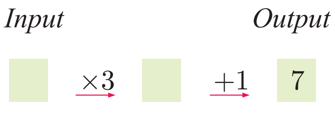
Consider the number crunching rule:

treble the input number and then add 1.

If the output number is 7, what was the input number?



We can answer this question using a **flow chart**:



What number with 1 added is 7?
Since $6 + 1 = 7$, the previous number must be 6.

What number when trebled is 6?
Since $2 \times 3 = 6$, the previous number must be 2.

So, the input number was 2.

Example 3 **Self Tutor**

Consider the rule:
treble the input number then add one.
Calculate the input numbers for the output numbers given.

	Input number	Output number
a		16
b		28

a *Input* *Output*

$$\boxed{} \xrightarrow{\times 3} \boxed{} \xrightarrow{+1} \boxed{16}$$
$$\boxed{} \xrightarrow{\times 3} \boxed{15} \xrightarrow{+1} \boxed{16}$$
$$\boxed{5} \xrightarrow{\times 3} \boxed{15} \xrightarrow{+1} \boxed{16}$$

$\therefore$ the input number was 5.

b *Input* *Output*

$$\boxed{} \xrightarrow{\times 3} \boxed{} \xrightarrow{+1} \boxed{28}$$
$$\boxed{} \xrightarrow{\times 3} \boxed{27} \xrightarrow{+1} \boxed{28}$$
$$\boxed{9} \xrightarrow{\times 3} \boxed{27} \xrightarrow{+1} \boxed{28}$$

$\therefore$ the input number was 9.

EXERCISE 5B.2

1 Given the following output numbers and rules, calculate the corresponding input numbers.

		Output numbers:
a	Rule: *The input number plus four.*	$\{5, 7, 15\}$
b	Rule: *Double the input number plus two.*	$\{2, 4, 10\}$
c	Rule: *Five times the input number minus three.*	$\{7, 12, 17\}$
d	Rule: *Add one to the input number then double the result.*	$\{2, 6, 12\}$
e	Rule: *Multiply the input number by itself then add one.*	$\{2, 5, 17\}$
f	Rule: *Multiply the input number by one more than itself.*	$\{2, 6, 20\}$

FINDING THE 'NUMBER CRUNCHING RULE'

Consider the following rules with input numbers and outputs:

Rule: *Two times the input number.*

Input number	*Output number*
1	2
2	4
3	6
4	8

When the input number is increased by 1, the output number is increased by 2.

Rule: *Three times the input number plus one.*

Input number	*Output number*
1	4
2	7
3	10
4	13

When the input number is increased by 1, the output number is increased by 3.

Rule: *Five times the input number minus two.*

Input number	*Output number*
1	3
2	8
3	13
4	18

When the input number is increased by 1, the output number is increased by 5.

We can use these observations to help find the 'number crunching rule' given a table of input numbers and their outputs.

In general, when we consider a table of input numbers and their outputs:

> If the output number increases by k each time the input number increases by 1, the rule contains 'the input number times k'.

Example 4

Self Tutor

Find the rule which connects these input numbers and their corresponding output numbers:

Input, n	*Output*, M
1	6
2	10
3	14
4	18

When the *Input number* is increased by 1, the *Output number* is increased by 4.

This suggests that the *Input numbers* have been multiplied by 4.

$4n$	4	8	12	16
M	6	10	14	18

The value of M is always 2 more than $4n$, so the rule is *four times the input number, plus two.*

So, $M = 4n + 2$.

EXERCISE 5B.3

1 For the following input numbers and output numbers, find the rule in the number crunching machine:

a

Input, n	*Output*, M
1	3
2	6
3	9
4	12

b

Input, n	*Output*, M
1	3
2	5
3	7
4	9

c

Input, n	*Output, M*
1	7
2	9
3	11
4	13

d

Input, n	*Output, M*
1	5
2	9
3	13
4	17

e

Input, n	*Output, M*
1	3
2	8
3	13
4	18

f

Input, n	*Output, M*
1	7
2	10
3	13
4	16

2 For each of the following tables, write a formula connecting the variables. Check your formula for all number pairs given.

a

a	1	2	3	4	5	6
N	4	6	8	10	12	14

b

x	1	2	3	4	5	6
y	4	5	6	7	8	9

c

c	1	2	3	4	5	6
K	7	11	15	19	23	27

d

d	1	2	3	4	5	6
Q	4	11	18	25	32	39

e

h	1	2	3	4	5	6
C	8	14	20	26	32	38

f

n	1	2	3	4	5	6
M	6	14	22	30	38	46

C SUBSTITUTING INTO FORMULAE

Having established a rule or formula connecting two variables, we can **substitute** the value of one variable to find the corresponding value of the other.

For example, suppose the cost C of hiring a volleyball court for h hours is given by the formula $C = 8h+3$ euros.

To hire the court for 1 hour, we substitute $h = 1$ into the formula.

The cost is $C = 8 \times 1 + 3 = €11$.

To hire the court for $2\frac{1}{2}$ hours, we substitute $h = 2\frac{1}{2}$ into the formula.

The cost is $C = 8 \times 2\frac{1}{2} + 3 = €23$.

DISCUSSION

When $h = 0$, $C = 8 \times 0 + 3 = €3$. Can you explain why it might cost you €3 before you start to use it?

Example 5 **Self Tutor**

If $D = 3x - 11$, find D when:

a $x = 10$ **b** $x = 23$ **c** $x = 2$

a When $x = 10$,
$D = 3 \times 10 - 11$
$\therefore$ $D = 30 - 11$
$\therefore$ $D = 19$

b When $x = 23$,
$D = 3 \times 23 - 11$
$\therefore$ $D = 69 - 11$
$\therefore$ $D = 58$

c When $x = 2$,
$D = 3 \times 2 - 11$
$\therefore$ $D = 6 - 11$
$\therefore$ $D = -5$

EXERCISE 5C

1 Find the value of y given the rule:

- **a** $y = 8x + 5$ when **i** $x = 3$ **ii** $x = 7$ **iii** $x = 10\frac{1}{2}$
- **b** $y = 21 - 4x$ when **i** $x = 0$ **ii** $x = 2\frac{1}{2}$ **iii** $x = 7$
- **c** $y = \dfrac{3x + 4}{2}$ when **i** $x = 2$ **ii** $x = 6$ **iii** $x = 11$
- **d** $y = 2(x + 3) - 1$ when **i** $x = 4$ **ii** $x = 0$ **iii** $x = 6\frac{1}{2}$

2 If $N = 3a + 7$, complete the table of values:

a	0	2	5	-1	-4
N					

3 If $K = 11 - 4n$, complete the table of values:

n	-5	-2	0	2	9
K					

4 The cost C of hiring a squash court for h hours is given by $C = 12h + 5$ dollars.
Find the cost of hiring a court for: **a** 1 hour **b** 30 mins **c** 1 hour 15 mins.

5 The volume of water in a tank t minutes after a tap is switched on, is given by $V = 5000 - 20t$ litres.

- **a** Find V when $t = 0$. What does this mean?
- **b** Find the volume of water left in the tank after:
  - **i** 5 minutes **ii** 1 hour **iii** $3\frac{1}{2}$ hours.

6 To add the whole numbers from 1 to n we can use the formula $S = \dfrac{n(n+1)}{2}$.

For example, $1 + 2 + 3 + 4 + 5 = \dfrac{5 \times 6}{2} = 15$.

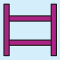
- **a** Check that the formula is correct for $n = 1$, 2, 3, and 4.
- **b** Use the formula to add all the whole numbers from 1 up to:
  - **i** 50 **ii** 200 **iii** 1000.

D USING PATTERNS

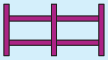
Many problems can be solved by observing patterns. We begin by looking at simple examples, and use them to find a **formula** connecting the variables. We use the formula to find the solution to the problem.

Example 6 **Self Tutor**

Sam builds steel fences in sections.



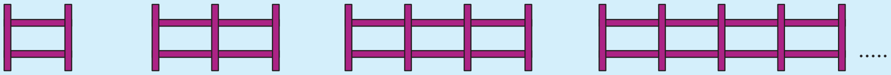
is called a 1-section and is made from 4 lengths of steel.



is called a 2-section and is made from 7 lengths of steel.

How many lengths of steel does Sam need to make a 63-section fence?

First we draw sketches of the next 2 or 3 cases to help establish a pattern.



1-section 4 lengths; 2-section 7 lengths; 3-section 10 lengths; 4-section 13 lengths

Let the number of sections be n and the number of steel lengths be S.

Number of sections (n)	1	2	3	4
Number of steel lengths (S)	4	7	10	13

+3 +3 +3

When n is increased by 1, S is increased by 3. We therefore compare $3n$ with S.

$3n$	3	6	9	12
S	4	7	10	13

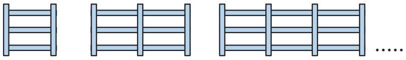
The S values are always 1 more than the $3n$ values. $\therefore \quad S = 3n + 1$

So, when $n = 63$, $S = 3 \times 63 + 1 = 190$.

$\therefore$ 190 lengths of steel are needed.

EXERCISE 5D

1 Consider again the **Opening Problem** on page **96**.



1-section 2-section 3-section

a Draw the next two section type fences.

b If S is the number of steel lengths Wiktor requires to make an n-section fence, copy and complete the following table:

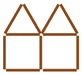
Section number (n)	1	2	3	4	5
Number of lengths (S)					

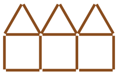
c Determine the formula which connects S and n.

d If Wiktor has an order for a 44-section fence, how many lengths of steel are required to make it?

2 Builder Ben is experimenting with toothpicks to investigate housing designs:


1 house


2 houses


3 houses

a Draw toothpick diagrams for 4 houses and 5 houses.

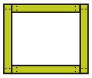
b If T is the number of toothpicks required to make h houses, copy and complete the following table:

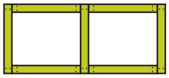
Houses (h)	1	2	3	4	5
Toothpicks (T)					

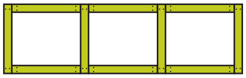
c Find the formula which connects T and h.

d If Ben wanted to build 25 houses in a row, how many toothpicks would he need?

3 Lana the landscaper uses railway sleepers to create garden beds.


1 bed


2 beds


3 beds

a Copy and complete the following table:

Garden beds (b)	1	2	3	4	5
Sleepers (S)					

b Find the formula that connects S and b.

c How many sleepers will Lana need to construct:

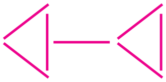
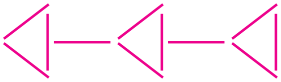
i 18 garden beds **ii** 37 garden beds?

4 Consider the pattern:  ,  , 

a Copy and complete the table of values:

Figure number (n)	1	2	3	4	5
Number of matchsticks (M)	2				

b Write a rule linking n and M.

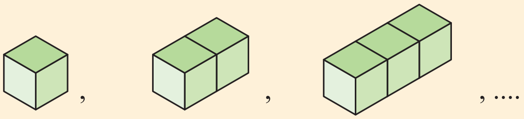
5 Consider the pattern:  ,  , 

a Copy and complete the table of values:

Figure number (n)	1	2	3	4	5
Number of matchsticks (M)	3				

b Write a rule linking n and M.

INVESTIGATION 1 THE CUBES PATTERN



, , , ...

What to do:

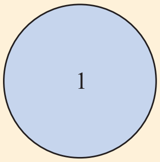
1. Use some cubes to form the pattern above.
2. For each member of the pattern count the number of cube faces that are exposed.
3. Copy and complete the table:

Number of cubes	1	2	3	4	5	6
Number of exposed cube faces	6	10				

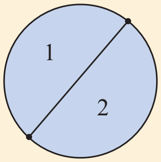
4. Predict the number of exposed cube faces for the next 3 members of this pattern.
5. Check your prediction by arranging cubes and counting.
6. Write a formula for the number of exposed cube faces when there are n cubes.
7. Without arranging cubes, predict the number of exposed cube faces when there are:
   a 10 cubes **b** 20 cubes **c** 100 cubes **d** 100 000 cubes.

INVESTIGATION 2 REGIONS OF A CIRCLE

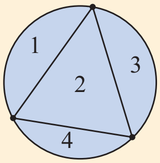
When points are marked on the circumference of a circle and each point is joined to every other point, the circle is divided into a number of regions.



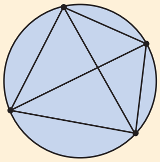
1 point
1 region



2 points
2 regions



3 points
4 regions



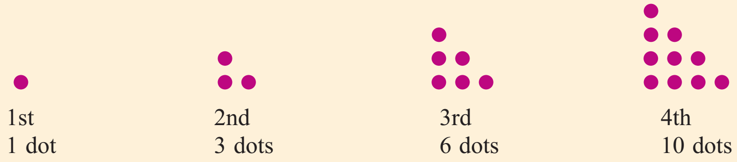
4 points
? regions

What to do:

1. Count the number of regions produced with 4 points.
2. Guess how many regions would be produced by 5, 6 and 7 points.
3. Check your answers to **2** by drawing the circles and counting the regions.
4. Were your answers to **2** correct?

INVESTIGATION 3 MAKING TRIANGLES

Consider the following triangles made out of dots.



What to do:

1 Construct the next 3 triangles. Record the number of dots in a table:

Triangle (n)	1st	2nd	3rd	4th	5th	6th	7th
Number of dots (D)							

2 Predict the number of dots in the 8th, 9th and 10th triangles.

3 Copy and complete: $\frac{1 \times 2}{2} = ...,$ $\frac{2 \times 3}{2} = ...,$ $\frac{3 \times 4}{2} = ...,$ $\frac{4 \times 5}{2} = ...$

Hence state a formula connecting D and n for the tabled values in **1**.

4 When Alan, Bonny, Claudia and Daniel shake hands in all possible ways, 6 handshakes take place.

We can represent these as AB, AC, AD, BC, BD, CD where AB represents the Alan - Bonny handshake.

Copy and complete:

Number of people (P)	2	3	4	5	6
Number of handshakes (H)					

- **a** State the connection between the number of handshakes and the dot triangles above.
- **b** State a formula connecting H and P.
- **c** How many handshakes can take place within a group of 20 people?

E PRACTICAL PROBLEMS

Milan is stacking boxes containing the latest release crime novel onto a wooden pallet ready to transport to a bookshop.

The pallet weighs 20 kg and each box of novels weighs 12 kg.

We can construct a table to show the total weight of the pallet and books as each carton of books is placed on the pallet.

Notice that each time the number of cartons increases by 1, the weight increases by 12 kg.

So, the weight of c cartons of novels would be $12c$ kg.

Number of cartons	*Total weight*
0	20 kg
1	32 kg
2	44 kg
3	56 kg
⋮	⋮

We then need to add the weight of the pallet, which is 20 kg.

So, the total weight W for a pallet with c cartons of novels is $W = 12c + 20$ kg.

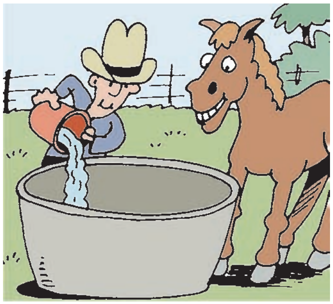
If Milan stacks 11 cartons of novels on the pallet the total weight would be
$12 \times 11 + 20$ kg $= 152$ kg.

EXERCISE 5E

1 Michelle has £2000 in a bank account. Each week she deposits a further £120.

- **a** Explain why the amount of money in her account after n weeks is given by $M = 2000 + 120n$ pounds.
- **b** How much money does she have in her bank account after:
  - **i** 3 weeks
  - **ii** 6 months
  - **iii** $1\frac{1}{2}$ years?

2 Lars notices that the water in his horse's drinking trough is only 2 cm deep. The trough is cylindrical. Each time Lars tips a bucket of water into the trough, the water level rises 1.5 cm.



- **a** Find how much the water level rises if b buckets of water are tipped into the trough.
- **b** What depth D cm of water is in the trough if b buckets of water have been emptied into it?
- **c** How deep is the water in the trough if Lars tips:
  - **i** 5 buckets
  - **ii** 18 buckets of water into it?

3 Students fill 600 mL bottles from a water cooler containing 50 litres of water.

- **a** If b bottles are filled, how much water is used?
- **b** How much water W litres is left in the cooler if b bottles have been filled?
- **c** How much water is left in the cooler if:
  - **i** 15 bottles
  - **ii** 37 bottles have been filled?

4 Jessie repairs washing machines. She charges a \$60 call-out fee and \$25 for each 15 minutes spent repairing the machine.

- **a** What will t lots of 15 minutes of labour cost?
- **b** Find the total charge C dollars for Jessie to come and spend t lots of 15 minutes repairing a washing machine.
- **c** Find the total charge for a call-out and repairs taking:
  - **i** 45 minutes
  - **ii** $1\frac{1}{4}$ hours.

5 A mayor makes a promise that if he is elected, his council will plant 5000 trees in the first year, and 2000 trees each year after that, as part of their 'green' policy. The mayor is elected.

- **a** How many trees have been planted at the end of:
  - **i** 2 years
  - **ii** 3 years
  - **iii** 4 years?
- **b** Write a formula for the number of trees T planted at the end of n years.
- **c** Find the number of trees planted after 10 years.

F NUMBER SEQUENCES

In this section we consider sequences of numbers which follow a pattern or rule. These rules often describe a member of the sequence in terms of the previous member.

Examine the number sequence 3, 7, 11, 15, 19, ...

We notice that
$$\begin{aligned} 3+4&=7\\ 7+4&=11\\ 11+4&=15\\ 15+4&=19 \end{aligned}$$

A **rule** connecting the members of the sequence is:

"the next number is equal to the previous number plus 4".

However, not all number sequences increase by a fixed amount from one member to the next. You will therefore need to practice your skills of observation to find the rules for each sequence.

Example 7 **Self Tutor**

For each of the number sequences following, state the next 3 members and write down the rule used to find the next member:

a 1, 7, 13, 19, 25, ... **b** 50, 47, 44, 41, 38, ... **c** 2, 10, 50, 250, ...

a
$$1 \xrightarrow{+6} 7 \xrightarrow{+6} 13 \xrightarrow{+6} 19 \xrightarrow{+6} 25$$

To get the next member we *add* 6,

$\therefore$ the next 3 members are: 31, 37 and 43.

Rule: 'The next member is equal to the previous one, plus 6.'

b
$$50 \xrightarrow{-3} 47 \xrightarrow{-3} 44 \xrightarrow{-3} 41 \xrightarrow{-3} 38$$

To get the next member we *take* 3,

$\therefore$ the next 3 members are: 35, 32 and 29.

Rule: 'The next member is equal to the previous one, minus 3.'

c
$$2 \xrightarrow{\times 5} 10 \xrightarrow{\times 5} 50 \xrightarrow{\times 5} 250$$

To get the next member we *multiply by* 5,

$\therefore$ the next 3 members are: 1250, 6250, 31 250.

Rule: 'The next member is equal to the previous one multiplied by 5.'

EXERCISE 5F

1 For each of the number sequences following, state the next 3 members and write down the rule used to find the next member:

a 1, 4, 7, 10, 13, ... **b** 11, 15, 19, 23, 27, ... **c** 2, 9, 16, 23, 30, ...
d 6, 12, 18, 24, ... **e** 13, 22, 31, 40, ... **f** 7, 20, 33, 46, ...
g 2, 1, 0, −1, ... **h** 8, 5, 2, ... **i** −11, −7, −3, ...

2 For each of the number sequences following, state the next 3 members and write down the rule used to find the next member:

a 38, 36, 34, 32, 30, ... **b** 29, 26, 23, 20, ... **c** 57, 51, 45, 39, ...
d 100, 97, 94, 91, ... **e** 250, 242, 234, 226, ... **f** 65, 61, 57, 53, 49, ...
g 1, 2, 4, 8, 16, ... **h** 2, 6, 18, 54, ... **i** 2, 8, 32, 128, ...
j 64, 32, 16, 8, 4, ... **k** 80, 40, 20, 10, ... **l** 243, 81, 27, 9, ...

3 Using the first number and rule given, state the next three numbers in each sequence:

a 7 'add 6' **b** 3 'add 9'
c 4 'add $1\frac{1}{2}$' **d** 56 'take 11'
e 150 'subtract 25' **f** 4 'multiply by 2 then add 3'
g 3 'times by 10 then subtract 4' **h** 97 'add one then divide by two'

4 Write down the number missing from each pattern:

a 3, 9, ☐ , 21, 27 **b** 12, ☐ , 36, 48, 60 **c** 75, 60, ☐ , 30, 15
d 3, 6, ☐ , 24, 48 **e** 6, 10, ☐ , 21, 28, 36 **f** 3, 9, 27, ☐ , 243
g 10, ☐ , 32, 43 **h** 2, 6, 24, ☐ , 720 **i** 100, 50, ☐ , $12\frac{1}{2}$
j 2, 5, 11, ☐ , 47 **k** 96, ☐ , 6, $1\frac{1}{2}$

5 State the next 3 members of these sequences. How are the next members found?

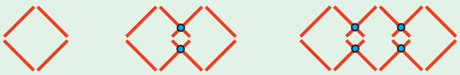
a 2, 3, 5, 8, 12, 17, ... **b** 1, 1, 2, 3, 5, 8, 13, ...

KEY WORDS USED IN THIS CHAPTER

- formula
- number sequence
- geometric pattern
- output number
- input number
- rule
- variable

REVIEW SET 5A

1 The following diamond pattern has dots marked where the diamonds cross:



a Complete the table of values:

Number of diamonds (n)	1	2	3	4	5
Number of dots (D)					

b Use the table of values to find the rule linking the number of dots D with the number of diamonds n.

c Use the rule to find the number of dots if there are 32 diamonds.

2 Find:
   a the output number if the input number is 27 and the rule is "*one third the input number minus two*"
   b the input number if the output number is 37 and the rule is "*twice the input number minus three*".

3 For $y = 4x - 2$, copy and complete the table of values:

x	1	3	6	10
y				

4 Consider the rule: *treble the input number then add five.*
Calculate the input numbers for the output numbers given.

Input	*Output*
	11
	20
	32

5 Write a formula connecting the variables in the table:

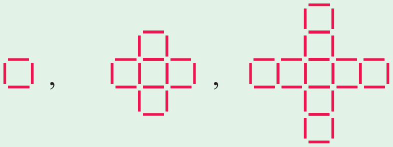
a

t	1	2	3	4
Q	1	7	13	19

b

m	1	2	3	4
G	4	11	18	25

6 The following patterns are made with matchsticks:



a Copy and complete the table of values:

Figure number (n)	1	2	3	4	5
Number of matchsticks (M)					

b Use the table of values to find the rule linking the number of matchsticks M with the figure number n.

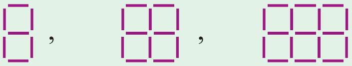
c Use the rule to find the number of matchsticks needed for the 20th figure.

7 Irena has \$263 in her bank, and she decides to take \$5 from it each week.
   a Find the amount of money that she will take from the bank in w weeks.
   b Find the total amount A (in dollars) left in the bank after w weeks.
   c How much money will be in the bank after:
      i 12 weeks **ii** 1 year?

8 Find the next two members of these sequences and state the rules used to find them:
   a 82, 75, 68, 61, ... **b** 3, 6, 12, 24, ... **c** 4, 2, 1, $\frac{1}{2}$, ...

REVIEW SET 5B

1 The following patterns are made with matchsticks:



a Copy and complete the table of values:

Figure number (n)	1	2	3	4	5
Number of matchsticks (M)					

b Use the table of values to find the rule linking the number of matchsticks M with the figure number n.

c Use the rule to find the number of matchsticks needed for the 50th figure.

2 Consider the rule: *double the input number then subtract three.* Calculate the input numbers for the output numbers given.

Input	*Output*
	7
	13
	25

3 For $y = 3x + 5$, copy and complete the table of values:

x	1	4	7	15
y				

4 Write a formula connecting the variables in the table:

a

r	1	2	3	4
D	3	8	13	18

b

s	1	2	3	4
N	3	7	11	15

5 The following patterns are made with matchsticks:

a Copy and complete the table of values:

Figure number (n)	1	2	3	4	5
Number of matchsticks (M)					

b Write a rule linking n and M.

c How many matchsticks would be required for the 30th figure?

6 The cost C of hiring a room for a function is given by $C = 150 + 30g$ euros where g is the number of guests. Find the cost of hiring the room for:

a 20 guests **b** 50 guests **c** 90 guests

7 **a** Write down the next two members of the following sequences and state the rules used to find them:

i 11, 8, 5, 2, ... **ii** 1, 2, 4, 7, 11, ...

b Find the missing member of: 2, 6, 12, ☐, 30, 42, ...
In a sentence explain how you found it.

8 A Youth Club hires a bus to take a group of children to the zoo. Their costs are \$80 for the bus, plus \$5 per person entrance to the zoo.

a Find the entry cost for p people to go into the zoo.

b Find the total cost C dollars for the Youth Club to take p people to the zoo.

c How much would it cost if **i** 12 people **ii** 27 people went to the zoo?

Chapter 6

Decimal numbers

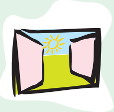
Contents:

Decimal numbers are widely used in everyday life. We see them frequently in money and in measurements of length, area, weight and so on.

HISTORICAL NOTE

- The earliest decimal system was probably invented by **Elamites** of **Iran** in the period 3500-2500 BC.
- The decimal system was developed in Ancient India and Arabia.
- The decimal point we use in this course was probably invented by **Bartholomaeus Pitiscus** in 1612.
- A comma is used instead of a decimal point in some European countries.

OPENING PROBLEM



If we divide 1 by 3 using a calculator, the decimal number 0.333 333 3....... is formed. We call this number 'zero point 3 recurring' as the line of 3s is endless.

It is said that all recurring decimals are **rational numbers** which result from one whole number being divided by another. If this is so, what fraction is equal to 'zero point 9 recurring' or 0.999 999 99......?

A PLACE VALUE

We have seen that the **number system** we use today is a **place value** system using base 10. In this chapter we extend the place value system to include parts of a whole.

In **Chapter 4** we introduced common fractions so that we could write numbers less than one.

If we restrict ourselves to fractions where the denominator is a power of 10, we can use the place value system to represent both whole and fractional numbers.

We introduce a mark called a **decimal point** to separate the whole number part from the fractional part.

For example: 731.245 represents $700+30+1+\frac{2}{10}+\frac{4}{100}+\frac{5}{1000}$

24.059 represents $20+4+\frac{5}{100}+\frac{9}{1000}$

When written as a sum like this, we say the number is in **expanded form**.

The **place value** table for 731.245 and 24.059 is:

	hundreds	tens	units		tenths	hundredths	thousandths
731.245	7	3	1	.	2	4	5
24.059		2	4	.	0	5	9

When a decimal number does not contain any whole number part, we write a zero in the ones place. This gives more emphasis to the decimal point.

For example, we write 0.75 instead of .75 .

Zeros are also very important within decimal numbers.

For example, $6.702 = 6 + \frac{7}{10} + \frac{2}{1000}$ whereas $6.72 = 6 + \frac{7}{10} + \frac{2}{100}$.

Example 1 **Self Tutor**

Write in expanded form: 7.802

$$7.802 = 7 + \frac{8}{10} + \frac{2}{1000}$$

Example 2 **Self Tutor**

Write in decimal form: $\frac{65}{1000}$

$$\frac{65}{1000} = \frac{60}{1000} + \frac{5}{1000}$$
$$= \frac{6}{100} + \frac{5}{1000}$$
$$= 0.065$$

EXERCISE 6A.1

1 Express the following in expanded form:

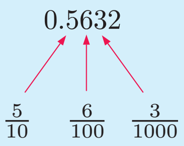
a 3.6
b 8.07
c 0.123
d 2.061
e 3.0071
f 0.000 54
g 3.058
h 0.0632
i 53.707
j 0.006 07

2 Write the following in decimal form:

a $\frac{2}{10}$
b $\frac{13}{100}$
c $\frac{241}{1000}$
d $\frac{83}{100}$
e $\frac{1}{10} + \frac{3}{100}$
f $\frac{37}{1000}$
g $\frac{3}{100} + \frac{7}{1000}$
h $\frac{6}{10} + \frac{5}{100} + \frac{9}{1000}$
i $\frac{3}{1000} + \frac{7}{10\,000}$
j $\frac{9}{100} + \frac{1}{1000}$
k $\frac{9}{10} + \frac{6}{100}$
l $\frac{7}{10} + \frac{5}{100} + \frac{1}{1000} + \frac{7}{10\,000}$

Example 3 **Self Tutor**

State the value of the digit 3 in 0.5632



$\therefore$ the 3 stands for $\frac{3}{1000}$.

3 State the value of the digit 5 in the following:

a 1523
b 3.518
c 53.07
d 87.0652
e 0.0512
f 53 077
g 81.954
h 3589.64

Example 4 **Self Tutor**

Express $4\frac{137}{1000}$ in decimal form.

$$\begin{aligned} 4\tfrac{137}{1000} &= 4 + \tfrac{100}{1000} + \tfrac{30}{1000} + \tfrac{7}{1000} \\ &= 4 + \tfrac{1}{10} + \tfrac{3}{100} + \tfrac{7}{1000} \\ &= 4.137 \end{aligned}$$

You should be able to see how to do this in one step.

4 Express in decimal form:

a $8\frac{8}{10}$ **b** $2\frac{57}{100}$ **c** $13\frac{18}{100}$ **d** $1\frac{461}{1000}$

e $7\frac{41}{1000}$ **f** $3\frac{7}{1000}$ **g** $5\frac{6}{10\,000}$ **h** $5\frac{39}{1000}$

i $\frac{648}{100}$ **j** $\frac{5666}{1000}$ **k** $\frac{681}{10}$ **l** $\frac{7061}{100}$

CONVERTING DECIMALS TO FRACTIONS

Our understanding of decimals and the place value system allows us to convert decimals to fractions. Some fractions can be cancelled down to their **simplest form** by dividing both the numerator and denominator by their **highest common factor (HCF)**.

For example, $0.36 = \frac{36}{100} = \frac{9}{25}$ when all common factors have been cancelled.

Example 5 **Self Tutor**

Write as a fraction in simplest form:

a 0.8 **b** 3.88 **c** 0.375

a $0.8 = \frac{8}{10} = \frac{4}{5}$

b $3.88 = 3 + \frac{88}{100} = 3\frac{22}{25}$

c $0.375 = \frac{375}{1000} = \frac{3}{8}$

EXERCISE 6A.2

1 Write as a fraction in simplest form:

a 0.3 **b** 0.9 **c** 1.2 **d** 2.5

e 1.7 **f** 3.2 **g** 0.15 **h** 0.16

i 0.02 **j** 0.07 **k** 0.04 **l** 0.125

2 Write as a fraction in simplest form:

a 0.27 **b** 0.84 **c** 0.004 **d** 0.015

e 0.0004 **f** 0.275 **g** 0.825 **h** 0.0025

i 0.625 **j** 0.000 05 **k** 4.08 **l** 0.075

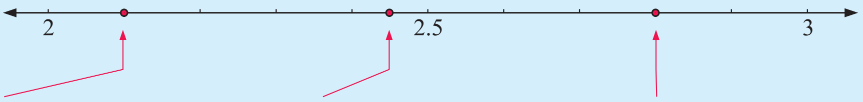
B ORDERING DECIMAL NUMBERS

Just like whole numbers, decimal numbers may be shown on a number line. To do this we generally divide each segment of the number line into **ten equal parts**.

Example 6 **Self Tutor**

Place the values 2.1, 2.45 and 2.8 on a number line.

Divide a number line from 2 to 3 into ten equal parts.



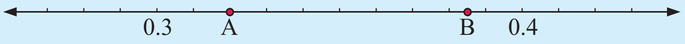
2.1 is placed $\frac{1}{10}$ of the way from 2 to 3.

2.45 lies half-way between 2.4 and 2.5 .

2.8 is placed $\frac{8}{10}$ of the way from 2 to 3.

Example 7 **Self Tutor**

Write down the values of A and B on the number line:



The segment between 0.3 and 0.4 is divided into 10 equal parts, so the number line shows 0.30, 0.31, 0.32, ..., 0.39, 0.40 .

A lies at 0.32
B lies half-way between 0.38 and 0.39, so B is 0.385 .

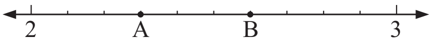
EXERCISE 6B

1 Place the following decimal numbers on separate number lines:

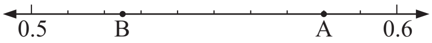
a 1.2, 1.35, 1.9
b 4.3, 4.75, 4.8
c 68.7, 68.2, 69.1
d 15.8, 16.9, 16.25
e 0.22, 0.26, 0.29
f 1.81, 1.85, 1.88

2 Write down the values of A and B on the following number lines:

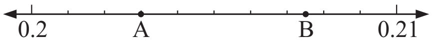
a



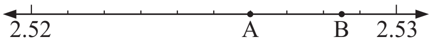
b



c



d



e

f

Example 8 **Self Tutor**

Insert $>$, $<$ or $=$ between the numbers 5.302 and 5.31 .

Both numbers have 5 wholes and three tenths.

5.302 has zero hundredths whereas 5.31 has one hundredth.

So, $5.302 < 5.31$

$>$ means "is greater than".

$<$ means "is less than".

3 Insert $<$, $>$ or $=$ between these pairs of numbers:

- **a** 3.63, 3.6
- **b** 7.07, 7.7
- **c** 0.008 76, 0.0786
- **d** 0.229, 0.292
- **e** 0.47, 0.5
- **f** 21.101, 21.011
- **g** 0.746, 0.467
- **h** 0.076, $\frac{67}{100}$
- **i** 0.306, 0.603
- **j** $\frac{150}{1000}$, 0.15
- **k** 7.5, 7.500
- **l** 0.7, $\frac{70}{100}$

Example 9 **Self Tutor**

Write the following decimal numbers in **ascending order** (from smallest to largest): 7.35, 7.28, 7.095

To help compare the numbers we write them with the same number of places after the decimal:

7.350, 7.280, 7.095

The numbers each have the same whole number part:

7.350, 7.280, 7.095

but different values in the tenths place:

7.350, 7.280, 7.095

So, 7.095, 7.28, 7.35 are in ascending order.

We can write zeros at the end of decimal numbers without changing the place value of the other digits.

4 Write in ascending order:

- **a** 2.36, 2.3, 2.036
- **b** 9.43, 9.34, 9.3, 9.04
- **c** 0.5, 0.495, 0.052
- **d** 18.7, 18.71, 18.6, 19.1
- **e** 8.055, 7.99, 8.1
- **f** 7.209, 7.092, 7.902, 7.29
- **g** 3.1, 3.09, 3.2, 3.009
- **h** 0.9, 0.09, 0.99, 0.099

5 Matthew's best four times for an 80 m sprint are 9.9 seconds, 9.09 seconds, 9.99 seconds and 9.89 seconds. Place these times in order from fastest to slowest.

6 On Monday the US dollar could be exchanged for 0.7211 euros. This means that each US dollar was worth a little more than 72 euro cents. For the rest of the week the exchange figures were: Tuesday 0.7122, Wednesday 0.7201, Thursday 0.7102, and Friday 0.7212 .
Place the exchange rates in order from highest to lowest.

C ADDING AND SUBTRACTING DECIMAL NUMBERS

When **adding** or **subtracting** decimal numbers, we write the numbers under one another so the decimal points are directly underneath each other.

When this is done, the digits in each place value will also lie under one another. We then add or subtract as for whole numbers.

Notice that the decimal points are placed directly underneath each other.

Example 10 **Self Tutor**

Find: $1.76 + 0.961$

$$\begin{array}{cccccc} & 1 & . & 7 & 6 & 0 \\ + & 0 & . & 9 & 6 & 1 \\ & 1 & & 1 & & \\ \hline & 2 & . & 7 & 2 & 1 \end{array}$$

Example 11 **Self Tutor**

Find: **a** $4.632 - 1.507$ **b** $8 - 0.706$

a

$$\begin{array}{cccccc} & 4 & . & 6 & \overset{2}{\cancel{3}} & \overset{12}{\cancel{2}} \\ - & 1 & . & 5 & 0 & 7 \\ \hline & 3 & . & 1 & 2 & 5 \end{array}$$

Place the decimal points directly under one another and subtract as for whole numbers.

b

$$\begin{array}{cccccc} & \overset{7}{\cancel{8}} & . & \overset{9}{\cancel{0}} & \overset{9}{\cancel{0}} & \overset{10}{\cancel{0}} \\ - & 0 & . & 7 & 0 & 6 \\ \hline & 7 & . & 2 & 9 & 4 \end{array}$$

We insert .000 after the 8 so we have the same number of decimal places in both numbers.

EXERCISE 6C

1 Find:

- **a** $0.3 + 0.6$
- **b** $0.8 + 0.23$
- **c** $0.17 + 1.36$
- **d** $0.2 + 0.9 + 2$
- **e** $0.076 + 0.61$
- **f** $11.56 + 8.072$
- **g** $0.071 + 0.477$
- **h** $12.66 + 1.302$
- **i** $0.0037 + 0.628$
- **j** $0.021 + 0.979$
- **k** $13.69 + 8.091$
- **l** $0.16 + 2.09 + 0.895$

2 Find:

- **a** $1.5 - 0.8$
- **b** $2.6 - 1.7$
- **c** $1 - 0.3$
- **d** $3 - 0.72$
- **e** $3.2 - 0.65$
- **f** $1 - 0.99$
- **g** $1 - 0.9999$
- **h** $1.6 - 0.9$
- **i** $5.2 - 3.6$
- **j** $0.083 - 0.0091$
- **k** $1.21 - 0.6$
- **l** $0.16 + 0.093 - 0.131$

3 Add:

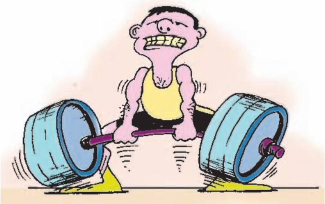
- **a** 38.76, 132.8 and 9.072
- **b** 18.61, 236.9 and 1072.4
- **c** 19.04, 360.8 and 0.0341
- **d** 0.76, 10.6, 108.77 and 0.862

4 Subtract:

- **a** 8.615 from 19.837
- **b** 14.86 from 28.79
- **c** 11.603 from 20
- **d** 9.674 from 68.3

5 A 20 m length of rope is cut into 4 pieces. Three of the pieces have lengths 5.62 m, 8.05 m, and 2.6 m. Find the length of the fourth piece.

6 A weightlifter snatches 135.8 kg, 142.9 kg, and 153.7 kg in consecutive lifts. Find the total mass lifted.



7 How much change would you expect from a $20 note if you purchased articles costing $8.63, $5.09 and $4.73?

8 Rosemary's Visa Card Statement is given alongside: What would be the total at the bottom of the statement?

Tescos	£130.80
Sports World	£288.00
The Red Lion Inn	£46.42
B&Q	£387.95
Morrisons	£59.46
Sainsburys	£14.95

9

Taxation	€507.90
Private Health Cover	€119.20
Superannuation	€95.62
Union Fees	€14.82

Each fortnight Alex is paid €1700 less the deductions given in the table alongside. What is Alex's actual take home pay each fortnight?

10 Continue the number sequences by writing the next three terms:

- **a** 0.1, 0.2, 0.3, ...
- **b** 0.2, 0.4, 0.6, ...
- **c** 0.03, 0.05, 0.07, ...
- **d** 0.05, 0.06, 0.07, ...
- **e** 5.2, 5.1, 5.0, ...
- **f** 0.11, 0.22, 0.33, ...
- **g** 7.4, 7.2, 7.0, ...
- **h** 3.2, 2.8, 2.4, ...
- **i** 0.6, 0.55, 0.5, ...

D MULTIPLYING AND DIVIDING BY POWERS OF 10

MULTIPLICATION

Consider multiplying 2.36

- by 100: $2.36 \times 100 = \frac{236}{{}_1\cancel{100}} \times \frac{\cancel{100}^{1}}{1}$
  $= 236$
- by 1000: $2.36 \times 1000 = \frac{236}{{}_1\cancel{100}} \times \frac{\cancel{1000}^{10}}{1}$
  $= 236 \times 10$
  $= 2360$

When we multiply by 100, the decimal point of 2.36 shifts 2 places to the **right**.

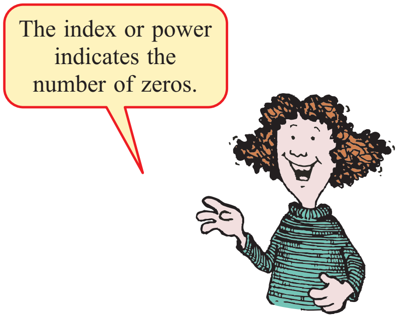
2.36 becomes 236.

When we multiply by 1000, the decimal point shifts 3 places to the **right**.

2.360 becomes 2360.

When multiplying by 10^n we shift the decimal point n places to the **right**.
The number becomes 10^n times **larger** than it was originally.

Remember
$$\begin{aligned}10^1 &= 10\\ 10^2 &= 100\\ 10^3 &= 1000\\ 10^4 &= 10\,000\\ &\vdots\end{aligned}$$



The index or power indicates the number of zeros.

Example 12 **Self Tutor**

Find:

a 9.8×10 **b** 0.0751×100 **c** $13.026 \times 10\,000$

a 9.8×10
$= 9.8 \times 10^1$ {$10 = 10^1$, so shift the decimal point 1 place right.}
$= 98$

b 0.0751×100
$= 0.0751 \times 10^2$ {$100 = 10^2$, so shift the decimal point 2 places right.}
$= 7.51$

c $13.026 \times 10\,000$
$= 13.0260 \times 10^4$ {$10\,000 = 10^4$, so shift the decimal point 4 places right.}
$= 130\,260$

EXERCISE 6D.1

1 **a** Multiply 8.7 by: **i** 10 **ii** 100 **iii** 1000 **iv** 10^5
b Multiply 0.073 by: **i** 10 **ii** 1000 **iii** 10^4 **iv** 10^6

2 Find:

a 38×10 **b** 9×100 **c** 3.2×10
d 0.8×10 **e** 0.71×100 **f** 2.8×100
g 0.6×100 **h** 0.83×100 **i** 1.89×10^4
j 0.053×10^3 **k** 0.0583×1000 **l** $0.187 \times 100\,000$

DIVISION

Consider dividing 3.7 by 100: $3.7 \div 100 = \frac{37}{10} \div \frac{100}{1}$

$$= \frac{37}{10} \times \frac{1}{100}$$

$$= \frac{37}{1000}$$

$$= 0.037$$

and by 1000: $3.7 \div 1000 = \frac{37}{10} \times \frac{1}{1000}$

$$= \frac{37}{10\,000}$$

$$= 0.0037$$

When we divide by 100, the decimal point in 3.7 shifts 2 places to the **left**.

003.7 becomes 0.037 .

When we divide by 1000, the decimal point in 3.7 shifts 3 places to the **left**.

0003.7 becomes 0.0037 .

When dividing by 10^n we shift the decimal point n places to the **left**.

The number becomes 10^n times **smaller** than it was originally.

Example 13 **Self Tutor**

Find: **a** $0.4 \div 10$ **b** $0.18 \div 1000$

a $0.4 \div 10$

$= 0.4 \div 10^1$ $\{10 = 10^1$, so shift the decimal point 1 place left$\}$

$= 0.04$

b $0.18 \div 1000$

$= 000.18 \div 10^3$ $\{1000 = 10^3$, so shift the decimal point 3 places left$\}$

$= 0.000\,18$

EXERCISE 6D.2

1 **a** Divide 0.9 by: **i** 10 **ii** 100 **iii** 10^4

b Divide 70.6 by: **i** 100 **ii** 10 000 **iii** 10^8

2 Find:

a $7 \div 10$ **b** $89 \div 10$ **c** $463 \div 10$

d $463 \div 100$ **e** $463 \div 1000$ **f** $463 \div 10\,000$

g $0.8 \div 10$ **h** $0.8 \div 1000$ **i** $0.73 \div 100$

j $0.07 \div 10$ **k** $0.07 \div 100$ **l** $0.083 \div 1000$

m $0.0023 \div 1000$ **n** $0.0028 \div 10\,000$ **o** $0.000\,51 \div 10\,000$

E MULTIPLYING DECIMAL NUMBERS

We can explain how decimal numbers are multiplied by first converting the decimals into fractions.

For example, consider the product 4×0.03.

If we first convert to fractions, we have

$$\begin{aligned} & 4 \times 0.03 \\ &= \tfrac{4}{1} \times \tfrac{3}{100} && \{\text{convert to fractions}\} \\ &= \tfrac{12}{100} && \{\text{multiply fractions}\} \\ &= 0.12. && \{\text{convert back to decimal}\} \\ &= 0.12 \end{aligned}$$

From this example we see that:

- we multiply the whole numbers 4 and 3
- then divide by a power of 10, in this case 100.

Now consider finding 0.4×0.05.

If we first convert to fractions, we have

$$\begin{aligned} & 0.4 \times 0.05 \\ &= \tfrac{4}{10} \times \tfrac{5}{100} && \{\text{convert to fractions}\} \\ &= \tfrac{20}{1000} && \{\text{multiply fractions}\} \\ &= 0.020. && \{\text{convert back to decimal}\} \\ &= 0.02 \end{aligned}$$

Once again we can see that

- we multiply the whole numbers 4 and 5
- then divide by a power of 10, in this case 1000.

With practice we do not need to convert the decimals to fractions first. We multiply the decimal numbers as though they were whole numbers, then divide by the appropriate power of 10.

Example 14 **Self Tutor**

Find: 0.3×0.07

$$\begin{aligned} & 0.3 \times 0.07 \\ &= (3 \times 7) \div 1000 \\ &= 21 \div 1000 \\ &= 0.021 && \{\text{shifting the decimal point 3 places left}\} \end{aligned}$$

We multiply 0.3 by 10 and 0.07 by 100, then balance by dividing by $10 \times 100 = 1000$.

EXERCISE 6E

1 Find the value of:

a 0.3×0.2
b 0.5×0.07
c 0.02×0.4
d $(0.4)^2$
e $(0.06)^2$
f 0.03×0.004
g 0.004×40
h 60×0.8
i 600×0.07
j 4000×0.6
k $0.04 \times 40\,000$
l $0.3 \times 0.5 \times 0.7$

2 Given that $87 \times 213 = 18\,531$, evaluate:

a 8.7×213 **b** 8.7×2.13 **c** 8.7×21.3

d 87×0.213 **e** 0.87×0.213 **f** 8.7×0.213

g 0.87×2.13 **h** 870×0.213 **i** $8.7 \times 0.002\,13$

3 Evaluate:

a 0.3×6 **b** 0.5×4.0 **c** 0.03×7

d 0.03×700 **e** 2.8×5 **f** 0.6×0.8

g 0.8×0.05 **h** 0.05×0.4 **i** $(0.2)^2$

j $(2.5)^2$ **k** 0.14×0.5 **l** $(0.03)^2$

m $(0.2)^3$ **n** $(0.3)^3$ **o** 25×0.0004

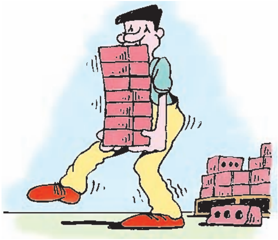
p $1 + 0.2 \times 0.3$ **q** $0.08 - 0.08 \times 0.2$ **r** $(0.3 - 1)^2$

4 **a** Find the cost of 72 books at €5.75 each.

b Find the cost of 8.6 m of plastic sheeting at $4.62 per metre.

5 In order to bake cakes for the school fair, I buy 180 kg of flour at $0.84 per kg and 25 kg of sugar at $1.17 per kg. How much money have I spent?

6 I load 450 bags of salt onto my lorry, each having mass 0.15 kg. Find the total mass of all bags.



7 House bricks have a mass of 4.3 kg each and I buy 2500 of them to build a wall around my courtyard.

a Find the total mass of the bricks.

b If my truck can carry only 2 tonnes at a time, how many truck loads are necessary to transport the bricks?

F DIVIDING DECIMAL NUMBERS

INVESTIGATION 1 DIVISION OF DECIMALS

What to do:

1 Copy and complete the following divisions. Look for patterns to use when the divisions involve decimals.

a $800 \div 200 = \square$, $80 \div 20 = \square$, $8 \div 2 = \square$, $0.8 \div 0.2 = \square$

b $800 \div 20 = \square$, $80 \div 2 = \square$, $8 \div 0.2 = \square$, $0.8 \div 0.02 = \square$

c $80 \div 200 = \square$, $8 \div 20 = \square$, $0.8 \div 2 = \square$, $0.08 \div 0.2 = \square$

2 In each set of divisions, what did you notice about the answers?

3 Did you find that in each set the division by the smallest *whole* number was the easiest?

From the **Investigation** you should have observed that multiplying or dividing both numbers in a division by the same factor *does not* change the result.

This observation leads to the following **rules of division**:

When **dividing** a **decimal number** by a **whole number**, carry out the division as normal, writing **decimal points under each other**.

Example 15

Self Tutor

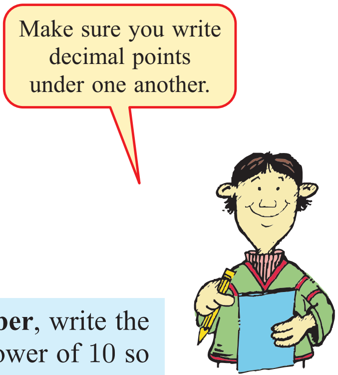
Find: **a** $32.5 \div 5$ **b** $0.417 \div 3$

a
$$\begin{array}{r} 5\,\overline{)\,3\,2\,.\,{}^{2}5} \\ 6\,.\,5 \end{array}$$

Answer: 6.5

b
$$\begin{array}{r} 3\,\overline{)\,0\,.\,4\,{}^{1}1\,{}^{2}7} \\ 0\,.\,1\;\;3\;\;9 \end{array}$$

Answer: 0.139



When **dividing** a **decimal number** by **another decimal number**, write the division as a fraction. Multiply top and bottom by the same power of 10 so the denominator becomes a whole number. Then perform the division.

Example 16

Self Tutor

Find: **a** $18 \div 0.06$ **b** $0.021 \div 1.4$

a
$$\begin{aligned} &18 \div 0.06 \\ &= \frac{18 \times 100}{0.06 \times 100} \\ &= \frac{1800}{6} \\ &= 300 \end{aligned}$$

b
$$\begin{aligned} &0.021 \div 1.4 \\ &= \frac{0.021 \times 10}{1.4 \times 10} \\ &= \frac{0.21}{14} \\ &= 0.015 \end{aligned}$$

$$\begin{array}{r} 0\,.\,0\;1\;5 \\ 14\,\overline{)\,0\,.\,2\;1\;0} \\ 1\;4\;\downarrow \\ \hline 7\;0 \\ 7\;0 \\ \hline 0 \end{array}$$

EXERCISE 6F

1 Evaluate:

a $8.4 \div 2$ **b** $15.6 \div 3$ **c** $20.4 \div 4$ **d** $0.15 \div 5$
e $0.126 \div 9$ **f** $1.61 \div 7$ **g** $49.8 \div 6$ **h** $3.04 \div 4$
i $0.616 \div 11$ **j** $0.0405 \div 3$ **k** $3.92 \div 8$ **l** $0.392 \div 7$

2 Calculate:

a $0.9 \div 0.3$ **b** $4.9 \div 0.7$ **c** $15 \div 0.5$ **d** $0.36 \div 0.6$
e $0.8 \div 0.16$ **f** $0.25 \div 0.05$ **g** $3.2 \div 0.08$ **h** $2.7 \div 0.003$
i $0.84 \div 0.12$ **j** $10.71 \div 0.17$ **k** $0.52 \div 0.013$ **l** $12.88 \div 0.023$

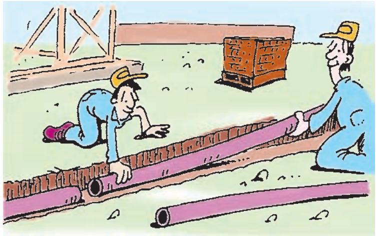
3 Evaluate:

a $0.36 \div 4$ **b** $3 \div 5$ **c** $13.2 \div 1.1$ **d** $0.08 \div 4$
e $0.08 \div 0.4$ **f** $0.08 \div 0.004$ **g** $1.2 \div 5$ **h** $1.2 \div 500$

i $1.2 \div 0.05$ **j** $0.12 \div 5000$ **k** $0.12 \div 50$ **l** $0.12 \div 0.5$
m $0.012 \div 0.0005$ **n** $19 \div 4$ **o** $3 \div 8$ **p** $3.5 \div 25$
q $0.035 \div 2.5$ **r** $0.049 \div 0.07$

4 Use your *calculator* to solve the following problems:

- **a** How many 5.6 m lengths of rope can be cut from a roll 151.2 m long?
- **b** If €4001.80 is distributed equally amongst 17 people, how much does each get?
- **c** 4 gold nuggets have mass 0.175 kg, 0.369 kg, 0.836 kg and 2.593 kg respectively. Find: **i** the total mass in kg **ii** the average mass of the 4 nuggets.
- **d** How many £1.25 packets of almonds can be bought for £25?
- **e** Determine the number of 2.4 m lengths of piping required to construct a 720 m drain.
- **f** How many tins of preserved fruit each costing \$2.55 can be purchased with \$58.65?



Example 17 **Self Tutor**

Use your calculator to simplify: $\dfrac{3.45 + 0.555}{0.03 \times 0.05}$

We must divide the whole of the numerator by the whole of the denominator, so we use the **bracket** keys:

Key in $\boxed{(}$ 3.45 $\boxed{+}$ 0.555 $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 0.03 $\boxed{\times}$ 0.05 $\boxed{)}$ $\boxed{=}$ *Answer*: 2670

5 Use your calculator to simplify:

a $\dfrac{2.7 \times 8}{0.3}$ **b** $\dfrac{2 + 0.75}{1.1}$ **c** $\dfrac{2.6 + 1.35}{0.05}$

d $\dfrac{0.25 \times 3.2}{0.02 \times 0.05}$ **e** $\dfrac{0.4 \times 0.06}{1 - 0.76}$ **f** $\dfrac{1.6 \times 0.25 \times 0.6}{4 \times 0.05 \times 0.16}$

G TERMINATING AND RECURRING DECIMALS

Every **rational number** can be written as either a **terminating** or a **recurring** decimal.

TERMINATING DECIMALS

Terminating decimals result when the rational number has a denominator which has no prime factors other than 2 or 5.

For example, $\frac{1}{8} = 0.125$ where the only prime factor of 8 is 2

$\frac{3}{20} = 0.15$ where the prime factors of 20 are 2 and 5.

Example 18 **Self Tutor**

Use division to write the following fractions as decimals:

a $\frac{1}{5}$ **b** $\frac{17}{40}$

a $\frac{1}{5}$

$= 0.2$

$$\begin{array}{r|l} 5 & 1\,.\,{}^{1}0 \\ \cline{2-2} & 0\,.\,2 \end{array}$$

b $\frac{17}{40}$

$= \frac{1.7}{4}$ {dividing top and bottom by 10}

$= 0.425$

$$\begin{array}{r|l} 4 & 1\,.\,7\,{}^{1}0\,{}^{2}0 \\ \cline{2-2} & 0\,.\,4\;\;2\;\;5 \end{array}$$

Another method of converting fractions to **terminating decimals** is to write the denominator as a **power of 10**. For example:

- for **halves** multiply by $\frac{5}{5}$ to convert to tenths
- for **fifths** multiply by $\frac{2}{2}$ to convert to tenths
- for **quarters** multiply by $\frac{25}{25}$ to convert to hundredths.

Example 19 **Self Tutor**

Write the following in decimal form, without carrying out a division:

a $\frac{3}{5}$ **b** $\frac{7}{25}$ **c** $\frac{5}{8}$

a $\frac{3}{5}$
$= \frac{3\times 2}{5\times 2}$
$= \frac{6}{10}$
$= 0.6$

b $\frac{7}{25}$
$= \frac{7\times 4}{25\times 4}$
$= \frac{28}{100}$
$= 0.28$

c $\frac{5}{8}$
$= \frac{5\times 125}{8\times 125}$
$= \frac{625}{1000}$
$= 0.625$

EXERCISE 6G.1

1 Use division to write the following fractions as terminating decimals:

a $\frac{9}{10}$ **b** $\frac{4}{5}$ **c** $\frac{1}{4}$ **d** $\frac{3}{5}$ **e** $\frac{1}{8}$

f $\frac{3}{40}$ **g** $\frac{7}{20}$ **h** $1\frac{1}{5}$ **i** $3\frac{1}{4}$ **j** $8\frac{31}{50}$

2 What would you multiply the following by to convert the denominators to powers of 10?

a $\frac{19}{20}$ **b** $\frac{11}{50}$ **c** $\frac{5}{8}$ **d** $\frac{19}{40}$ **e** $\frac{5}{200}$

3 Write the following as decimals without carrying out a division:

a $\frac{3}{4}$ **b** $\frac{7}{5}$ **c** $\frac{17}{50}$ **d** $\frac{93}{200}$ **e** $\frac{13}{125}$

f $\frac{81}{500}$ **g** $\frac{5}{4}$ **h** $3\frac{11}{25}$ **i** $1\frac{19}{125}$ **j** $2\frac{7}{20}$

RECURRING DECIMALS

Recurring decimals repeat the same sequence of numbers without stopping. Recurring decimals result when the denominator of a rational number has one or more prime factors other than 2 or 5.

For example, $\frac{3}{7} = 0.428\,571\,428\,571\,428\,571\ \ldots\ldots$

We indicate a recurring decimal by writing the full sequence once with a line over the repeated section. We can also indicate it using dots.

For example, $\frac{1}{3} = 0.\overline{3}$ or $0.\dot{3}$ and $\frac{3}{7} = 0.\overline{428\,571}$ or $0.\dot{4}28\,57\dot{1}$

Example 20 **Self Tutor**

Write as decimals: **a** $\frac{7}{9}$ **b** $\frac{5}{11}$

a $\frac{7}{9}$

$= 0.7777\ldots\ldots$

$= 0.\overline{7}$

$$\begin{array}{r|l} 9 & 7\,.\,0\,{}^{7}0\,{}^{7}0\,{}^{7}0\ \ldots\ldots \\ \cline{2-2} & 0\,.\,7\ \ 7\ \ 7\ \ 7\ \ldots\ldots \end{array}$$

b $\frac{5}{11}$

$= 0.454545\ldots\ldots$

$= 0.\overline{45}$

$$\begin{array}{r|l} 11 & 5\,.\,0\,{}^{6}0\,{}^{5}0\,{}^{6}0\,{}^{5}0\ \ldots\ldots \\ \cline{2-2} & 0\,.\,4\ \ 5\ \ 4\ \ 5\ \ 4\ \ldots\ldots \end{array}$$

Some decimals take a long time to recur. For example,

$\frac{1}{17} = 0.\overline{0588235294117647}$

EXERCISE 6G.2

1 Write as recurring decimals:

a $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{3}{7}$ **d** $\frac{7}{11}$ **e** $\frac{5}{6}$

2 Use your calculator to write as recurring decimals:

a $\frac{2}{15}$ **b** $\frac{9}{14}$ **c** $\frac{1}{13}$ **d** $\frac{23}{45}$ **e** $\frac{23}{54}$

INVESTIGATION 2 CONVERTING RECURRING DECIMALS TO FRACTIONS

Although it is quite easy to convert fractions into recurring decimal form, it is not always as easy to do the reverse process.

What to do:

1 Write down decimal expansions for $\frac{2}{9}$, $\frac{5}{9}$ and $\frac{7}{9}$.

a From your results, predict the fraction equal to: **i** $0.\overline{4}$ **ii** $0.\overline{8}$

b Copy and complete: $0.aaaaaa\ldots\ldots = 0.\overline{a} =$.

2 Use your calculator to write down decimal expansions for $\frac{23}{99}$, $\frac{47}{99}$ and $\frac{83}{99}$.

a From your results, predict the fraction equal to: **i** $0.\overline{17}$ **ii** $0.\overline{53}$

b Copy and complete: $0.abababab\ldots\ldots = 0.\overline{ab} =$.

3 Predict fractions equal to **a** $0.\overline{171}$ **b** $0.\overline{3628}$ **c** $0.\overline{12345}$

4 Sometimes the recurring part of a decimal occurs after one or more places from the decimal point. For example, $0.7\overline{3} = 0.7333333......$
Find the fraction equal to:

a $0.7\overline{3}$ **b** $0.9\overline{2}$ **c** $0.7\overline{6}$ **d** $0.a\overline{b}$

H DECIMAL APPROXIMATIONS

We are often given measurements as decimal numbers. In such cases we **approximate** the decimal by **rounding off** to the required accuracy.

We have previously seen how to round off whole numbers. For example:

$$\begin{aligned} &3628 \\ &\approx 3630 \quad \text{(to the nearest 10)} \\ &\approx 3600 \quad \text{(to the nearest 100)} \\ &\approx 4000 \quad \text{(to the nearest 1000)} \end{aligned}$$

We round off decimal numbers in the same way. For example:

$$\begin{aligned} &0.3872 \\ &\approx 0.387 \quad \text{(to 3 decimal places)} \\ &\approx 0.39 \quad \text{(to 2 decimal places)} \\ &\approx 0.4 \quad \text{(to 1 decimal place)} \end{aligned}$$

RULES FOR ROUNDING OFF DECIMAL NUMBERS

- If the digit after the one being rounded is **less than 5**, i.e., 0, 1, 2, 3 or 4, then we round **down**.
- If the digit after the one being rounded is **5 or more**, i.e., 5, 6, 7, 8 or 9, then we round **up**.

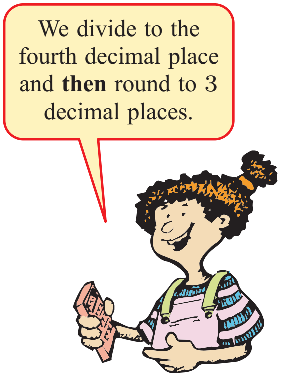
Example 21 Self Tutor

Find $\frac{36}{17}$ correct to 3 decimal places.

Rather than do long division, we can use a calculator.

Pressing 36 $\boxed{\div}$ 17 $\boxed{=}$, the result is 2.117 647 059

So, $\frac{36}{17} \approx 2.118$ (to 3 decimal places)



We divide to the fourth decimal place and **then** round to 3 decimal places.

ROUNDING DECIMAL NUMBERS USING A CALCULATOR

Most calculators have a FIX mode for rounding numbers to a certain number of decimal places.

To activate the FIX mode, press MODE , select the FIX option, and then specify the number of decimal places required.

While you remain in FIX mode, all answers will be given to this number of decimal places.

EXERCISE 6H

1 **a** Write 0.7690 correct to:
- **i** 1 decimal place
- **ii** 2 decimal places
- **iii** 3 decimal places.

b Write 0.07149 correct to:
- **i** 1 decimal place
- **ii** 2 decimal places
- **iii** 4 decimal places.

2 Find decimal approximations for:

a 8.7 to the nearest integer
b 15.63 to the nearest tenth
c $0.\overline{63}$ to 2 decimal places
d $0.\overline{46}$ to 3 decimal places
e $0.\overline{7}$ to 4 decimal places
f $0.\overline{8}$ to 5 decimal places.

3 Use a calculator to evaluate correct to the number of decimal places shown in the square brackets:

a $\frac{37}{7}$ [1]
b 2.4×3.79 [1]
c $(0.8)^2$ [1]
d $(0.72)^2$ [2]
e $\frac{8}{23}$ [2]
f $0.3 \div 1.7$ [2]
g $(0.043)^2$ [3]
h $\frac{2.3}{0.6}$ [3]
i $\frac{14}{13}$ [3]

4 Darren's batting average is calculated as 53.6853. Round this to one decimal place.

5 Romandy Gold makes an annual profit of £136.748 million. Round this figure to one decimal place.

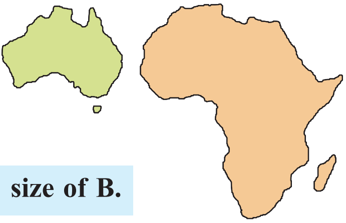
6 Jordan shoots 32.587 points per basketball game. Round this to two decimal places.

7 Tye calculates the interest due on her savings account to be \$78.1983. Round this to the nearest cent.

8 Use your calculator to round to the number of decimal places in brackets:

a 4.6517 [2]
b 15.1387 [3]
c 8.6604 [2]
d 98.99 [1]
e 15.9962 [2]
f 21.019 [1]
g 1.784 96 [4]
h 17.499 [0]
i 0.006 52 [1]
j $39 \div 17$ [4]
k $56.9 \div 11.7$ [3]
l $(0.367)^2$ [3]
m $(8.391)^3$ [3]
n $0.637 \times (0.21)^2$ [4]
o $\frac{21.62}{(8.37)^2}$ [3]

I COMPARING SIZES



How much bigger is Africa than Australia?
To answer questions like this we need to obtain data from an atlas or textbook or from the internet. If we know the areas of the two continents, we can use:

The number of times A is bigger than B = size of A ÷ size of B.

Using the internet, we find that Africa is about 30.22 million km^2 and Australia is about 7.68 million km^2.
The number of times Africa is bigger than Australia is $30.22 \div 7.68 \approx 3.94$
So, Africa is almost four times bigger than Australia.

EXERCISE 6I

1 How many times higher is:
- **a** Mount Everest (8848 m) than Mount Hunter (4445 m) in Alaska?
- **b** Mount Manaslu (8163 m) in Nepal than Mount Yasus (3500 m) in Africa?
- **c** Mount Konkur (7719 m) in China than Mount Logan (5951 m) in Canada?

2 How many times bigger is:
- **a** Asia (45.84 million km^2) than Antarctica (14.00 million km^2)
- **b** Africa (30.22 million km^2) than South America (17.84 million km^2)
- **c** Asia (45.84 million km^2) than Australia (7.68 million km^2)?

3 How many times longer is:
- **a** the river Nile (6650 km) in Africa than the river Murray (2520 km) in Australia
- **b** the river Amazon (6400 km) in South America than the river Snake (1670 km) in the USA
- **c** the river Yangtze (6300 km) in China than the river Rhine (1320 km) in Europe?

4 Jon's property is 4.63 times larger than Helen's. If Helen's property is 368.7 ha, find the area of Jon's property.

KEY WORDS USED IN THIS CHAPTER

- decimal number
- highest common factor
- recurring decimal
- decimal point
- place value
- simplest form
- expanded form
- rational number
- terminating decimal

LINKS
click here

LEAP YEARS

Areas of interaction:
Human ingenuity, Environment

REVIEW SET 6A

1 **a** Convert 0.3 to a fraction. **b** State the value of the digit 6 in 17.3264.

c Multiply 8.46 by 1000. **d** Write $7 + \frac{4}{10} + \frac{3}{100}$ in decimal form.

e Find 70.2×100. **f** State the value of the digit 2 in 0.362.

g Evaluate $0.02 \div 100$. **h** Write 2.64 as a fraction in simplest form.

2 Express 23.452 as a fraction in simplest form.

3 Evaluate: **a** $0.62 + 2.531$ **b** 0.28×0.43

4 Write: **a** 23.549 to 2 decimal places **b** 0.4723 to the nearest hundredth

c $0.\overline{54}$ to 2 decimal places.

5 If you purchased articles costing $11.63, $13.72 and $21.40, how much change would you receive from a $50 note?

6 A race track is 3.2 km long. How many laps are needed to complete a 360 km race?

7 If a man's height is 1.6 times that of his daughter, who is 125 cm tall, determine the height of the man.

8 New Zealand has an area of 268.68 thousand km^2. It has a human population of 4.3 million, and is home to 40.1 million sheep. How many:

a sheep are there per person **b** people are there per km^2?

REVIEW SET 6B

1 **a** Write 0.3826 correct to 3 decimal places.

b State the value of the digit 5 in 46.054.

c Express 4.012 as a fraction in simplest form.

d Given that $82 \times 76 = 6232$, evaluate 8.2×0.076.

e Divide 0.42 by 100. **f** Write $\frac{6}{11}$ as a recurring decimal.

2 Evaluate: **a** $0.375 + 2.54$ **b** 2.8×2.4

3 Insert $<$, $>$ or $=$ between these pairs of numbers:

a 2.01 and 2.101 **b** 0.966 and 0.696

4 Find: **a** 0.4398 to 3 decimal places **b** 3.2849 to the nearest hundredth

5 Evaluate: **a** $3 \times 2.6 - 0.3$ **b** $\dfrac{3 \times 0.3 + 2.6}{7}$

6 Evaluate without a calculator:

a $23.42 + 361.023$ **b** $0.72 \div 0.012$ **c** $0.5 - 0.1 \times 0.2$

7 Solve the following problems:

a By how much does 132 exceed 130.876?

b If €107.25 is shared equally between 13 people, how much does each receive?

c In one day a truck delivers 48 tonnes of sand to a building site. The first 3 loads were 11.25 tonnes, 13.76 tonnes and 12.82 tonnes. How much sand was delivered in the fourth and final load?

Chapter 7

Percentage

Contents:

OPENING PROBLEM

The following football premiership table shows the standings of the teams after the first 4 rounds of the season:

Premiership Table

Team	Played	Won	Drawn	Lost	Goals for	Goals against	Percentage	Points
Falcons	4	3	1	-	7	3	70.0	10
Lions	4	2	1	1	5	3	62.5	7
West	4	2	1	1	6	4	60.0	7
Central	4	1	3	-	3	1	75.0	6
Eagles	4	2	-	2	4	4	50.0	6
South	4	1	2	1	2	3	40.0	5
East	4	-	1	3	2	6	25.0	1
North	4	-	1	3	1	7	12.5	1

Teams are awarded three points for a win and one point for a draw.

Consider the following questions:

- Which column is first used to order the teams?
- Which column is next used to separate teams on equal points?
- How are the percentages calculated?

A UNDERSTANDING PERCENTAGES

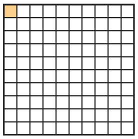
We use **percentages** to compare a portion with a whole amount which we call 100%.

Percentages are commonly used to describe interest rates, sale prices, test results, inflation, changes in profit levels, employment levels and much more.

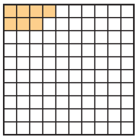
% reads **per cent**, which means '**in every hundred**'.

If an object is divided into one hundred equal parts then each part is called 1 per cent or 1%.

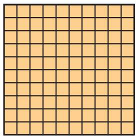
Thus, $\frac{1}{100} = 1\%$



$\frac{7}{100} = 7\%$



$\frac{100}{100} = 100\%$



Fractions and decimals are also used to indicate parts of quantities.

All fractions and decimals may be converted into percentage form by first writing them as fractions with a **denominator of 100**.

In general, $$\frac{x}{100} = x\%$$

Example 1 **Self Tutor**

Convert to percentages: **a** $\frac{1}{4}$ **b** 0.87 **c** 3

a

$\frac{1}{4}$

$= \frac{1\times 25}{4\times 25}$

$= \frac{25}{100}$ {25 out of 100}

$= 25\%$

b

0.87

$= \frac{87}{100}$ {87 out of 100}

$= 87\%$

c

3

$= 3 \times \frac{100}{100}$

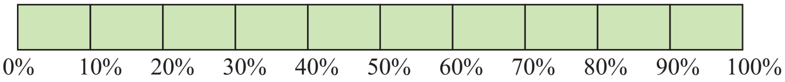
$= \frac{300}{100}$

$= 300\%$

It is very common to estimate percentages. It is important you are able to imagine the fraction that a particular percentage describes.

We often hear statements like: 'contains 25% real juice'
'80% of the students passed their examination'
'the unemployment rate is 5%'
'40% off sale!'

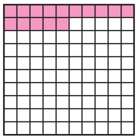
We need to remember that 100% describes a whole and the whole is made up of 10 lots of 10%.



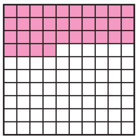
EXERCISE 7A

1 What percentage is represented by the following shaded diagrams?

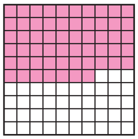
a



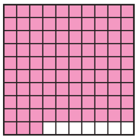
b



c



d



2 Write the following as fractions with denominator 100, and then convert to percentages:

a $\frac{4}{10}$	**b** 2	**c** $\frac{3}{50}$	**d** $\frac{17}{25}$
e $\frac{9}{20}$	**f** $\frac{4}{5}$	**g** 0.73	**h** 0.31
i 0.6	**j** 0.3	**k** 1.6	**l** 2.13
m $\frac{1}{2}$	**n** 1	**o** $\frac{3}{4}$	**p** $3\frac{1}{2}$

3 A computer shows the progress when downloading a document by shading a bar until the download is complete.
Which computer shows the download to be:

a 5% complete
b 90% complete
c 48% complete
d 23% complete?

A

B

C

D

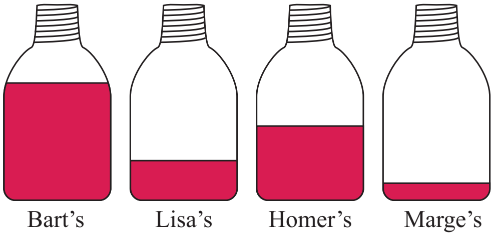
4 Draw four 10 cm bars and shade them so they show the following percentages of a completed download:

a 97% **b** 71% **c** 8% **d** 39%

5 Bart has poured some raspberry cordial into the drink bottles and will fill them up with water. Who likes their cordial mix:

a 50% cordial **b** 25% cordial

c 10% cordial **d** 80% cordial?



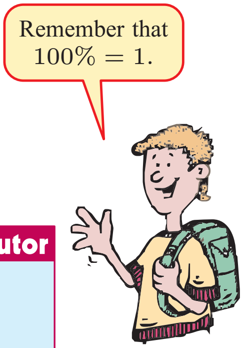
B INTERCHANGING NUMBER FORMS

Not all fractions can be easily written with a denominator of 100, so they cannot be easily expressed as a percentage.

CONVERTING FRACTIONS AND DECIMALS INTO PERCENTAGES

To convert fractions and decimals into percentage form, we multiply the fraction or decimal by **100%**.

Since $100\% = \frac{100}{100} = 1$, multiplying by 100% is the same as multiplying by 1. We therefore do not change the value of the number.



Example 2 **Self Tutor**

Convert to percentages by multiplying by 100%:

a $\frac{3}{4}$ **b** $\frac{1}{12}$ **c** 0.064

a
$$\begin{aligned}&\tfrac{3}{4}\\&= \tfrac{3}{4} \times 100\%\\&= \tfrac{300}{4}\%\\&= 75\%\end{aligned}$$

b
$$\begin{aligned}&\tfrac{1}{12}\\&= \tfrac{1}{12} \times 100\%\\&= \tfrac{100}{12}\%\\&= 8\tfrac{4}{12}\%\\&= 8\tfrac{1}{3}\%\end{aligned}$$

c
$$\begin{aligned}&0.064\\&= 0.064 \times 100\%\\&= 6.4\%\end{aligned}$$

EXERCISE 7B

1 Convert into percentage form by multiplying by 100%:

a $\frac{3}{5}$ **b** 0.7 **c** 2.4 **d** $2\frac{1}{2}$

e 4.025 **f** $\frac{1}{3}$ **g** 0.83 **h** 6

i 0.004 **j** 0.067 **k** $\frac{2}{9}$ **l** $1\frac{3}{8}$

CONVERTING PERCENTAGES INTO FRACTIONS

We can easily write percentages as fractions with a **denominator of 100**. The fraction can then be written in its lowest terms.

Example 3 Self Tutor

Express as common fractions in lowest terms: **a** 30% **b** 120% **c** $2\frac{1}{2}\%$

a 30%

$= \frac{30}{100}$

$= \frac{30 \div 10}{100 \div 10}$ {HCF is 10}

$= \frac{3}{10}$

b 120%

$= \frac{120}{100}$

$= \frac{120 \div 20}{100 \div 20}$ {HCF is 20}

$= \frac{6}{5}$

c $2\frac{1}{2}\%$

$= \frac{2.5 \times 2}{100 \times 2}$ {remove the decimal}

$= \frac{5}{200}$

$= \frac{5 \div 5}{200 \div 5}$ {HCF is 5}

$= \frac{1}{40}$

CONVERTING PERCENTAGES INTO DECIMALS

To convert a percentage into a decimal, we **divide by 100%**.

This means that we shift the decimal point 2 places to the left.

Example 4 Self Tutor

Express as a decimal: **a** 31% **b** $12\frac{1}{2}\%$ **c** 150%

a 31%

$= \frac{31}{100}$

$= 0.31$

b $12\frac{1}{2}\%$

$= \frac{12.5}{100}$

$= 0.125$

c 150%

$= \frac{150}{100}$

$= 1.5$

To divide by 100, move the decimal point 2 places to the left.

2 Convert into percentage form:

a $\frac{3}{10}$
b 4
c $\frac{7}{50}$
d $\frac{11}{25}$
e $\frac{17}{20}$
f $\frac{2}{5}$
g 0.67
h 0.85
i 1
j 0.7
k 0.2
l 1.3
m 3.17
n $\frac{1}{2}$
o $\frac{3}{4}$
p 0.031
q 0.0014
r 0.105
s 0.056
t $\frac{5}{16}$

3 Express as common fractions in lowest terms:

a 10%
b 25%
c 60%
d 85%
e 5%
f 15%
g 42%
h 100%
i 500%
j 1%
k 135%
l 12%

4 Write as common fractions in lowest terms:

a $7\frac{1}{2}\%$ **b** $33\frac{1}{4}\%$ **c** $12\frac{1}{2}\%$ **d** $33\frac{1}{3}\%$

e $\frac{1}{4}\%$ **f** $\frac{1}{2}\%$ **g** $6\frac{1}{4}\%$ **h** $7\frac{2}{5}\%$

5 Write as a decimal:

a 40% **b** 23% **c** 50% **d** 17% **e** 8.3% **f** 150%

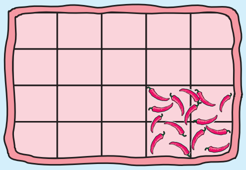
g 200% **h** 36.8% **i** 5000% **j** 0.01% **k** 117.9% **l** 86.7%

6 Copy and complete this table of common conversions:

Percentage	*Fraction*	*Decimal*	*Percentage*	*Fraction*	*Decimal*
100%			10%		
75%			5%		
50%			$33\frac{1}{3}\%$		
25%			$66\frac{2}{3}\%$		
20%			$12\frac{1}{2}\%$		

Example 5 **Self Tutor**

A rectangular pizza is cut into 20 pieces as shown. Chilli has been put on 4 pieces.



a What fraction of the pizza does not have chilli?

b What percentage of the pizza does not have chilli?

c What percentage of the pizza does have chilli?

a There are 20 pieces and 16 do not have chilli.

$\therefore$ $\frac{16}{20} = \frac{4}{5}$ does not have chilli.

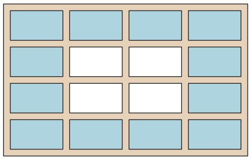
b $\frac{4}{5} \times 100\%$ does not have chilli.

$\therefore$ 80% does not have chilli.

c If 80% does not have chilli, then 20% does have chilli. $(80\% + 20\% = 100\%)$

Percentages that add up to 100% are called *complementary percentages*.

7 A window consists of 16 panes of glass as shown. The outer 12 panes are coloured blue.



a What fraction of the window is blue?

b What percentage of the window is blue?

c What percentage of the window is not blue?

8 A bus has 25 seats. 10 seats are occupied by passengers.

a What fraction of the seats are occupied?

b What percentage of the seats are occupied?

c What percentage of the seats are unoccupied?

C ONE QUANTITY AS A PERCENTAGE OF ANOTHER

Percentages are often used to **compare quantities**. It is therefore useful to be able to express one quantity as a percentage of another.

We may only compare quantities with something in common. For example, expressing "5 bicycles as a percentage of 45 cars" is *not possible*, but expressing "5 bicycles as a percentage of 50 *vehicles*" does make sense as bicycles are a type of vehicle. We say we can only compare "**like with like**".

Also, when we compare quantities we must make sure they have the **same units**.

For example, if we are asked to express "35 cm as a percentage of 7 m" we would normally convert the larger unit to the smaller one. So, we would find "35 cm as a percentage of 700 cm".

To express one quantity as a percentage of another, we first write them as a fraction and then convert the fraction to a percentage.

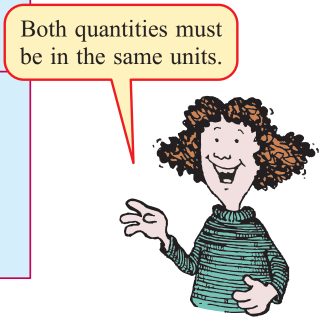
Example 6 **Self Tutor**

Express the first quantity as a percentage of the second:
45 minutes, 3 hours

$$\frac{45\text{ minutes}}{3\text{ hours}} = \frac{45\text{ minutes}}{3 \times 60\text{ minutes}} \quad \{\text{write as a fraction with the same units}\}$$

$$= \tfrac{45}{180} \times 100\% \quad \{100\% = 1\}$$

$$= 25\%$$



Example 7 **Self Tutor**

Express the first quantity as a percentage of the second:

a 2 km, 800 m

b 5.6 cm, 80 mm

a

$$\frac{2\text{ km}}{800\text{ m}} \quad \{\text{write as a fraction}\}$$

$$= \frac{2 \times 1000\text{ m}}{800\text{ m}} \quad \{\text{same units}\}$$

$$= \tfrac{2000}{800} \times 100\% \quad \{100\% = 1\}$$

$$= 250\%$$

b

$$\frac{5.6\text{ cm}}{80\text{ mm}}$$

$$= \frac{56\text{ mm}}{80\text{ mm}}$$

$$= \tfrac{56}{80} \times 100\%$$

$$= 70\%$$

EXERCISE 7C

1 Express the first quantity as a percentage of the second:

a 10 km, 50 km
b €2, €8
c 3 m, 4 m
d 300 m, 0.7 km
e 60 cents, $2
f 2 L, 400 mL
g 8 months, 2 years
h 50 g, 1.2 kg
i 5 mm, 8 cm
j 24 min, 2 hours
k 0.5 m, 25 cm
l £2.50, £1.50
m 5 mg, 2 g
n 3 weeks, 15 days
o 45 seconds, 2 min

Example 8 **Self Tutor**

Express as a percentage:

a A test mark of 17 out of a possible 25.
b Out of 1600 vehicles sold in a month, 250 of them were vans.

a

$$\begin{aligned}&\frac{17 \text{ marks}}{25 \text{ marks}}\\ &= \frac{17 \times 4}{25 \times 4}\\ &= \frac{68}{100}\\ &= 68\%\end{aligned}$$

b

$$\begin{aligned}&\frac{250 \text{ vans}}{1600 \text{ vehicles}} && \{\text{vans are vehicles, so units ok}\}\\ &= \tfrac{250}{1600} \times 100\% && \{100\% = 1\}\\ &= \tfrac{25\,000}{1600}\%\\ &= 15.625\%\end{aligned}$$

2 Express as a percentage:

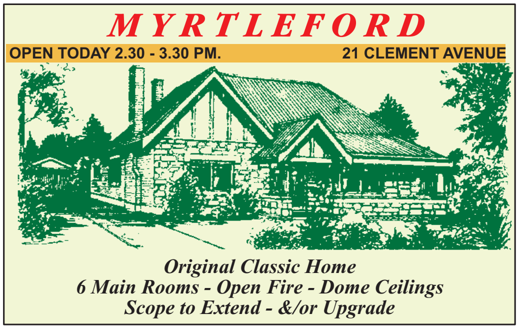
a 64 marks out of a possible 80 marks
b 23 marks out of a possible 40 marks
c 332 books sold out of a total of 500 printed
d 140 Toyota vehicles out of 800 cars sold
e an archer scores 97 out of a possible 125 points.

3 Express as a percentage:

a 24 of 60
b 220 mL of 2 L
c €1.20 of €300
d $8.50 of $2000
e 350 g of 1 kg
f 16 hours of 1 day.

4 A house was bought for £450 000 and 3 years later was valued at £540 000. Find:

a the new value as a percentage of the old value
b the increase in value over the three year period
c the increase in value as a percentage of the original value.



INVESTIGATION 1 SCORING SPREADSHEET

The spreadsheet in this investigation is used to record basketball scores. It will calculate each player's score as a percentage of the team's total score. In a basketball game, players had the following scores:

Eric Chong	23 points	Phan Nguyen	10 points	Raj Khoo	28 points
Bruce Tsang	37 points	KJ Khaw	17 points	John Foo	2 points

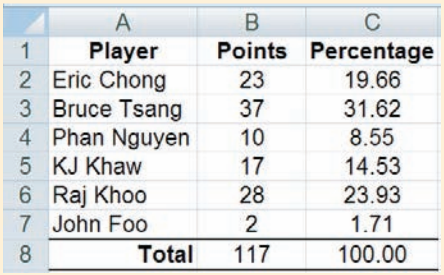
What to do:

SPREADSHEET

1. Open the spreadsheet and enter the data as shown:

	A	B	C
1	Player	Points	Percentage
2	Eric Chong	23	
3	Bruce Tsang	37	
4	Phan Nguyen	10	
5	KJ Khaw	17	
6	Raj Khoo	28	
7	John Foo	2	
8	Total		

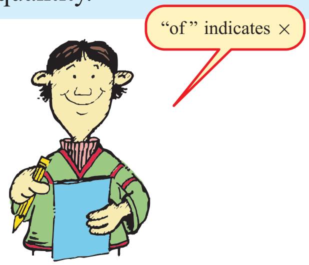
2. Find the total score by entering the formula =sum(B2:B7) in cell B8.
3. In cell C2, enter the formula =B2/B8 ∗ 100. Fill this formula down to C7. Use the 'Increase Decimal' icon ←.0 .00 on the toolbar to set the number of decimal places to 2.
4. Activate cell B8 and fill the formula across to C8. What do you notice?
5. In over-time, Eric scored an extra 7 points and John scored 4 more points. Change the points on your spreadsheet. How has each player's percentage changed?
6. Use the 'borders' ▦▾ icon, the 'bold' **B** icon, and the 'centre' ☰ icon to format your spreadsheet as shown:



	A	B	C
1	Player	Points	Percentage
2	Eric Chong	23	19.66
3	Bruce Tsang	37	31.62
4	Phan Nguyen	10	8.55
5	KJ Khaw	17	14.53
6	Raj Khoo	28	23.93
7	John Foo	2	1.71
8	Total	117	100.00

D FINDING PERCENTAGES OF QUANTITIES

To find a **percentage of a quantity**, we first convert the percentage to a **decimal**. We then **multiply** to find the required fraction **of** the quantity.



For example, 25% of \$20

$= 0.25 \times \$20 \qquad \{25\% = \frac{25}{100} = 0.25\}$

$= \$5$

Example 9 **Self Tutor**

Find: **a** 35% of \$5000 **b** 12.4% of 6 m (in cm)

a 35% of \$5000
$= 0.35 \times \$5000$ $\{35\% = \frac{35}{100} = 0.35\}$
$= \$1750$

Calculator: $\underbrace{35 \boxed{\div} 100}_{35\%} \boxed{\times} 5000 \boxed{=}$

b 12.4% of 6 m
$= 0.124 \times 600$ cm $\{12.4\% = \frac{12.4}{100} = 0.124\}$
$= 74.4$ cm

EXERCISE 7D

1 Find:

a 20% of \$36
b 36% of €4200
c 5% of 18 m (in cm)
d 125% of £600
e 22% of 1 tonne (in kg)
f 72% of 3 hours (in min)
g $7\frac{1}{2}\%$ of 12 hours (in min)
h 3.8% of 12 m (in mm)
i 95% of 5 tonnes (in tonnes)
j 108% of 5000 kg (in tonnes)

2 My percentage for mathematics last term was 85%. If my scores out of 20 marks for the first nine tests were 14, 18, 16, 16, 18, 15, 16, 20 and 19, find my score for the last test out of 20 marks.

3 An orchard produces 150 tonnes of apples in one season. There are 30 tonnes of second grade apples, 8% are unfit for sale, and the rest are first grade. If first grade apples sell for €1640 per tonne and second grade for €1250 per tonne, find the total value of the apple harvest.

E THE UNITARY METHOD IN PERCENTAGE

Suzanna's doctor has told her she is obese, and needs to lose 10% of her body weight. This means she needs to lose 9 kg. How can we find out Suzanna's current weight?

The **Unitary Method** can be used to solve this problem. We first find 1%, then multiply it by 100 to find the whole amount.

The unitary method is also used to find other percentages of a quantity.

Example 10 **Self Tutor**

a Find 100% of a quantity if 6% of the quantity is 42.

b Find 71% of a quantity if 40% of the quantity is 480.

a 6% corresponds to 42
$\therefore$ 1% corresponds to 7 {dividing by 6}
$\therefore$ 100% corresponds to 700 {multiplying by 100}

b 40% corresponds to 480
$\therefore$ 1% corresponds to 12 {dividing by 40}
$\therefore$ 71% corresponds to 852 {multiplying by 71}

EXERCISE 7E

1 Find 100% if:

a 20% is 240 mL **b** 36% is 288 g **c** 7% is $126
d 30% is 12 kg **e** 57% is €513 **f** 45% is 36 seconds

2 Find:
a 60% of a quantity if 15% of the quantity is 45
b 72% of a quantity if 8% of the quantity is 64
c 86% of a quantity if 14% of the quantity is 420
d 12% of a quantity if 75% of the quantity is 250.

3 36% of students at a school use public transport. If 144 students use public transport, how many students attend the school?

4 An alloy contains 16% zinc and the rest is pure copper. If 20 kg of zinc is used to make the alloy, how much:

a alloy is produced **b** copper is used?

5 John scored 60% for a test. If his actual score was 72, what was the maximum mark possible for the test?

6 Suzanna's doctor told her to lose 9 kg, which was 10% of her body weight.

a How much did Suzanna weigh?

b If Suzanna lost 12% of her body weight, what is her new weight?

F PERCENTAGE INCREASE OR DECREASE

There are many situations where quantities are either increased or decreased by a certain percentage.

For example:
- a worker has her salary increased by 15%
- the price of goods increases by 4% due to inflation
- a person on a diet reduces his weight by 10%
- a store has a 25% discount sale.

We will examine *two methods* for dealing with **percentage increase** or **percentage decrease**.

METHOD 1: WITH TWO STEPS

Using this method we
- find the **increase** then **add** it to the original quantity

or
- find the **decrease** then **subtract** it from the original quantity.

We see that by using this method *two steps* are necessary.

Example 11 Self Tutor

a Increase $5000 by 20%.

b Decrease $80 000 by 12%.

a

$$\begin{aligned} & 20\% \text{ of } \$5000 \\ &= 0.2 \times \$5000 \\ &= \$1000 \\ \therefore\ & \text{the new amount} \\ &= \$5000 + \$1000 \\ &= \$6000 \end{aligned}$$

b

$$\begin{aligned} & 12\% \text{ of } \$80\,000 \\ &= 0.12 \times \$80\,000 \\ &= \$9600 \\ \therefore\ & \text{the new amount} \\ &= \$80\,000 - \$9600 \\ &= \$70\,400 \end{aligned}$$

METHOD 2: WITH ONE STEP USING A "MULTIPLIER"

The original amount is 100% of the quantity.

- If we **increase** an amount by 20%, then this is **added** and in total we have $100\% + 20\% = 120\%$ of the original.
  So, to **increase** an amount by 20% in **one step**, we multiply the original by 120%.
  The value 120% or 1.2 is called the **multiplier**.
- If we **decrease** an amount by 20%, then this is **subtracted** and we are left with $100\% - 20\% = 80\%$ of the original.
  So, to **decrease** an amount by 20% in **one step**, we multiply the original by 80%.
  The value 80% or 0.8 is called the **multiplier**.

Example 12 Self Tutor

a Increase $5000 by 20%.

b Decrease $80 000 by 12%.

a To increase by 20%, we multiply by $100\% + 20\% = 120\%$

$$\begin{aligned} \therefore\ & \text{the new amount} \\ &= 120\% \text{ of } \$5000 \\ &= 1.2 \times \$5000 \\ &= \$6000 \end{aligned}$$

b To decrease by 12%, we multiply by $100\% - 12\% = 88\%$

$$\begin{aligned} \therefore\ & \text{the new amount} \\ &= 88\% \text{ of } \$80\,000 \\ &= 0.88 \times \$80\,000 \\ &= \$70\,400 \end{aligned}$$

EXERCISE 7F

1 Perform the following operations using two steps:

- **a** increase €8000 by 15%
- **b** decrease 96 kg by 5%
- **c** increase \$15 500 by 12%
- **d** decrease \$25 000 by 30%
- **e** increase 78 kg by 3%
- **f** decrease £85 by 22%.

2 Perform the following using a multiplier:

- **a** increase 800 m by 15%
- **b** increase 30 tonnes by 10%
- **c** increase £4000 by $17\frac{1}{2}\%$
- **d** decrease €4200 by 10%
- **e** decrease 9000 hectares by 15%
- **f** decrease 56 000 tonnes by 27%.

3 What is the overall effect of increasing \$100 by 10% and then decreasing the result by 10%? The answer is not \$100.

4 When decreasing a quantity by 10% in two successive years, the overall result is to decrease the original by 19%. Use multipliers to explain why this occurs.

5 Find the overall effect of increasing a quantity by 20% each year for 3 successive years.

G FINDING A PERCENTAGE CHANGE

If we increase or decrease a quantity then a **change** occurs. The change is the final amount minus the original amount.

change = final amount − original amount

For example:

- if €50 increases to €60, the change is €60 − €50 = €10
- if \$50 decreases to \$42, the change is \$42 − \$50 = −\$8

We need to be careful when we calculate the change. Always read the question carefully because the original quantity is not necessarily the first quantity mentioned.

Consider the following examples:

- *I now weigh* 65 *kg, whereas only two months ago I weighed* 59 *kg.*

  original amount = 59 kg and final amount = 65 kg

  ∴ change = final amount − original amount
  = 65 kg − 59 kg
  = 6 kg

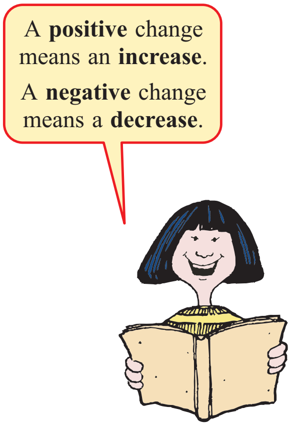
  The change is a 6 kg increase.

- *My old job paid* €15 *an hour whereas my new one only pays* €13.50 *an hour.*

  original amount = €15 and final amount = €13.50

  ∴ change = final amount − original amount
  = €13.50 − €15
  = −€1.50

  The change is a €1.50 per hour decrease.



PERCENTAGE CHANGE

If we compare the change to the original amount and express this as a percentage then we have calculated the **percentage change**.

$$\textbf{percentage change} = \frac{\textbf{change}}{\textbf{original}} \times 100\%$$

Example 13 **Self Tutor**

Find the percentage increase or decrease in the change from:

a \$120 to \$150 **b** 400 m to 250 m.

a original amount = \$120
final amount = \$150

$$\begin{aligned}\text{change} &= \text{final} - \text{original}\\ &= \$150 - \$120\\ &= \$30\end{aligned}$$

$\therefore$ percentage change

$$\begin{aligned}&= \frac{\text{change}}{\text{original}} \times 100\%\\ &= \tfrac{30}{120} \times 100\%\\ &= 25\%\end{aligned}$$

$\therefore$ there is a 25% increase.

b original amount = 400 m
final amount = 250 m

$$\begin{aligned}\text{change} &= \text{final} - \text{original}\\ &= 250\text{ m} - 400\text{ m}\\ &= -150\text{ m}\end{aligned}$$

$\therefore$ percentage change

$$\begin{aligned}&= \frac{\text{change}}{\text{original}} \times 100\%\\ &= \tfrac{-150}{400} \times 100\%\\ &= -37.5\%\end{aligned}$$

$\therefore$ there is a 37.5% decrease.

EXERCISE 7G

1 Describe the change if:

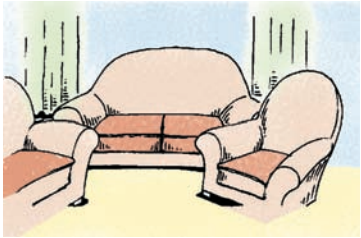
- **a** 32 cm is stretched to 40 cm
- **b** the cost drops from £136 to £119
- **c** 42 kg is decreased to 39 kg
- **d** 56 m is increased to 62 m
- **e** I now charge \$45 per hour whereas before it was \$38 per hour
- **f** my old truck carried 15 tonnes, but the new one carries 18 tonnes.

2 Find the percentage increase or decrease for the following changes:

- **a** 48 cm to 60 cm
- **b** €160 to €96
- **c** 2.5 tonnes to 4 tonnes
- **d** 1 hour to 24 minutes
- **e** 1.5 kg to 800 g
- **f** 2.4 litres to 4.2 litres

3 The average price for an apartment in a given suburb is £134 000, whereas two years ago it was £112 000. State the percentage change in price over this period.

4 During a sale, the price of a lounge suite dropped from \$1600 to \$1250. What was the percentage decrease in price?



H BUSINESS APPLICATIONS

PROFIT AND LOSS

In order to run a successful business, shopkeepers must sell their products at prices that are greater than what they paid for them. When they achieve this, they make a **profit**.

However, sometimes products must be sold at prices less than those for which they were bought. In this case they make a **loss**.

- The **cost price** is the price at which the business buys the article.
- The **selling price** is the price at which the business sells the article.
- The **profit** or **loss** is the difference between the cost price and selling price of the item.
- A **profit** occurs if the selling price is greater than the cost price.
- A **loss** occurs if the selling price is less than the cost price.

PERCENTAGE PROFIT OR LOSS

- **percentage profit** $= \dfrac{\textbf{profit}}{\textbf{cost price}} \times \textbf{100\%}$
- **percentage loss** $= \dfrac{\textbf{loss}}{\textbf{cost price}} \times \textbf{100\%}$

Example 14 **Self Tutor**

I bought a stereo system for \$500 and then sold it for \$660. Find:

a the profit or loss

b the percentage profit or loss.

a cost price = \$500
selling price = \$660
Selling price > cost price,
so a profit is made.
The profit = \$660 − \$500
= \$160

b percentage profit

$= \dfrac{\text{profit}}{\text{cost price}} \times 100\%$

$= \frac{160}{500} \times 100\%$

$= 32\%$

Example 15 **Self Tutor**

A microscope was purchased for \$600 and sold two months later for \$450.

Find the loss as a percentage of the cost price.

cost price = \$600, selling price = \$450

Selling price < cost price, so a loss is made.

The loss = \$600 − \$450 = \$150

Percentage loss $= \dfrac{\text{loss}}{\text{cost price}} \times 100\%$

$= \dfrac{150}{600} \times 100\%$

$= 25\%$

EXERCISE 7H.1

1 Copy and complete the following table:

	Cost price	*Selling price*	*Profit or loss?*	*How much profit or loss?*
a	\$45	\$60		
b	£125	£95		
c	\$255	\$199		
d	€2225	€2555		

2 Copy and complete the following table:

	Cost price	*Selling price*	*Profit or loss?*
a	\$60		\$25 profit
b		€195	€35 loss
c	£275		£95 loss
d		\$297	\$135 profit

3 For each of the following transactions, find:

i the profit or loss **ii** the percentage profit or loss.

- **a** I bought a CD set for \$50 and then sold it for \$30.
- **b** Jon bought a car for £5000 and then sold it for £6250.
- **c** Jodie bought a bicycle for €200 and then sold it for €315.
- **d** Hilda sold for \$816 a refrigerator which cost her \$680.
- **e** Frank sold for €422.50 a kitchen sink which cost him €325.

4 A car was purchased for \$28 000 and sold 2 years later for \$21 000. Find the loss as a percentage of the cost price.

5 An agent buys a refrigerator for €800 and sells it for €960. Find the profit as a percentage of the cost price.

6 A second hand car dealer purchased a motor vehicle for \$22 500. It was sold one week later for \$27 000. Find the profit made on the sale of the car, then express this profit as a percentage of the cost price.

7 A retailer buys forty pairs of running shoes for a total price of €3400. He then sells the shoes for €123.25 per pair. Calculate the total profit and express this profit as a percentage of the cost price.

8 A newsagent buys 120 magazines for £1.20 each. If only 72 of the magazines are sold for £1.95 each, determine whether the newsagent made a profit or a loss. Express the profit or loss as a percentage of the cost price.

9 Prior to Oktoberfest, a department store orders 2000 painted beer steins for €4.60 each. In the first week it sells 1450 of the steins for €12.20 each, and in the second week the rest are placed on special at €8.85. At the end of the second week 50 steins remain unsold. Determine the total profit, and express this profit as a percentage of the cost price.

Example 16 **Self Tutor**

Find the selling price for goods purchased for \$150 and sold at a 20% profit.

For a 20% profit we must increase \$150 by 20%.

Two step method:

$$\begin{aligned}\text{Profit} &= 20\% \text{ of } \$150\\ &= 0.2 \times \$150\\ &= \$30\end{aligned}$$

So, the selling price

$$\begin{aligned}&= \$150 + \$30\\ &= \$180\end{aligned}$$

or

Using the multiplier:

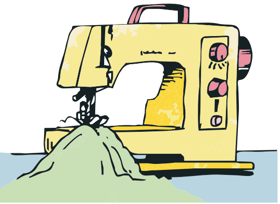
To increase by 20% we multiply by 120%.

$$\begin{aligned}\text{So, the selling price } &= 120\% \text{ of } \$150\\ &= 1.20 \times \$150\\ &= \$180\end{aligned}$$

10 Find the selling price for goods bought for:

- **a** £500 and sold for a 20% profit
- **b** \$350 and sold at a 25% loss
- **c** £4500 and sold for a profit of 8%
- **d** €8000 and sold at a loss of 35%

11 Jacki bought a sewing machine for \$560 and sold it for a profit of 18%. At what price did she sell the sewing machine?



DISCOUNT

In order to attract customers or to clear old stock, many businesses reduce the price of items from the **marked price** shown on the price tag.

The amount of money by which the cost of the item is reduced is called **discount**. Discount is often stated as a percentage of the marked price or original selling price. It is thus a **percentage decrease**.

The price at which the item is sold is called the **selling price**.

selling price = marked price − discount

Example 17 **Self Tutor**

If the marked price of a video projector is \$1060 and a 22% discount is offered, find the actual selling price.

We have to decrease \$1060 by 22%.

To decrease by 22% we multiply by $100\% - 22\% = 78\%$.

So, selling price $= 78\%$ of marked price
$= 78\%$ of \$1060
$= 0.78 \times \$1060$ {"of" indicates × }
$= \$826.80$

EXERCISE 7H.2

1 **a** If the marked price of a golf set is €1300 and a 12% discount is offered, find the actual selling price.

b A furniture distributor advertises kitchen tables at a marked price of €480. It offers a 15% discount for the first 20 customers. What is the actual selling price if you are the first customer?

2 Copy and complete the following table:

	Marked price	*Discount*	*Selling Price*	*Discount as % of marked price*
a	€125	€25		
b	£240			26%
c	€1.85			20%
d		55 cents	\$2.45	
e	\$142		\$127	
f		¥600		10%

I SIMPLE INTEREST

When a person **borrows** money from a lending institution such as a bank or a finance company, the borrower must repay the loan in full, and also pay an additional charge which is called **interest**.

If the charge is calculated each year or **per annum**, as a fixed percentage of the **original amount** of money borrowed, then the charge is called **simple interest**.

Consider the following example:

€4000 is borrowed for 3 years at 10% per annum simple interest.

The simple interest charge for 1 year is 10% of €4000 = €400.

The simple interest for 3 years is €400 × 3 = €1200,

and so the borrower must repay €4000 + €1200 = €5200.

Example 18 🔊 **Self Tutor**

Find the simple interest payable on a loan of $60 000 borrowed at 9% p.a. for:

a 4 years **b** 5 months

$$\begin{aligned}\text{The simple interest charge for 1 year} &= 9\% \text{ of } \$60\,000\\ &= 0.09 \times \$60\,000\\ &= \$5400\end{aligned}$$

a The simple interest for 4 years
$= \$5400 \times 4$
$= \$21\,600$

b 5 months is $\frac{5}{12}$ of 1 year
so simple interest for 5 months
$= \frac{5}{12} \times \$5400$
$= \$2250$

Example 19 🔊 **Self Tutor**

Find the total amount needed to repay a loan of $40 000 at 9% p.a. simple interest over 5 years.

$$\begin{aligned}\text{The simple interest charge for 1 year} &= 9\% \text{ of } \$40\,000\\ &= 0.09 \times \$40\,000\\ &= \$3600\\ \therefore \quad \text{the simple interest charge for 5 years} &= \$3600 \times 5\\ &= \$18\,000\\ \therefore \quad \text{the total to be repaid} &= \$40\,000 + \$18\,000\\ &= \$58\,000\end{aligned}$$

EXERCISE 7I

1 Find the simple interest charged when:

- **a** $5000 is borrowed for 1 year at 12% per annum simple interest
- **b** £2500 is borrowed for 2 years at 8% p.a. simple interest
- **c** €40 000 is borrowed for 6 months at 11% p.a. simple interest
- **d** $250 000 is borrowed for 9 months at 20% p.a. simple interest.

2 Find the total amount needed to repay a loan of:

- **a** €2400 for 3 years at 10% p.a. simple interest
- **b** $8000 for 18 months at 12% p.a. simple interest
- **c** £7500 for $2\frac{1}{2}$ years at 8% p.a. simple interest
- **d** $23 000 for 4 months at 15% p.a. simple interest

3 Kyle borrows $25 000 at 6% p.a. simple interest for four years. Calculate the monthly repayment required to pay this loan off in 48 equal instalments.

ACTIVITY SPEAKER PRICES

SPEAKER SPECIALS

MASUDA MINI 4" 2 WAY SPEAKERS
Comes with wall mounting brackets.
\$129 **SAVE \$70**

PETERSON SPEAKERS
100 watt 20 cm 3 way speakers.
\$200 **SAVE \$99**

INFINITY 125 WATT SPEAKERS
125 watt studio monitor speaker with high output polycall speaker. 5 year warranty.
\$745 **SAVE \$54**

What to do:

1 For each of the items in the "Speaker Specials" shown, find the:

a original price **b** discount price **c** percentage discount.

2 Tim bought a pair of 'Infinity' speakers. He paid 20% of the price from his savings and borrowed the remainder at 12% p.a. simple interest for two years. Find the:

a interest payable **b** total price Tim will pay for the speakers.

INVESTIGATION 2 THE OPENING PROBLEM

What to do:

1 Set up a spreadsheet of the premiership table in the **Opening Problem** on page **136** but *do not* type in the percentage column.

SPREADSHEET

2 The formula for calculating the percentage is:
percentage = goals for ÷ (goals for + goals against) × 100
Use this formula to create your own percentage column correct to one decimal place.

3 The results of the matches played in round 5 are:
Lions 2 defeated Falcons 0; Eagles 2 drew with Central 2;
West 0 lost to South 1; East 4 drew with North 4.
Enter this information on your spreadsheet.
For example, the Falcons row should now read:

	Played	Won	Drawn	Lost	Goals for	Goals against	Percentage	Points
Falcons	5	3	1	1	7	5	58.3	10

4 Produce a new up-to-date premiership table with the teams in correct rank order. There is an easy way of doing this without having to retype the table. Find out what it is.

KEY WORDS USED IN THIS CHAPTER

- cost price
- final amount
- per annum
- selling price
- decimal
- loss
- percentage
- simple interest
- denominator
- multiplier
- percentage change
- unitary method
- discount
- original amount
- profit

LINKS
click here

ELECTIONS

Areas of interaction:
Approaches to learning

REVIEW SET 7A

1 Write as a percentage:

a 0.6 **b** $\frac{1}{2}$ **c** 0.01 **d** 2

2 **a** Write 107% as a decimal. **b** Write 44% as a fraction in lowest terms.

c Write $\frac{4}{25}$ with denominator 100 and hence express it as a percentage.

d Convert 0.35 to a percentage.

e Find 72% of a quantity if 11% of the quantity is 550.

f Find 40% of €2000. **g** Express 9 hours as a percentage of one day.

3 **a** Increase £300 by 12%. **b** Decrease 460 tonnes by 25%.

4 In a school of 800 students, 63% have black hair, 19% have brown hair, and the remainder have fair hair. Find:

a the percentage of fair-haired students **b** how many students have brown hair.

5 I bought a bicycle for $640 and sold it for a profit of 20%. Find the selling price of the bicycle.

6 A greengrocer bought 200 kg of oranges for $180 and sold them for $1.14 per kilogram. Find: **a** his profit **b** his percentage profit on the cost price.

7 Find the simple interest when €4500 is borrowed for 3 years at 14% p.a.

REVIEW SET 7B

1 **a** Convert 0.085 to a percentage. **b** Convert $\frac{7}{40}$ to a percentage.

c Express 72 cents as a percentage of $1.80. **d** Find 65% of €6.00.

e Convert 2.5% to a decimal. **f** Decrease 250 by 20%.

g Find the percentage increase when 60 cm is extended to 84 cm.

h Find 70% of a quantity if 15% of the quantity is 45.

2 A house was bought for $385 000 and sold for a profit of 10%. For how much was it sold?

3 Find the percentage profit when a car bought for £7000 is sold for £8470.

4 If 45% of a person's mass is 36 kg, what is the person's total mass?

5 A dress was bought for €160 and sold for €136. Find the percentage loss on the cost price.

6 A washing machine has a selling price of £420, but a 15% discount is given. Find the actual price paid for the washing machine.

7 30% of a farmer's crop was barley and the rest was wheat. If he planted 750 hectares with barley, what area was planted with wheat?

8 Find the total amount of money that needs to be repaid on a loan of $6000 for 4 years at 9% p.a. simple interest.

PUZZLE MATHWORD MAZES

In the following mazes we start at the arrow marked **in** and follow the letters vertically or horizontally (but not diagonally) to the arrow marked **out**. Our path is made up of mathematical words. All letters are used and no letter is used more than once.

Write the words you discover in the blank squares provided in the same order as you find them. This will help you see the length of the next word to be found.

Problem 1: This problem is partially solved for you.

in →	D	E	M	E	T	R	
	R	G	N	U	S	E	
	E	G	O	M	L	E	
	E	A	H	T	G	N	
	P	T	P	A	L	L	→ out
	E	N	A	R	L	E	

D	E	G	R	E	E		
P	E	N	T	A	G	O	N
M	E	T	R	E			

Problem 2: Complete the Mathword maze.

in →	C	A	P	O	M	V	O	H	E	X	
	I	E	R	L	A	R	L	E	C	A	
	R	C	A	Y	G	G	U	M	A	G	
	E	L	N	N	O	O	L	E	F	O	
	C	V	E	I	O	D	E	L	L	N	→ out
	O	E	T	N	R	D	U	A	A	R	
	N	E	E	G	E	E	Q	L	P	A	

Make up a Mathword maze of your own.

Chapter 8

Algebra: Expressions and evaluation

Contents:

In this chapter we discover how to build up algebraic expressions. We will use **symbols** to represent numbers of physical quantities.

OPENING PROBLEM

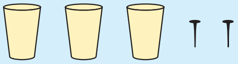
Cassi picks S strawberries and places them into n punnets, which are small open boxes. She places the same number of strawberries in each punnet and there are 4 left over. How can we write this information in algebraic form?

A BUILDING EXPRESSIONS

Suppose we have some cups  and some nails . We can place a certain number of nails into each cup, then write expressions for the total number of nails present.

Example 1 **Self Tutor**

Suppose we have 3 cups with nails in them plus 2 nails left over.



Find how many nails are present if:

a Lucy has put 5 nails into each cup **b** Li has put 8 nails into each cup

c Larry has put c nails into each cup.

a If Lucy has put 5 nails into each cup, then there are $3 \times 5 + 2$ nails.

b If Li has put 8 nails into each cup, then there are $3 \times 8 + 2$ nails.

c If Larry has put c nails into each cup, then there are $3 \times c + 2 = 3c + 2$ nails.

In the following exercise you should find that:

- Algebraic expressions can be **evaluated** or have their values calculated if the value of the variable is known.
- Algebraic expressions with the same value can be written in different ways, and therefore at least one of them can be **simplified**.

EXERCISE 8A.1

1 Suppose we have 2 cups with nails in them plus 5 nails left over.

Find how many nails are present if Kelly has put in each cup:

a 3 nails **b** 4 nails **c** b nails **d** p nails.

2 Suppose we have 4 cups with nails in them plus 3 nails left over.

Find how many nails are present if Mike has put in each cup:

a 2 nails **b** 5 nails **c** a nails **d** n nails.

3 Suppose we have b bags which contain some apples. Find how many apples are present if each bag contains:

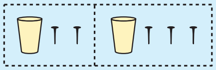
a x apples and there are 3 left over **b** y apples and there are 4 left over

c t apples and there are 7 left over **d** m apples and there are n left over.

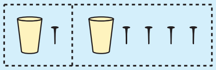
Example 2 **Self Tutor**

Suppose we have two cups each containing c nails. Represent the following groupings algebraically:

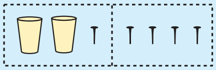
a



b



c



What equal expressions can be made from these different groupings?

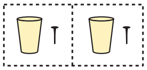
a $(c+2)+(c+3)$ **b** $(c+1)+(c+4)$ **c** $(2c+1)+4$

As the total number of nails in each case is the same,

$(c+2)+(c+3)=(c+1)+(c+4)=(2c+1)+4=2c+5.$

4 **a** Write algebraic expressions for the following, assuming there are n nails in each cup:

i



ii

iii

b Write down a set of equal expressions to represent the situation in **a**.

5 For each of the following diagrams find *three* different ways of representing the total number of nails present. Assume that there are c nails in each cup.

a

b

c

6 Without drawing pictures of cups and nails, which of these are true statements?

a $(c+3)+(c+2)=2c+5$ **b** $(2c+1)+(c+4)=3c+5$

c $(c+4)+(c+4)=2c+8$ **d** $3(c+2)=3c+5$

7 Suppose we have 4 cups and 7 nails:

a If there are n nails in each cup, what expression represents the total number of nails?

b Find the total number of nails present if: **i** $n=3$ **ii** $n=10$ **iii** $n=25$.

Example 3 **Self Tutor**

Evaluate $3c+7$ for: **a** $c=4$ **b** $c=10$

a If $c=4$ then

$3c+7$

$=3\times4+7$

$=12+7$

$=19$

b If $c=10$ then

$3c+7$

$=3\times10+7$

$=30+7$

$=37$

8 Evaluate:

a $2w+5$ when $w=4$

b $3z+4$ when $z=3$

c $5y-6$ when $y=2$

d $2y-8$ when $y=10$

e $17-4a$ when $a=1$

f $12-4b$ when $b=3$

g $3(k+2)$ when $k=5$

h $4(h-5)$ when $h=9$

i $2x+7$ when $x=30$

j $5y-6$ when $y=4$

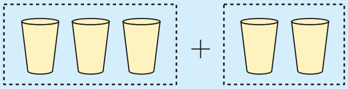
Example 4 **Self Tutor**

By drawing diagrams, write simpler algebraic expressions for:

a $3c+2c$ **b** $3c+2+2c+4$ **c** $2(2c+3)$

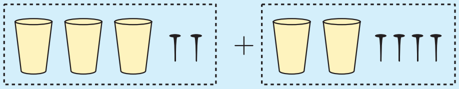
We let each cup contain c nails.

a $3c+2c$ can be represented by:



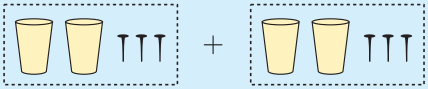
In total there are 5 cups of c nails, so $3c+2c=5c$.

b $3c+2+2c+4$ can be represented by:



In total there are 5 cups of c nails plus another 6 nails, so $3c+2+2c+4=5c+6$

c $2(2c+3)$ is 2 lots of $(2c+3)$ or $2c+3+2c+3$



In total there are 4 cups of c nails plus another 6 nails, so $2(2c+3)=4c+6$

9 By drawing diagrams, write simpler algebraic expressions for:

a $2c+c$

b $5c+2c$

c $c+c+2$

d $2c+1+c+3$

e $2(c+4)$

f $c+1+c+2$

g $3(c+2)$

h $4(c+1)$

Example 5

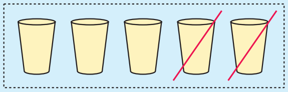
Self Tutor

By drawing diagrams, write simpler algebraic expressions for:

a $5c - 2c$ **b** $4c + 3 - 2c - 1$ **c** $(3c + 4) - (c + 1)$

Let c represent the number of nails in a cup.

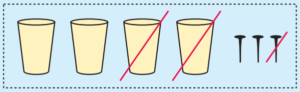
a $5c - 2c$ is



which is $3c$.

So, $5c - 2c = 3c$.

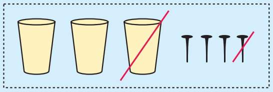
b $4c + 3 - 2c - 1$ is



which is $2c + 2$.

So, $4c + 3 - 2c - 1 = 2c + 2$.

c $(3c + 4) - (c + 1)$ is



which is $2c + 3$.

So, $(3c + 4) - (c + 1) = 2c + 3$

10 By drawing diagrams if necessary, write simpler algebraic expressions for:

a $4c - 3c$ **b** $5c - c$ **c** $3c - 3c$

d $5c + 3 - 2c$ **e** $4c + 5 - c - 1$ **f** $3c + 5 - 2c - 3$

g $(4c + 3) - (c + 2)$ **h** $(3c + 4) - (2c + 3)$ **i** $(7c + 4) - (3c + 2)$

11 Match equivalent expressions in the following:

a $3(p + 4)$ **b** $7p$ **A** $3p + 4p$ **B** $2(p + 5)$

c $2(3 + 4p)$ **d** $p + 5 + p + 5$ **C** $7p + 7$ **D** $8p + 6$

e $6p + 6$ **f** $7(p + 1)$ **E** $3p + 12$ **F** $p + 2 + 5p + 4$

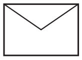
EXPRESSIONS WITH TWO VARIABLES

So far we have considered expressions containing only one unknown quantity or **variable**.

We now consider expressions with two different variables. In order for expressions to be equal for *all* values of the variables, the numbers of each of the variables must be the same in both expressions.

In the following drawings: ● represents 1 button.

represents c buttons in a cup



represents e buttons in an envelope.

Example 6 **Self Tutor**

Write algebraically what the following represents:

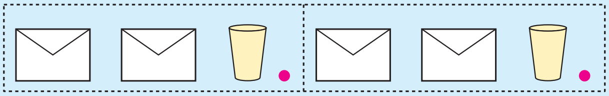
The number of buttons can be represented algebraically as $2e + c + 3$.

Example 7 **Self Tutor**

Illustrate the following pairs of algebraic expressions. What can you conclude?

$$2(2e + c + 1) \quad \text{and} \quad 4e + 2c + 2$$

$2(2e + c + 1)$ means two lots of $2e + c + 1$.



$4e + 2c + 2$ means:

The numbers of buttons and each type of container is the same in each case, so no matter what the values of c and e are, the total number of buttons must be the same.

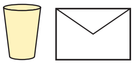
We can hence write $2(2e + c + 1) = 4e + 2c + 2$

EXERCISE 8A.2

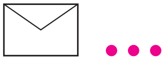
1 Write algebraically what the following represent:

a

b

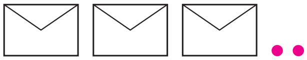


c

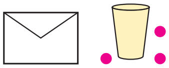


d

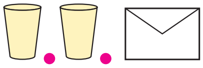
e



f



g



2 Illustrate the following pairs of algebraic expressions. What can you conclude?

a $3c + 3e$ and $3(c + e)$

b $2c + 3 + e$ and $3 + 2c + e$

c $2(c + e + 1)$ and $2c + 2e + 2$

d $2(c + 1) + 3(e + 2)$ and $2c + 3e + 8$

B KEY WORDS IN ALGEBRA

Before we look further into algebra, we need to define some *key words*:

A **numeral** is a symbol used to represent a known number.

For example: 5, 0, -7 and $\frac{2}{3}$ are all numerals.
the number $\pi \approx 3.141592\,654$ is also a numeral.

A **variable** is an unknown quantity which we represent by a letter or **pronumeral**.

For example: the *number of carrots in my garden* is represented by c
the *speed of a cyclist* is represented by s.

An **expression** is an algebraic form consisting of numerals, variables, and operation signs such as $+$, $-$, $\times$, $\div$ and $\sqrt{}$.

For example: $2x + 5$ and $-3(2x - 1)$

An **equation** is an algebraic form containing an $=$ sign.

For example: $3x - 7 = 8$ and $\dfrac{x}{3} = 10$

The **terms** of an expression or equation are the algebraic forms separated by $+$ and $-$ signs, the signs being included.

For example: the terms of $3x + 2y + 8$ are $3x$, $2y$, and 8.
the terms of $2x - 3y - 5$ are $2x$, $-3y$, and -5.

Like terms are terms with exactly the same variable form.

For example: $3x$ and $7x$ are like terms.
8 and 7 are like terms.
$7x$ and 7 are unlike terms.
x and x^2 are unlike terms.
x and xy are unlike terms.

The **constant term** of an expression is the term which does not contain a variable.

For example: the constant term in $5x + 6$ is 6
the constant term in $-7 + 3x^2$ is -7
there is no constant term in $x^2 + x$.

The **coefficient** of any term is its numerical part, including its sign.

For example: the coefficient of p in $2p + 4$ is 2
the coefficient of r in $7 - 6r$ is -6
the coefficient of x^2 in $x^2 + x$ is 1 since x^2 is $1 \times x^2$.

EXERCISE 8B

1 Are the following statements true or false? Correct the statements which are false.

a $\dfrac{3a + b}{2}$ is an equation.

b $\dfrac{a}{b} + c$ is an expression.

c $2x + 3 = 5$ is an equation.

d $2x + 3 - 8y$ is an equation.

e $2x + 4 - 3y$ has 3 terms.

f The coefficient of x in $2y - 5x + 8$ is 5.

g The constant term in $5x - 11y - 6$ is 6.

2 Write down the coefficient of x in each of the following:

- **a** $4x$
- **b** $6+7x$
- **c** $5y-2+2x$
- **d** $3x-2y+4$
- **e** $5+2y+x$
- **f** $-2x+y-6$
- **g** $x-5$
- **h** $5-x$
- **i** $8-3x$

3 State the number of terms in each expression in question **2**.

4 Consider the expression $4x+6y-5+x$.

- **a** How many terms are there in this expression?
- **b** What is the constant term?
- **c** What is the coefficient of the second term?
- **d** What are two like terms in this expression?

5 State the like terms in:

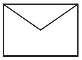
- **a** $3x+8+6x+4$
- **b** $x+2y+5x-y$
- **c** $3x+y+x+2y+5$
- **d** $2+t+6+4t$
- **e** $6b+ab+a$
- **f** $6+ab+a+2ab$

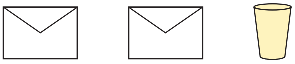
C SIMPLIFYING EXPRESSIONS

Consider again the pictures of containers and things we used previously:


represents 1 button

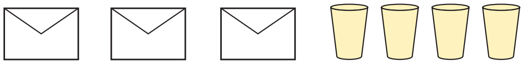

represents c buttons in a cup


represents e buttons in an envelope

In this case  represents $2e+c$ buttons

and  represents $e+3c$ buttons.

Adding these items together gives 

which can be rearranged to give 

Using symbols this means $2e+c+e+3c=3e+4c$.

The algebraic expression on the left has been **simplified** by collecting together the symbols which were the same. This is called **simplifying by collecting like terms**.

$2e$ and e are like terms that add to $3e$,

and c and $3c$ are like terms that add to $4c$.

We say we have
'3 lots of e'
and '4 lots of c'.

Example 8 **Self Tutor**

Simplify:

a $c+c+c+d+d$ **b** $(n+n)-(m+m+m)$

a $c+c+c+d+d$
$= 3c+2d$

b $(n+n)-(m+m+m)$
$= 2n-3m$

Example 9 **Self Tutor**

Simplify by collecting like terms:

a $4x+3x$ **b** $3x-2x$ **c** $8x-x$ **d** $3x+2$

a $4x+3x=7x$ $\{4x$ and $3x$ are like terms$\}$

b $3x-2x=1x=x$ $\{3x$ and $-2x$ are like terms$\}$

c $8x-x=7x$ $\{8x-x$ is really $8x-1x\}$

d $3x+2$ cannot be simplified as $3x$ and 2 are not like terms.

EXERCISE 8C

1 Simplify:

a $x+x$ **b** $c+c+c$ **c** $a+a+b+b$

d $a+a+a+b$ **e** $3+x+x+y$ **f** $a+a+b+b+b$

g $g+g+2+g$ **h** $3-(a+a)$ **i** $y+y+y+y-4$

j $6-(b+b+b)$ **k** $4+t+t+s+s+s$ **l** $2\times(m+m)$

2 If possible, simplify by collecting like terms:

a $a+3a$ **b** $2x+3x$ **c** $a+b$ **d** $2x+y$

e $2a+7a$ **f** $8d-3d$ **g** $4x+3$ **h** $6m-m$

i $6n-5n$ **j** $3x-7$ **k** $3a-3a$ **l** $x+x+2$

m $x+x-2$ **n** $a+a-a$ **o** $4n+4m$ **p** $13d-3d$

q $13d-3$ **r** $p+p+p$ **s** $a+a+a+a$ **t** $4x+3x+6x$

Example 10 **Self Tutor**

Simplify by collecting like terms:

a $2x-4x$ **b** $-3x-x$ **c** $3x+x-7x$

a $2x-4x$
$= -2x$

b $-3x-x$
$= -4x$

c $3x+x-7x$
$= 4x-7x$
$= -3x$

3 If possible, simplify:

a $2x-5x$
b $3a-5a$
c $2d-2$
d $n-2n$
e $x+x-2x$
f $3a+a-7a$
g $-a-a-a$
h $-2x-2x-3x$
i $2x+3x-5$
j $4x+2xy$
k $5x-10x$
l $5x-10x+5$
m $-12x-8$
n $-12x-8x$
o $-b-3b$
p $-5m+2m$
q $-2a+5a$
r $2a-5a$
s $3x+2x-x$
t $3x+2x-5$

Example 11 **Self Tutor**

Simplify by collecting like terms:

a $3a+b+4a+2b+1$ **b** $3a+6b-a-2b$

a
$$\begin{aligned} &3a+b+4a+2b+1 \\ =\ &3a+4a+b+2b+1 \\ =\ &7a+3b+1 \end{aligned}$$

b
$$\begin{aligned} &3a+6b-a-2b \\ =\ &3a-a+6b-2b \\ =\ &2a+4b \end{aligned}$$

4 If possible, simplify by collecting like terms:

a $a+3a+2b+4b$
b $2x+3x+4p+p$
c $5g+3h+3g+2h$
d $6x+5y+2x+y$
e $a+b-a-b$
f $2p+3q-p+5$
g $5x+4z-x-2z$
h $2y+1+3y+3$
i $6x-4y+2x+8y$
j $4m+4+p$
k $8a+8b+8c$
l $7x-11y+4x-y$

5 Give one of the following responses to the questions below:

A always **B** never **C** sometimes.

In each case explain your answer.

a Could $3x+5=3y+5$?
b Could $3(x+2y)=3x+6y$?
c Could $x+y=x+y+4$?
d Could $y-x=x-y$?
e Could $2x+3y=3+2x$?
f Could $4x+3y-4=3y$?

6 Pat says that $4-(p+p+p)$ is the same as $4-p+p+p$.

a By replacing p by 3 in both expressions, show that Pat is wrong.
b Simplify each expression algebraically.

D ALGEBRAIC PRODUCTS

PRODUCT NOTATION

To simplify written expressions, mathematicians have agreed to follow several rules when writing algebraic products.

> Except when confusion may arise, we agree to leave out × signs between multiplied quantities.

For example, instead of $2 \times a$ or $a \times 2$ we write $2a$.

Notice that the numeral 2 is written before the variable.

In a product where there are two or more variables, we agree to write them in **alphabetical order**.

For example, $2 \times b \times a$ is written as $2ab$ and $d \times 3 \times c$ is written as $3cd$.

Example 12 **Self Tutor**

Write in product notation:

a $a \times b$ **b** $b \times 3a$ **c** $2 \times a + b \times 3$ **d** $2 \times (a + b)$

a $a \times b$
$= ab$

b $b \times 3a$
$= 3ab$

c $2 \times a + b \times 3$
$= 2a + 3b$

d $2 \times (a + b)$
$= 2(a + b)$

INDEX NOTATION

Just as $2 \times 2 \times 2 \times 2 = 2^4$, we write $a \times a \times a \times a = a^4$.

In this case, a is the base and 4 is the index or power or exponent.

Example 13 **Self Tutor**

Simplify using index notation:

a $2 \times a \times a \times a \times b \times b$ **b** $m \times m - 5 \times n \times n$

a $2 \times a \times a \times a \times b \times b$
$= 2a^3b^2$

b $m \times m - 5 \times n \times n$
$= m^2 - 5n^2$

When a product is written out in full, we say it is in **expanded form**.

For example: a^2b is in index form. $a \times a \times b$ is in expanded form.

EXERCISE 8D

1 Simplify using product notation:

a $5 \times d$
b $d \times 5$
c $a \times 3$
d $5 \times 2a$
e $6 \times m$
f $a \times 6b$
g $7c \times 3$
h $3c \times 2d$
i $p \times q \times 7$
j $a \times b \times c$
k $d \times b \times h$
l $a \times b + 3$

2 Simplify using product notation:

a $a \times c + b$
b $2 \times a + 3 \times b$
c $a \times b - c$
d $a - b \times c$
e $b - c \times 2$
f $a \times b + c \times d$
g $6 - b \times c \times 3$
h $5 \times (a + 2)$
i $3 \times (k + 2)$
j $(a + 4) \times 7$
k $(b - a) \times 4$
l $a \times b \times (c + 2)$

3 Write in expanded form:

a a^2
b b^4
c $2n^2$
d $3m^3$
e $11m^2n$
f $7ab^3$
g $(2a)^2$
h $(3b)^3$
i $a^3 + 2b^2$
j $2d^2 - 4n^3$

4 Write in simplest form:

a $2 \times p \times p$
b $t \times t \times t$
c $c \times c \times c \times 5$
d $2 \times a \times b \times b \times b$
e $p \times p + q$
f $2 \times r \times r \times s \times s$
g $p \times p \times q \times q \times r$
h $3 \times a \times 3 \times a$
i $5 \times x \times x \times 5 \times x$
j $a + a \times a$
k $x \times x \times x + 2$
l $a \times a - 2 \times a$
m $2 \times a \times a \times a + b$
n $6 \times a - 2 \times b \times b$
o $2 \times a \times b + c \times c \times c$
p $s \times t \times t - 3 \times t$
q $3 \times b + n \times n \times 4$
r $3 \times r \times r \times r + 4 \times r$
s $3 \times a + a + a + a$
t $3 \times a + a + a \times a$
u $3 \times (a + a + a) \times a$

5 Write in simplest form:

a $2x \times x$
b $x \times 2x$
c $2x \times 5x$
d $5x \times 2x$
e $3x + 4x$
f $3x \times 4x$
g $4x \times 3x$
h $4x - 3x$
i $x^2 \times 2$
j $2 \times x^2$
k $2x \times -5$
l $-4 \times 5x$
m $-2x \times -3x$
n $-5x \times -4x$
o $5x^2 \times -3$
p $-3x^2 \times 4x$

E EVALUATING ALGEBRAIC EXPRESSIONS

Algebraic expressions containing two or more variables can also be **evaluated** if the variables are assigned values.

To **evaluate** an algebraic expression, we **substitute** the given numbers for the variables and then calculate the result.

Example 14 **Self Tutor**

If $x = 3$ and $y = 2$, evaluate: **a** $2x - y$ **b** $3(3y + 5x)$

a
$$\begin{aligned} &2x - y \\ &= 2 \times 3 - 2 \quad \{\text{substituting}\} \\ &= 6 - 2 \\ &= 4 \end{aligned}$$

b
$$\begin{aligned} &3(3y + 5x) \\ &= 3(3 \times 2 + 5 \times 3) \\ &= 3(6 + 15) \\ &= 3 \times 21 \\ &= 63 \end{aligned}$$

NEGATIVE SUBSTITUTION

Variables do not always take positive values. They can also be assigned negative values. To avoid confusion with signs, we usually write negative substitutions in brackets.

Example 15 **Self Tutor**

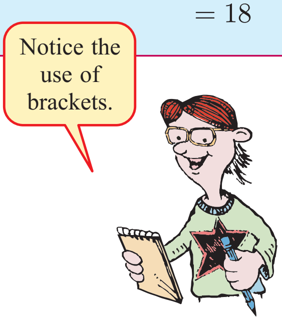
If $x = 5$ and $y = -4$, find the value of:

a $2x + 3y$ **b** $3x - 2y$ **c** $2(x - y)$

a $$\begin{aligned} &2x + 3y \\ &= 2 \times 5 + 3 \times (-4) \\ &= 10 + -12 \\ &= 10 - 12 \\ &= -2 \end{aligned}$$

b $$\begin{aligned} &3x - 2y \\ &= 3 \times 5 - 2 \times (-4) \\ &= 15 - -8 \\ &= 15 + 8 \\ &= 23 \end{aligned}$$

c $$\begin{aligned} &2(x - y) \\ &= 2(5 - (-4)) \\ &= 2(5 + 4) \\ &= 2 \times 9 \\ &= 18 \end{aligned}$$



EXERCISE 8E

1 If $x = 5$ and $y = 6$, find the values of the following expressions:

a $4x$ **b** $x + 2y$ **c** $2(x + y)$
d $3y - 3x$ **e** $3(x - y)$ **f** $3x - y$
g $2(5x - 2y)$ **h** $5(y - x) + 2$ **i** $2(3x - 2y)$
j $2y - 5$ **k** $16 - 2x$ **l** $5 + 2y - 3x$

2 If $a = 3$, $b = 2$ and $c = 5$, find the value of:

a $a + b$ **b** $2a$ **c** c^2 **d** bc
e $c - b$ **f** $b - c$ **g** $2ac$ **h** $2a^2$
i $3bc$ **j** $4ab^2$ **k** $a(b + c)$ **l** $ab + ac$

3 If $m = 4$, $n = 2$, $g = 0$ and $h = 5$, evaluate:

a $3m + n$ **b** $m + 3n$ **c** $3(m + n)$ **d** $3m + 3n$
e m^2 **f** $2m^2$ **g** $(2m)^2$ **h** $mn - gh$
i $3n$ **j** $(3n)^2$ **k** $2n^2$ **l** $2g - 6$
m $gm - nh$ **n** $5n^3$ **o** hn^2 **p** $(hn)^2$

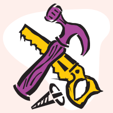
4 Given $p = 2$, $q = -3$, $r = -1$, and $s = -5$, evaluate:

a q^2 **b** $2pq$ **c** $3p^2$ **d** r^3
e qrs **f** $q^2 + s$ **g** $s - q^2$ **h** $pq + rs$
i $p^2 - q^2$ **j** $p + q^2$ **k** $(p + q)^2$ **l** $2p - 3qs$

5 If $a = 3$, $b = 6$, and $c = -7$, find the value of:

a $2a - b$ **b** $\dfrac{b}{a}$ **c** $a - c$ **d** ac
e $2(b - a)$ **f** $c(a - b)$ **g** c^2 **h** $a^2 - b^2$

ACTIVITY "THINK OF A NUMBER" GAMES



What to do:

- Play in pairs - player A and player B
- Player A chooses a number while player B calls out the steps in each game.
- Choose a few different values in each game and take it in turns to be player A or B.
- Discuss the results. Letting the number be x, write an algebraic expression to describe the steps in each game.

Game 1

Step 1: Think of a number.
Step 2: Double it.
Step 3: Add 7.
Step 4: Take away 1.
Step 5: Divide by 2.
Step 6: Subtract the original number.
Step 7: State the output.

Game 2

Step 1: Think of a number.
Step 2: Add nine.
Step 3: Multiply by two.
Step 4: Subtract eighteen.
Step 5: Divide by two.
Step 6: Subtract the original number.
Step 7: State the output.

Game 3

Step 1: Think of a number.
Step 2: Add one.
Step 3: Multiply by two.
Step 4: Subtract the original number.
Step 5: State the output.

Game 4

a *Step 1:* Think of a number.
Step 2: Add four.
Step 3: Multiply by three.
Step 4: State the output.

b *Step 1:* Use the same number as **a**.
Step 2: Multiply by three.
Step 3: Add four.
Step 4: State the output.

KEY WORDS USED IN THIS CHAPTER

- coefficient
- expanded form
- numeral
- constant term
- expression
- pronumeral
- equation
- like terms
- variable

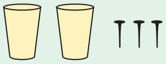
LINKS
click here

BARYCENTRES IN SPACE

Areas of interaction:
Human ingenuity

REVIEW SET 8A

1 Suppose we have 2 cups with nails in them plus 3 nails left over.



Find how many nails are present if Alex has put in each cup:

a 2 nails **b** 5 nails **c** n nails.

2 Simplify:

a $4c - 2c$ **b** $2a + 5 + 5a$ **c** $12p - 4 - 7p$ **d** $9y - y + 6 + 2y$

3 How many terms are in the expression $2x^3 - 6x + 2y$?

4 What is the coefficient of x^3 in the expression $7 - 3x + 4x^3$?

5 Find like terms in the following:

a $3x^2 - x + 6 + x^2 + 3x$ **b** $2a - 3b + 4 - a + 1$

c $2c - c^2 + 3c + 1$ **d** $2c + cd - 4c + dc - 4d$

6 If possible, simplify by collecting like terms:

a $5x - 10y + 6x$ **b** $5 - m + 5r - m$

c $6ac + 6a - 4c$ **d** $7e - 8f - 2e + 5f$

7 Simplify using product notation:

a $8x \times 5$ **b** $(a - c) \times 7$

8 Write in simplest form:

a $3 \times a \times a \times b \times b \times b$ **b** $2 \times x + x \times x + x \times x \times x$

c $2x^2 \times 3x$

9 If $p = 7$ and $q = 2$ evaluate:

a $pq - 3q$ **b** $p(2q - 3)$

10 If $a = -2$, $b = 3$ and $c = -4$, find the value of $ab - bc$.

REVIEW SET 8B

1 Suppose we have 5 bags which contain some oranges. Find how many oranges are present if each bag contains:

a y oranges and there are 2 left over **b** t oranges and there are r left over

2 How many terms are in the expression $3x^2 - 2x + 5$?

3 For the expression $5x^2 - 3x + 2y - 5y^2$, state the coefficient of y^2.

4 State the like terms in:

a $5x + 6y - 2x + 3$ **b** $a^2 + 4 + 2a^2 - 1$

c $6t^2 - 2t + 5$

5 Simplify by collecting like terms:

a $8x + 2x - 10$ **b** $3a + 4 + 2a - 3$

c $2c - 3d + 5d - 6c$ **d** $-8t + 4q + qt + 5t$

6 Write in simplest form:

a $5 \times p \times p - p \times p \times p$ **b** $2 \times x \times x \times 2 \times x \times x \times x$

c $-3x \times 4x$ **d** $6x \times 2x^2$

7 If $x = 5$ and $y = 3$ evaluate:

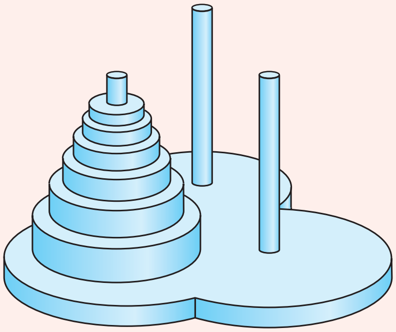
a $xy + 2y$ **b** $(x + 2y)^2$

8 If $x = -3$, $y = 4$ and $z = -5$, evaluate:

a xy **b** $x - y$ **c** $2y - 5x$ **d** $xy - z^2$

ACTIVITY THE TOWER OF HANOI

This problem allegedly gets its name from the Tower of Bramah in Hanoi, where the Lord Bramah instructed his priests to place 64 golden discs of varying diameters on one of three diamond needles. The priests were to transfer them as fast as possible to another needle, one disc at a time, never putting a large disc on a small disc, and never putting the discs aside rather than on a needle. When the discs were completely transferred, the world would come to an end.



What to do:

If you do not have a commercial version of the game, you can make your own using coins or cardboard discs with increasing diameters.

Start with 7 discs arranged in a pyramid on one of the three needles (or piles). The object of the puzzle is to move the pyramid of discs to another needle by moving only one disc at a time and not placing a larger disc on a smaller disc.

Hint: Begin with a small number of discs and build up a table of results like this:

Number of discs	1	2	3	4	5	6	7
Minimum number of moves	1	3					

You should be able to complete the puzzle with 5 discs in 31 moves.

1 How many moves are needed to shift the 6 disc pyramid?

2 How many moves are needed to shift the 7 disc pyramid?

3 Find a formula to express the minimum number of moves required if n discs are used.

4 Read again the Tower of Hanoi legend which says the world will end when all 64 discs have been moved. If the priests can move 5 discs per minute, how many years would it take to move all 64 discs?

Chapter 9

Length and area

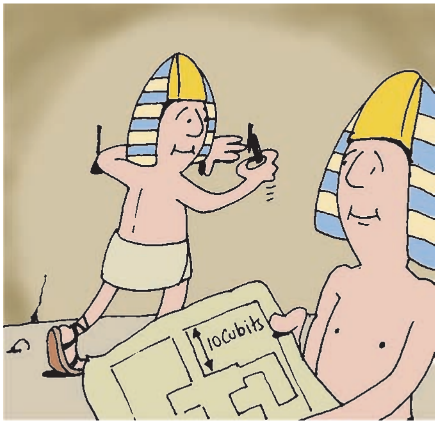
Contents:

Measurements of length, area, volume, capacity, mass, and time enable us to answer questions such as:

- How far is it to work?
- How big is the swimming pool?
- How long does the bus take to get to school?
- How much does the parcel weigh?
- How many litres of petrol did you buy?

We need to take measurements so that quantities can be accurately compared.

Measurement skills are needed for many jobs. Builders need to measure lengths, butchers need to weigh their products, landscapers need to calculate areas and volumes, and architects need to be able to draw accurate scale plans.



The earliest units of measurement were related to parts of the body. For example, a **span** was the width of your outstretched hand, and a **cubit** was the distance from your elbow to your fingertip. Noah's Ark is said to have been 300 cubits long.

Before 1500 AD, people used the **yard** or distance from your nose to your fingertip, and the **pace** or stride length.

All of these measures are called **natural units** of measure. They are not useful for accurate measuring because they vary from person to person. As a consequence, **standard units** of measure were developed to allow accurate comparisons to be made.

In this course we use the **Metric system** of units developed in France in 1789. This system is used in many countries and includes the **metre** as the standard unit of length, and the **kilogram** as the standard unit of mass. It is an easy system to work with because it uses base 10 for all conversions. Common prefixes are added when naming related units.

Greek prefixes are used to make the base units larger.

For example,

kilo	means	1000
mega	means	1 000 000

Latin prefixes are used to make the base units smaller.

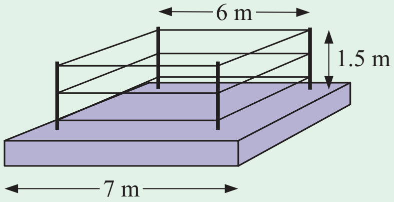
For example,

centi	means	$\frac{1}{100}$
milli	means	$\frac{1}{1000}$

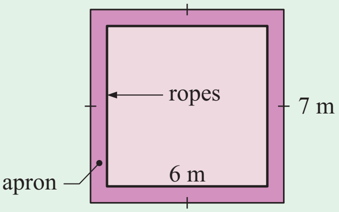
The metric system is more correctly called **Le Système International d'Unités** or **SI** for short.

OPENING PROBLEM

A boxing ring has the dimensions shown. There are 3 ropes on each side of the ring, and each rope is 6 m long. The ropes are connected to the corner posts which are 1.5 m high.



The ring is 7 m long on all sides. The part of the ring outside the ropes is called the apron.



Can you find the:

a total length of the ropes
b total length of the corner posts
c area of the ring inside the ropes
d area of the apron?

A LENGTH

The **metre** (m) is the base unit for length in the metric system.

HISTORICAL NOTE

Originally the **metre** was to be one ten-millionth of the distance from the north pole to the equator along the line of longitude through Paris, France. After difficult and exhaustive surveys, a piece of platinum alloy was prepared to this length and called the **standard metre**.

The standard metre was kept at the International Bureau of Weights and Measures at Sevres, near Paris. Thus the standard metre was not easily accessible to scientists around the world.

From 1960 to 1983 the metre was defined as 1 650 763.73 wavelengths of orange-red light from the isotope Krypton 86, measured in a vacuum.

In 1983 the metre was redefined as the distance light travels in a vacuum in $\frac{1}{299\,792\,458}$ of a second.

From the metre, other units of length were devised to measure smaller and larger distances:

1 kilometre (km) = 1000 metres (m)	(about 3 laps of a football pitch)
1 metre (m) = 100 centimetres (cm)	(about an adult's stride length)
1 centimetre (cm) = 10 millimetres (mm)	(about the width of a fingernail)

EXERCISE 9A.1

1 Give the unit you would use to measure:

a the length of a paper clip **b** the height of a flagpole

c the length of a journey by train **d** the width of a plank

e the length of a bee **f** the length of a swimming pool

2 Choose the correct answer.

a The width of a driveway would be:

A 3 cm **B** 3 m **C** 30 m **D** 300 mm

b The length of an ant would be:

A 4 km **B** 40 cm **C** 4 mm **D** 400 m

c The distance from Adelaide to Melbourne would be:

A 8000 m **B** 80 cm **C** 8 mm **D** 800 km

3 Which measurement is most likely to be correct?

a The length of a canoe is:

A 5 mm **B** 5 cm **C** 5 m **D** 5 km

b The height of a doorway is:

A 200 cm **B** 20 mm **C** 20 m **D** 0.2 km

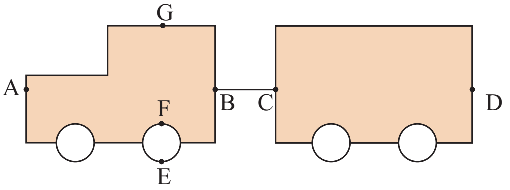
c The distance by road from Brussels to Amsterdam is:

A 180 m **B** 18 000 cm **C** 1800 m **D** 180 km

d The width of a hair is:

A 0.6 m **B** 0.6 mm **C** 0.6 cm **D** 0.06 m

4 The diagram shows a model train. Use your ruler to measure the following lengths in millimetres:



a the length of the engine [AB]

b the length of the carriage [CD]

c the distance between the engine and the carriage [BC]

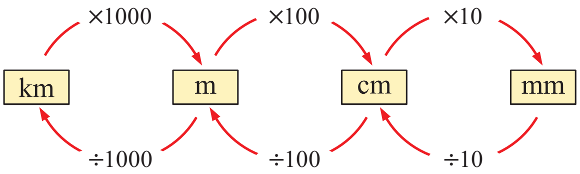
d the total length [AD] **e** the height of a wheel [EF] **f** the train's height [EG].

CONVERTING UNITS

Often we need to convert from one unit of length to another. The following general rules are helpful.

- When we convert from a **larger** unit to a **smaller** unit there will be more of the smaller units, so we must **multiply**.
  For example, $2 \text{ m} = (2 \times 100) \text{ cm} = 200 \text{ cm}$.
- When we convert from a **smaller** unit to a **larger** unit there will be less of the larger units, so we must **divide**.
  For example, $2000 \text{ m} = (2000 \div 1000) \text{ km} = 2 \text{ km}$.

In the SI system of units we must multiply or divide by powers of 10. The conversion diagram below will help you.



Example 1 **Self Tutor**

Express in centimetres: **a** 3.2 m **b** 423 mm **c** 6 km

a
$$\begin{aligned} &3.2\text{ m}\\ &= (3.2 \times 100)\text{ cm}\\ &= 320\text{ cm}\end{aligned}$$

b
$$\begin{aligned} &423\text{ mm}\\ &= (423 \div 10)\text{ cm}\\ &= 42.3\text{ cm}\end{aligned}$$

c
$$\begin{aligned} &6\text{ km}\\ &= (6 \times 1000)\text{ m}\\ &= 6000\text{ m}\\ &= (6000 \times 100)\text{ cm}\\ &= 600\,000\text{ cm}\end{aligned}$$

EXERCISE 9A.2

1 Express in centimetres:

a 8 m **b** 16.4 m **c** 40 mm **d** 6000 mm

e 0.2 m **f** 5 km **g** 0.4 km **h** 20 km

2 Express in metres:

a 5 km **b** 2000 km **c** 600 cm **d** 36 500 mm

3 Express the following in millimetres:

a 6 m **b** 7 cm **c** 250 cm **d** 748 m

4 Express the following in kilometres:

a 10 000 m **b** 45 000 cm **c** 4 000 000 mm **d** 200 m

5 Convert to the units shown:

a 7 cm = ... mm **b** 500 cm = ... m **c** 4000 m = ... km

d 50 mm = ... cm **e** 6 m = ... cm **f** 8 km = ... m

g 6.4 cm = ... mm **h** 340 cm = ... m **i** 5200 m = ... km

j 25 mm = ... cm **k** 3.8 m = ... cm **l** 1500 m = ... km

Example 2 **Self Tutor**

Find the sum of 3 km + 350 m + 220 cm in metres.

$$\begin{aligned} &3\text{ km} + 350\text{ m} + 220\text{ cm}\\ &= 3000\text{ m} + 350\text{ m} + 2.2\text{ m} \qquad \{\text{converting to metres}\}\\ &= 3352.2\text{ m}\end{aligned}$$

6 Find the sum of:

- **a** 2 km + 130 m + 120 cm in metres
- **b** 5 km + 850 m + 450 cm in metres
- **c** 8 m + 70 cm + 4 mm in centimetres
- **d** 6 m + 40 cm + 32 mm in centimetres

Example 3 **Self Tutor**

Calculate the number of 50 m laps Ian swims in his 4.2 km pool session.

We first convert the 4.2 km to m.

$$\begin{aligned} 4.2\text{ km} &= (4.2 \times 1000)\text{ m} \\ &= 4200\text{ m} \\ \therefore \quad \text{number of laps} &= \frac{4200\text{ m}}{50\text{ m}} \\ &= 84 \end{aligned}$$

7 Calculate the number of:

- **a** 250 cm lengths of wire which can be cut from a 40 m spool
- **b** 400 mm lengths of dress material which can be cut from a roll of length 8 m
- **c** 6 m pipes required to lay a 240 km pipeline
- **d** 12.5 m lengths of rope which can be cut from a 25 km length
- **e** 2.6 m lengths of cable which can be cut from a 13 km length.

B PERIMETER

In English the word **perimeter** refers to the boundary of a region.

For example:

- the boundary line of a hockey field is the playing *perimeter*
- the boundary of a prison is protected by a *perimeter* fence.

However, in mathematics the word *perimeter* refers to the distance around a figure.

The **perimeter** of a closed figure is a measurement of the distance around the boundary of the figure.

POLYGONS

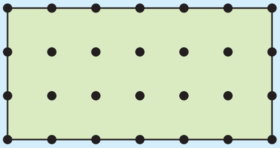
The perimeter of a **polygon** can be found by adding the lengths of all the sides.

When we draw polygons, sides with the same length are given the same markings.

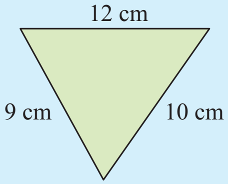
Example 4 **Self Tutor**

Find the perimeter of the following:

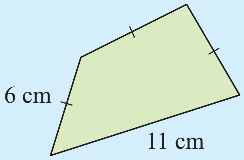
a



b



c



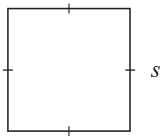
a Perimeter
$= 6 + 3 + 6 + 3$ units
$= 18$ units

b Perimeter
$= 9 + 10 + 12$ cm
$= 31$ cm

c Perimeter
$= 11 + (3 \times 6)$ cm
$= 11 + 18$ cm
$= 29$ cm

We can derive formulae for the perimeters of common polygons:

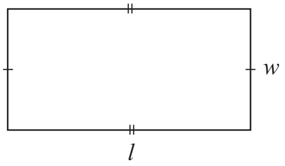
SQUARE



Since all 4 sides are equal in length,
perimeter $=$ 4 × side length,
$\therefore \quad P = 4 \times s$

So, $\boxed{\boldsymbol{P = 4s}}$

RECTANGLE



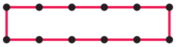
perimeter $=$ length $+$ width $+$ length $+$ width
$= 2 \times$ (length $+$ width)
$\therefore \quad P = 2 \times (l + w)$

So, $\boxed{\boldsymbol{P = 2(l + w)}}$

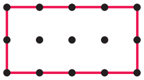
EXERCISE 9B

1 Find the perimeter of the following figures:

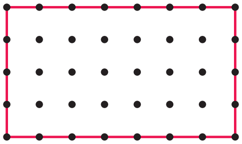
a



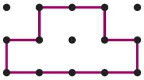
b



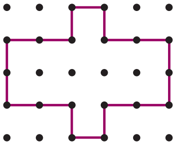
c



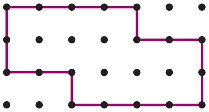
d



e

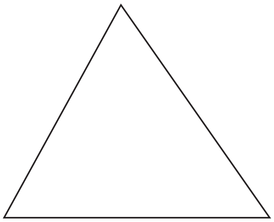


f

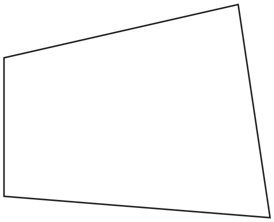


2 Measure with your ruler the lengths of the sides of the figures below. Hence find the perimeter of each figure.

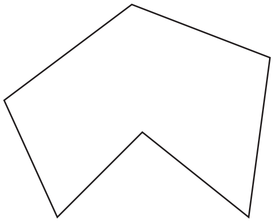
a



b

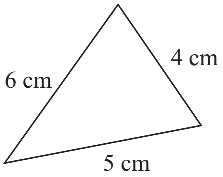


c

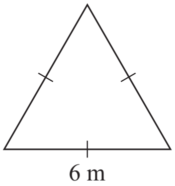


3 Find the perimeter of each of the following figures:

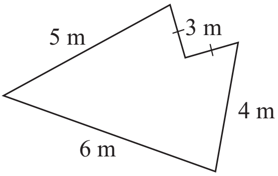
a



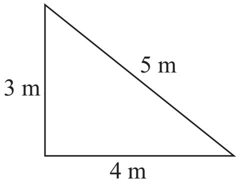
b



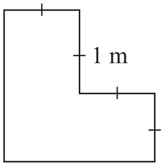
c



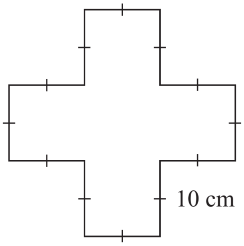
d



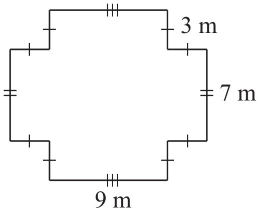
e



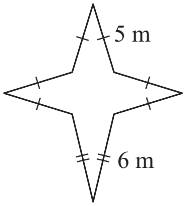
f



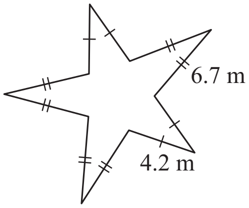
g



h



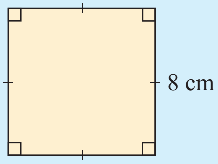
i



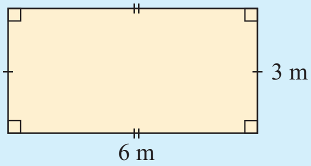
Example 5 **Self Tutor**

Find the perimeters of the following figures:

a



b



a

$P = 4s$

$\therefore \quad P = 4 \times 8$ cm

$\therefore \quad P = 32$ cm

b

$P = 2(l + w)$

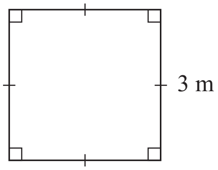
$\therefore \quad P = 2(6 + 3)$ m

$\therefore \quad P = 2 \times 9$ m

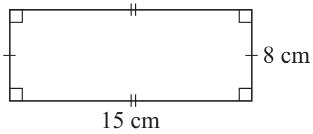
$\therefore \quad P = 18$ m

4 Use the formulae to find the perimeters of the following figures:

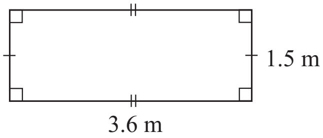
a



b



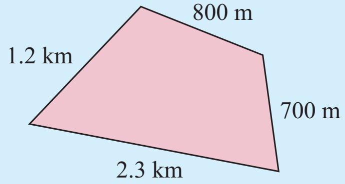
c



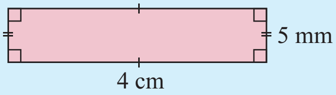
Example 6 **Self Tutor**

Calculate the perimeters of the following figures:

a



b



a $P = 2.3 \text{ km} + 1.2 \text{ km} + 800 \text{ m} + 700 \text{ m}$ {adding all sides}

$\therefore\ P = 2300 \text{ m} + 1200 \text{ m} + 800 \text{ m} + 700 \text{ m}$ {converting to m}

$\therefore\ P = 5000 \text{ m}$

b $P = 2(l + w)$ {perimeter of a rectangle}

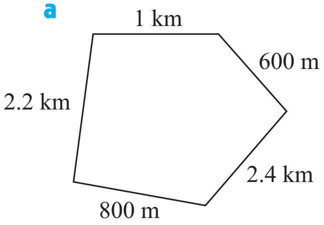
$\therefore\ P = 2(40 + 5) \text{ mm}$ {converting to mm}

$= 2 \times 45 \text{ mm}$

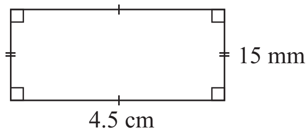
$= 90 \text{ mm}$

5 Calculate the perimeters of the following figures:

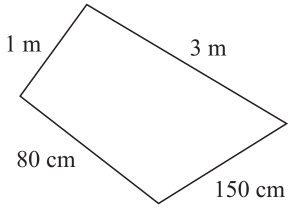
a



b

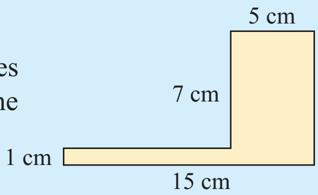


c



Example 7 **Self Tutor**

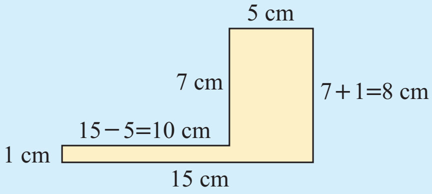
Find the lengths of the unknown sides and hence calculate the perimeter of the figure:



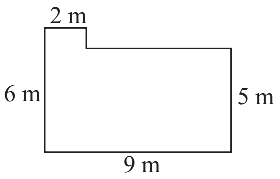
We use the known lengths to calculate the other side lengths:

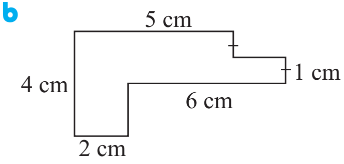
Now $P = 5 + 8 + 15 + 1 + 10 + 7 \text{ cm}$

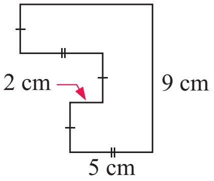
$\therefore\ P = 46 \text{ cm}$



6 Find the lengths of the unknown sides and hence calculate the perimeters of the figures below. Assume that all corners are right angles.

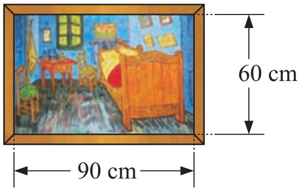
a


b


c


7 A rectangular painting 60 cm high and 90 cm wide is to be given a 5 cm wide wooden frame. Find the perimeter of:

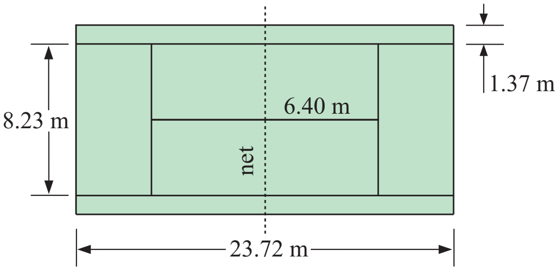
a the painting **b** the frame.



8 A tennis court has its lines marked as shown alongside.

a Find the total length of the lines.

b Find the length of the outer perimeter of the tennis court.



9 A rectangular swimming pool is 22 metres by 8 metres. It is surrounded by a path one metre wide.

a Draw and label a diagram of the pool and path.

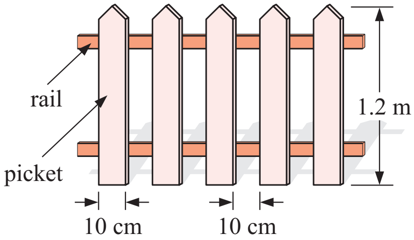
b Find the perimeter around the outside edge of the path.

10 An equilateral triangular field has sides of length 380 metres. Find the cost of fencing the field with three strands of wire if the wire costs £1.18 per metre.

11 A long distance runner trains by running around a 450 m by 1.25 km rectangular block. If he completes 12 laps in training, how far has he run?

12 How long would it take a farmer to check the fences of a 680 m by 340 m rectangular paddock if 50 m can be checked every minute?

13 Jason plans to build a picket fence 30 m long with the design shown. There is a 2 m long post every 2 m, to which the rails are attached. The timber for the pickets costs $1.80 per metre, for the rails costs $2.50 per metre, and for the posts costs $4.50 per metre. Find:



a the number of posts and hence the total length of timber required for the posts

b the total length of rails needed

c the number of pickets needed and the length of timber needed to make these pickets

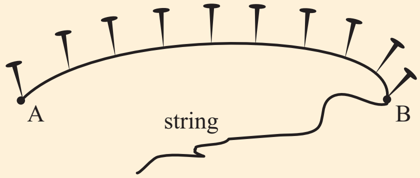
d the total cost of the fence.

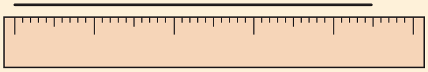
INVESTIGATION 1 LENGTHS OF CURVES

Finding the length of a curve or the perimeter of a curved figure can be quite difficult. At best we can only estimate such measurements. In this investigation you will consider a method for estimating the length of a curve.

What to do:

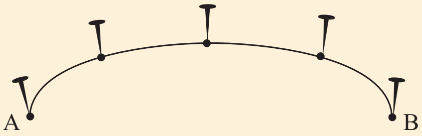
Pins can be placed along the curve and used to guide a piece of string. The length of string required can then be measured with your ruler.

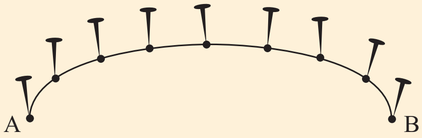




PRINTABLE WORKSHEET

1 Click on the icon and print the two curves. Use the method described above to estimate their lengths, placing the pins in the positions shown.





2 Write up your findings by considering the following questions:

- **a** Which estimate do you think is most accurate?
- **b** Do the pins have to be equally spaced?
- **c** Where is it most important to have the pins?
- **d** How could you improve your estimate of the length of this curve?

3 Draw a curved figure of your own choosing. Use the method above to obtain a good estimate of its perimeter.

C AREA

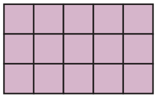
All around your school there are many flat surfaces such as paths, floors, ceilings, walls, and courts for playing sport. All of these surfaces have boundaries which define their shape.

Area is the amount of **surface** inside a region.

Information on cans of paint and bags of fertiliser refer to the **area** they can cover.

The **area** of a closed figure is the number of square units it encloses.

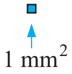
For example, the rectangle alongside has an area of 15 square units.



If each square was 1 cm by 1 cm we would have 15 square centimetres.
If each square was 1 m by 1 m we would have 15 square metres.

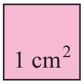
In the metric system, the units we use for the measurement of area are related to the units we use for length.

1 **square millimetre** (mm^2) is the area enclosed by a square of side length 1 mm.



The area of a computer chip might be measured in mm^2.

1 **square centimetre** (cm^2) is the area enclosed by a square of side length 1 cm.

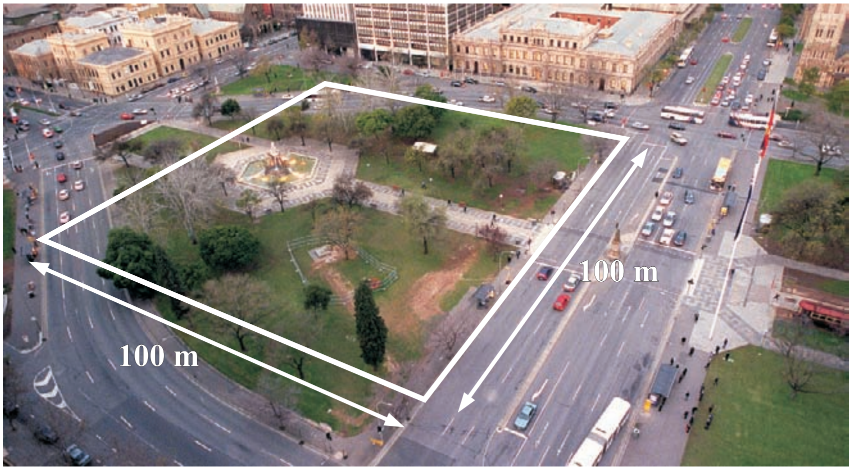


The area of a book cover might be measured in cm^2.

1 **square metre** (m^2) is the area enclosed by a square of side length 1 m.

The area of a brick paving is measured in m^2.

1 **hectare** (ha) is the area enclosed by a square of side length 100 m.



Courtesy The Advertiser and David Cronin

Larger areas are often measured in ha.

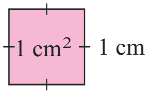
1 **square kilometre** (km^2) is the area enclosed by a square of side length 1 km.

The area of a country or continent would be measured in km^2.

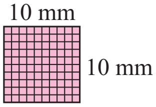
CONVERSION OF AREA UNITS

We can use the length unit conversions to help us convert from one area unit to another.

We have already seen that a square with sides of length 1 cm has area 1 cm^2:



We could also measure the sides of this square in millimetres:



Each of the small squares has an area of 1 mm^2.

There are $10 \times 10 = 100$ square millimetres in this square, so $1\text{ cm}^2 = 100\text{ mm}^2$

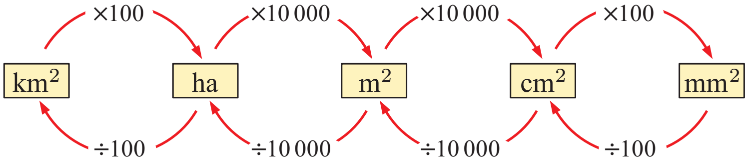
Likewise,

$$\begin{aligned} 1\ \text{m}^2 &= 1\ \text{m} \times 1\ \text{m} \\ &= 100\ \text{cm} \times 100\ \text{cm} \\ &= 10\,000\ \text{cm}^2 \end{aligned} \qquad \begin{aligned} 1\ \text{ha} &= 100\ \text{m} \times 100\ \text{m} \\ &= 10\,000\ \text{m}^2 \end{aligned} \qquad \begin{aligned} 1\ \text{km}^2 &= 1\ \text{km} \times 1\ \text{km} \\ &= 1000\ \text{m} \times 1000\ \text{m} \\ &= 1\,000\,000\ \text{m}^2 \\ &= 100 \times 10\,000\ \text{m}^2 \\ &= 100\ \text{ha} \end{aligned}$$

$1\ \text{cm}^2 = 100\ \text{mm}^2$ $\quad$ $1\ \text{ha} = 10\,000\ \text{m}^2$

$1\ \text{m}^2 = 10\,000\ \text{cm}^2$ $\quad$ $1\ \text{km}^2 = 100\ \text{ha}$

AREA UNIT CONVERSIONS

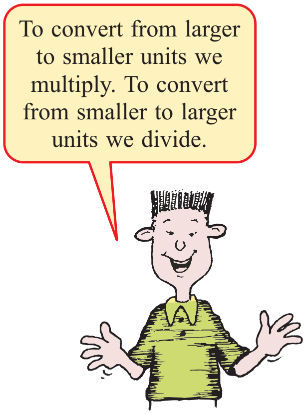


Example 8 **Self Tutor**

Convert: **a** 2.2 ha to m^2 **b** 540 mm^2 to cm^2

a $\begin{aligned} &2.2\ \text{ha} \\ &= (2.2 \times 10\,000)\ \text{m}^2 \\ &= 22\,000\ \text{m}^2 \end{aligned}$

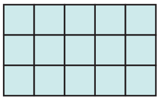
b $\begin{aligned} &540\ \text{mm}^2 \\ &= (540 \div 100)\ \text{cm}^2 \\ &= 5.4\ \text{cm}^2 \end{aligned}$



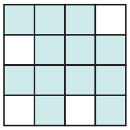
EXERCISE 9C

1 Determine the shaded area of the following, giving your answers in square units:

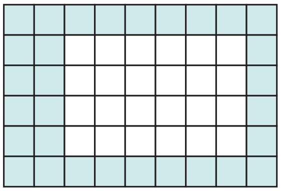
a



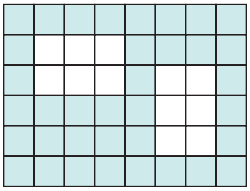
b



c



d



2 Convert:

a 1.2 m^2 to cm^2 **b** 0.8 km^2 to ha **c** 97 cm^2 to mm^2

d 5 ha to m^2 **e** 15 000 m^2 to ha **f** 47 600 cm^2 to m^2

g 1600 ha to km^2 **h** 7.9 cm^2 to mm^2 **i** 0.53 km^2 to ha

j 5600 m^2 to ha **k** 2.37 m^2 to cm^2 **l** 0.0038 m^2 to mm^2

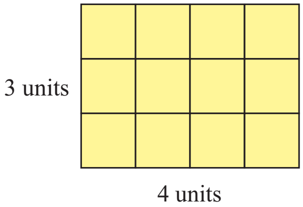
3 A piece of paper has an area of 630 cm^2. Express this area in mm^2.

4 Farmers Ahmad, Badan and Dahari own blocks of land with the following areas:
Ahmad 1.2 km^2, Badan 212 ha, Dahari 560 000 m^2.
Which farmer owns the: **a** largest **b** smallest block of land?

D AREAS OF POLYGONS

Dividing shapes into unit squares and then counting these unit squares is not a very convenient way of calculating many areas. Formulae have therefore been devised to find the areas of polygons such as rectangles, triangles, parallelograms, and trapezia.

RECTANGLES



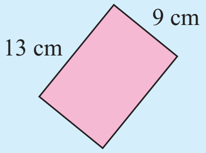
Consider a rectangle 4 units long and 3 units wide. Clearly the area of this rectangle is 12 units^2, and we can find this by multiplying $4 \times 3 = 12$.

Area of rectangle = length × width

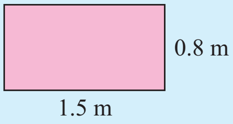
Example 9 Self Tutor

Find the areas of the following rectangles:

a



b



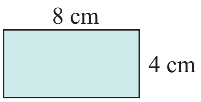
a Area
$= \text{length} \times \text{width}$
$= 13 \text{ cm} \times 9 \text{ cm}$
$= 117 \text{ cm}^2$

b Area
$= \text{length} \times \text{width}$
$= 1.5 \text{ m} \times 0.8 \text{ m}$
$= 1.2 \text{ m}^2$

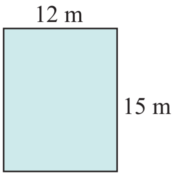
EXERCISE 9D.1

1 Find the areas of the following rectangles:

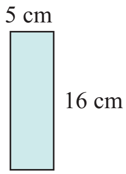
a



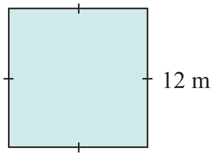
b



c



d



2 **a** Estimate the area of the floor of your classroom in square metres.

b Measure the dimensions of your classroom. Calculate the area of the floor to the nearest square metre. How close was your estimate?

3 **a** Estimate the area of the cover of your maths textbook in square centimetres.

b Measure the length and width of your textbook in cm and calculate its area. How close was your estimate?

Example 10

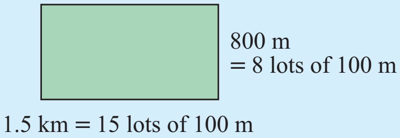
Self Tutor

A rectangular paddock is 800 m by 1.5 km.

a Find the area of the paddock in hectares.

b If fertiliser costs \$50 per hectare, how much will it cost to fertilise the paddock?

a To calculate the area in hectares, we convert the measurements to 'lots of 100 m'.



$$\begin{aligned}\text{Area} &= l \times w \\ &= 1.5 \text{ km} \times 800 \text{ m} \\ &= 1500 \text{ m} \times 800 \text{ m} \\ &= 15 \text{ lots of } 100 \text{ m} \times 8 \text{ lots of } 100 \text{ m} \\ &= 120 \text{ ha}\end{aligned}$$

b $$\begin{aligned}\text{Cost} &= 120 \text{ ha} \times \$50 \\ &= \$6000\end{aligned}$$

4

©iStockphoto.com

A corn field is 2 km by 3.2 km. Find:

a the area of the property in hectares

b the total cost of sowing a crop if it costs \$80 per hectare.

5 A football pitch is 50 m by 90 m.

a Find the area of the pitch.

b How long will it take to mow the whole pitch if 60 square metres can be mown each minute?

6 Floor tiles are 20 cm by 30 cm.

a How many tiles would you need to tile a floor 5 m by 6 m?

b If each tile costs €6.85, find the total cost of the tiles.

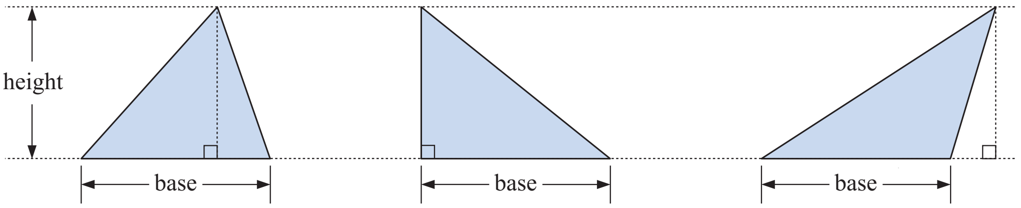
7 A driveway is 23 m long and 3 m wide. Find the total cost of concreting the driveway if the materials cost £20 per square metre and the cost of hiring labour to do the concreting is £25 per square metre.

8 What would be the cost of resurfacing a 50 m by 32 m gymnasium floor with a rubberised compound costing \$35.60 a square metre?

AREAS OF OTHER POLYGONS

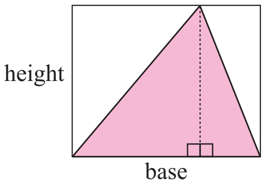
The formulae for the areas of triangles, parallelograms, and trapezia can be derived from the formula for the area of a rectangle.

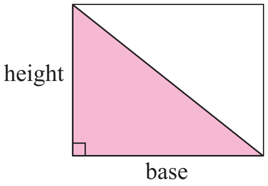
TRIANGLES



Area of triangle $= \frac{1}{2}$ (base $\times$ height)

The first two cases are demonstrated easily by drawing a rectangle with the same base and height as the triangle.



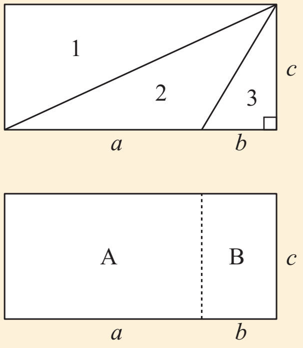


$$\text{Area} = \tfrac{1}{2}\ (\text{area of rectangle})$$
$$= \tfrac{1}{2}\ (\text{base} \times \text{height})$$

INVESTIGATION 2 AREA OF A TRIANGLE

The rectangle alongside has been divided into three triangles.

The second rectangle has the same dimensions as the one above it. However, this rectangle has been divided into two smaller rectangles A and B.



What to do:

1 In terms of a and b and c, write down formulae for:

a area A **b** area B **c** area 3

2 Notice that area 2 + area 3 = area 1

$= \frac{1}{2}$ of the complete rectangle

$= \frac{1}{2}$ of area A $+$ $\frac{1}{2}$ of area B

Use your formulae in **1** to copy and complete:

area 2 + area 3 $= \frac{1}{2}$ ...... $+$ $\frac{1}{2}$ ......

$\therefore$ area 2 + ... = ... + ...

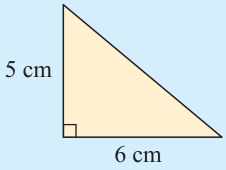
$\therefore$ area 2 = ...

$= \frac{1}{2}$(base $\times$ height)

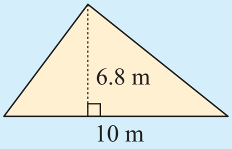
Example 11 **Self Tutor**

Find the areas of the following triangles:

a



b



a Area $= \frac{1}{2}(\text{base} \times \text{height})$

$\therefore \quad A = \frac{1}{2}(6 \times 5)\ \text{cm}^2$

$\therefore \quad A = 15\ \text{cm}^2$

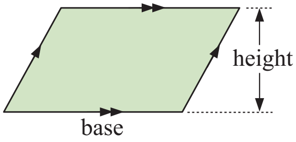
b Area $= \frac{1}{2}(\text{base} \times \text{height})$

$\therefore \quad A = \frac{1}{2}(10 \times 6.8)\ \text{m}^2$

$\therefore \quad A = \frac{1}{2} \times 68\ \text{m}^2$

$\therefore \quad A = 34\ \text{m}^2$

PARALLELOGRAMS



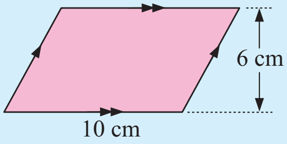
Area of parallelogram = base × height

We can demonstrate this formula by cutting out a triangle from one end of the parallelogram and shifting it to the other end. The resulting shape is a rectangle with the same base and height as the parallelogram.

Perform this demonstration for yourself using paper and scissors.

Example 12 **Self Tutor**

Find the area of:

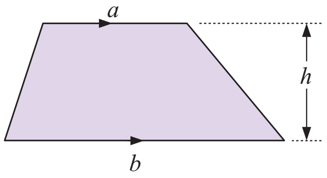


Area = base × height

$\therefore \quad A = 10\ \text{cm} \times 6\ \text{cm}$

$\therefore \quad A = 60\ \text{cm}^2$

TRAPEZIA



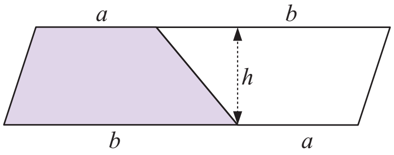
Area = average length of parallel sides × distance between parallel sides

or **Area** $= \left(\dfrac{a+b}{2}\right) \times h$

We can demonstrate this result using a second trapezium of exactly the same shape. We place the trapezia together to form a parallelogram.

DEMO

$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \text{ of area of parallelogram}\\ &= \tfrac{1}{2} \times \text{base} \times \text{height}\\ &= \tfrac{1}{2} \times (a+b) \times h\end{aligned}$$

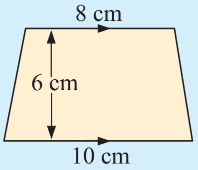


Perform this demonstration for yourself using paper and scissors.

Example 13

Self Tutor

Find the area of the trapezium:



$$\text{Area} = \left(\frac{a+b}{2}\right) \times h$$

$$\therefore \quad A = \left(\frac{8+10}{2}\right) \times 6 \text{ cm}^2$$

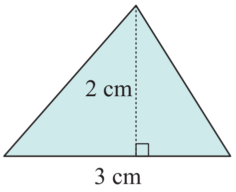
$$\therefore \quad A = 9 \times 6 \text{ cm}^2$$

$$\therefore \quad A = 54 \text{ cm}^2$$

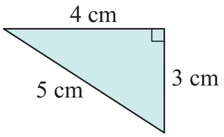
EXERCISE 9D.2

1 Find the areas of the following triangles:

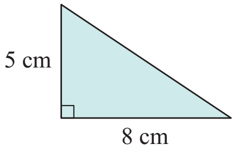
a



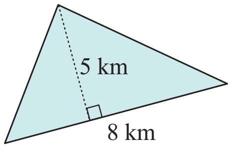
b



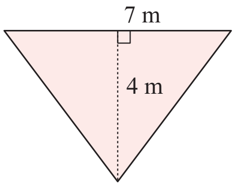
c



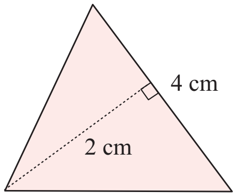
d



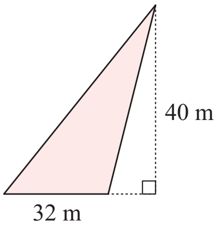
e



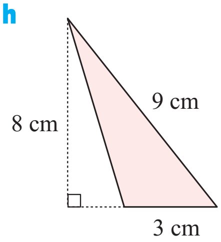
f



g

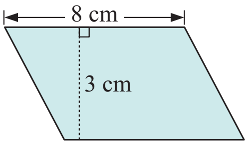


h

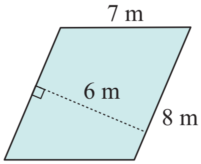


2 Find the areas of the following parallelograms:

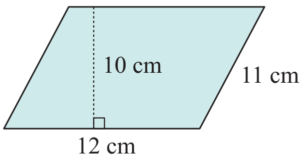
a



b

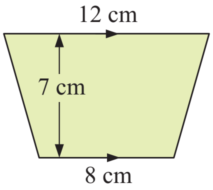


c

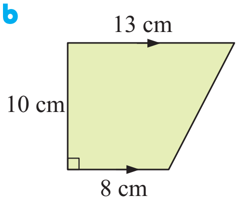


3 Find the areas of the following trapezia:

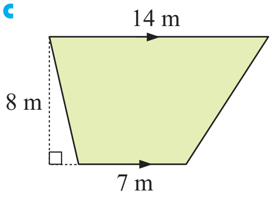
a



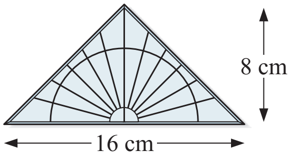
b



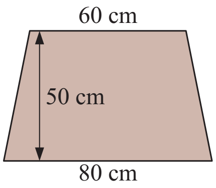
c



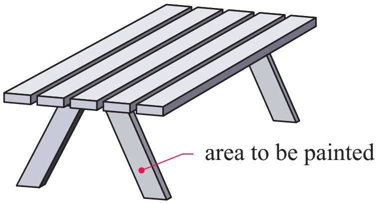
4 Find the area of the set square:



5 A student's desk is 80 cm wide at the front and 60 cm wide at the back. The desk is 50 cm deep. Find the area of the desk.



6

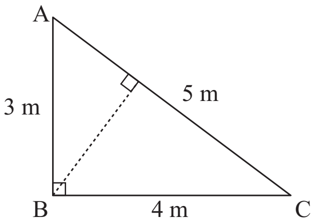


The surface of the park bench leg indicated must be repainted due to graffiti. The leg is 10 cm wide at the base, and the top of the leg is 60 cm above the ground. Find the area that needs to be repainted.

7 A right angled triangle has sides of length 3 m, 4 m and 5 m.

a Find the area of the triangle ABC.

b A perpendicular is drawn from side [AC] to B. Let this perpendicular have length x m. Find x.



E AREAS OF COMPOSITE SHAPES

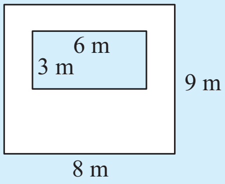
The figures in this section are called **composite shapes**. They are made up or **composed** of two or more standard shapes. Their areas can be calculated using addition and subtraction of the areas of the standard shapes.

Example 14

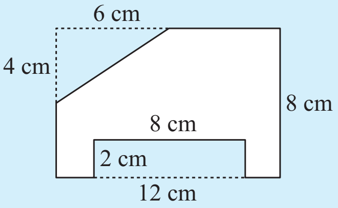
Self Tutor

Find the areas of the unshaded regions:

a



b



a Area

$= \text{area large rectangle}$

$\quad - \text{area small rectangle}$

$= (9 \times 8 - 6 \times 3) \text{ m}^2$

$= (72 - 18) \text{ m}^2$

$= 54 \text{ m}^2$

b Area

$= \text{area large rectangle} - \text{area } \Delta$

$\quad - \text{area small rectangle}$

$= (12 \times 8 - \frac{1}{2} \times 6 \times 4 - 8 \times 2) \text{ cm}^2$

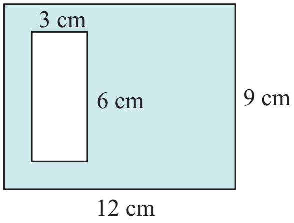
$= (96 - 12 - 16) \text{ cm}^2$

$= 68 \text{ cm}^2$

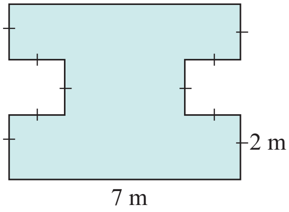
EXERCISE 9E

1 Find the areas of the following shaded regions:

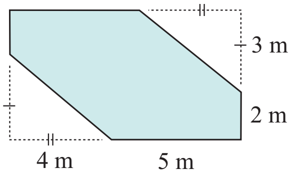
a



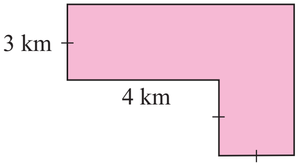
b



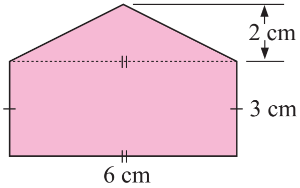
c



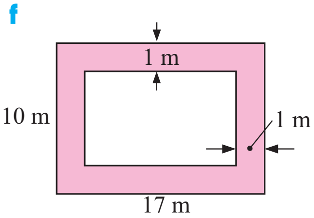
d



e

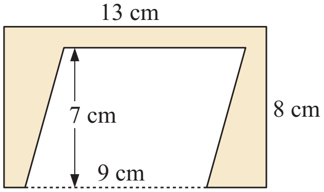


f

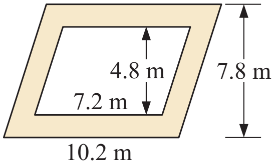


2 Find the shaded areas in the following figures:

a



b

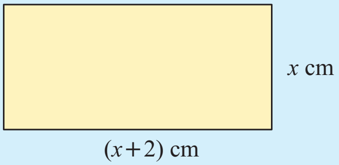


3 A rectangular lawn 18 m by 30 m is surrounded by a concrete path 1 m wide. Draw a diagram of the situation and find the total area of concrete.

Example 15 **Self Tutor**

For the rectangle given, find a formula for the:

a perimeter P **b** area A

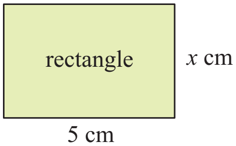


a
$$P = (x+2) + x + (x+2) + x \text{ cm}$$
$$\therefore \quad P = x + 2 + x + x + 2 + x \text{ cm}$$
$$\therefore \quad P = 4x + 4 \text{ cm}$$

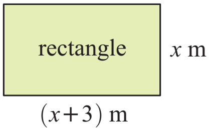
b
$$A = \text{length} \times \text{width}$$
$$\therefore \quad A = (x+2) \times x \text{ cm}^2$$
$$\therefore \quad A = x(x+2) \text{ cm}^2$$

4 Find a formula for **i** the perimeter P **ii** the area A of:

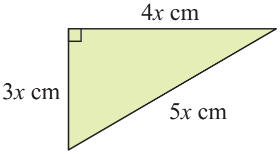
a



b

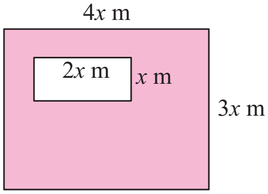


c

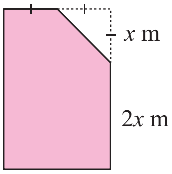


5 Find the shaded area A in terms of x for:

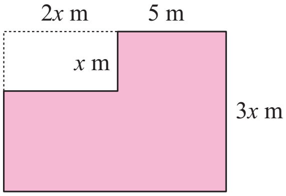
a



b



c

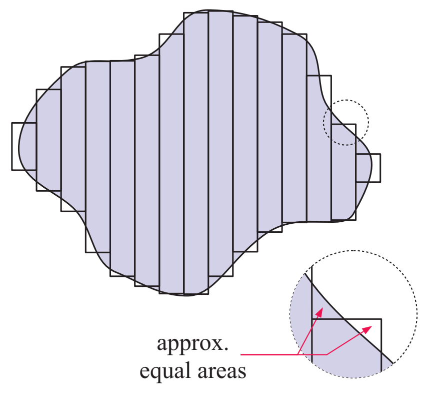


INVESTIGATION 3 AREAS BY "STRIPS"

In this investigation we will estimate the areas of regions with curved boundaries. We will do this by dividing the region into "strips" or rectangles of unit width.

The area of each rectangle is simply worked out by measuring its height. The total area is the sum of the area of all the rectangles.

In using this method we assume that the errors at the top and bottom of each strip will be small.

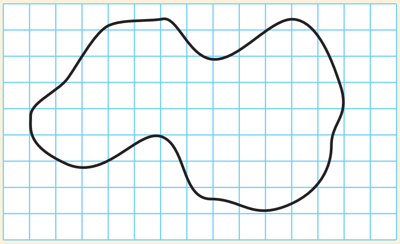


What to do:

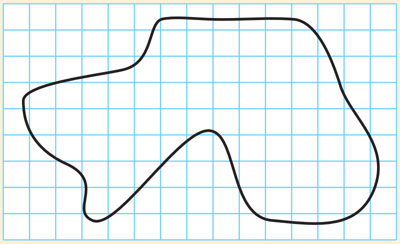
PRINTABLE SHAPES

1 Use the method above to estimate the areas of the following shapes:

a



b

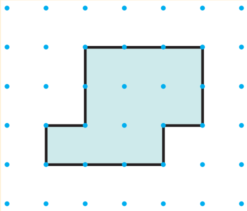


2 Compare your answers with those obtained by other members of your class.

3 What are the advantages and disadvantages of this method? Can you suggest where it is least accurate?

INVESTIGATION 4 PICK'S RULE FOR AREAS

Below is a diagram of a polygon whose **vertices** or corner points lie on **lattice points**. The lattice points which lie on the figure are referred to as **boundary points**.



The lattice points which lie inside the figure are referred to as **interior points**.

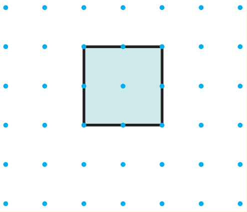
The figure alongside has 14 boundary points and 3 interior points.

Suppose $B =$ the number of boundary points,
and $I =$ the number of interior points.

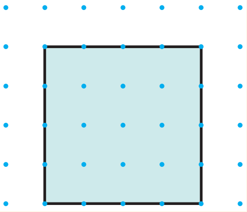
What to do:

1 For the figures below, copy and complete the table which follows:

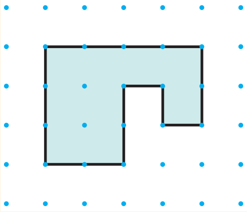
a



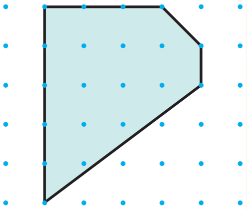
b



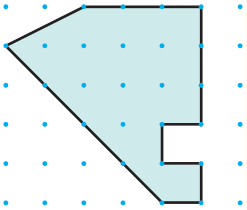
c



d



e



f

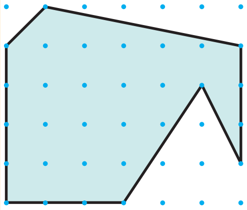


Figure	B	$\frac{B}{2}$	I	$\frac{B}{2}+I$	*Area* A
a					
b					
c					
d					
e					
f					

2 Look at the last two columns of your table. Suggest a rule connecting A, B and I.

3 Draw several polygons of your own on grid paper and check that your rule in **2** is correct.

This result is called **Pick's Rule**.

KEY WORDS USED IN THIS CHAPTER

- area
- composite shape
- hectare
- kilogram
- metre
- Metric system
- parallelogram
- perimeter
- Pick's Rule
- polygon
- rectangle
- trapezium
- triangle

LINKS
click here

POPULATION DENSITY

Areas of interaction:
Health and social education

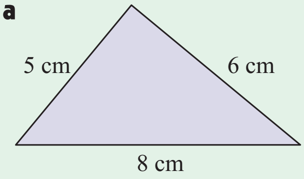
REVIEW SET 9A

1 Convert:

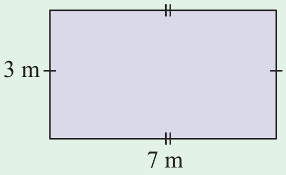
a 7.43 km to m
b 16.3 m to cm
c 1500 m to km
d 469 cm to m
e 94.38 mm to cm
f 2500 mm to m

2 Find the perimeter of:

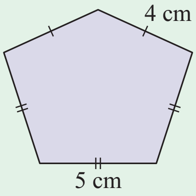
a



b

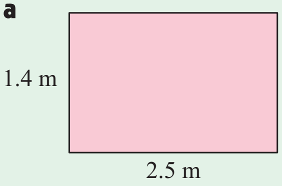


c



3 Find the area of:

a



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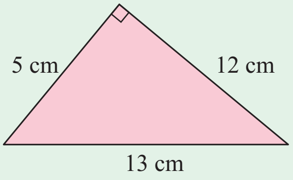
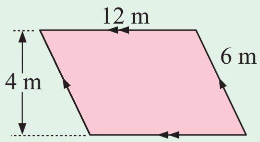


c



4 A piece of wire 640 cm long is cut into equal lengths. These are then bent into squares 4 cm by 4 cm. How many squares can be made?

5 The school sports fields are laid out in a rectangle 220 m by 160 m. Find the cost of fertilising the grass if 1 kg of fertiliser covers 80 square metres and fertiliser costs $25 for a 40 kg bag.

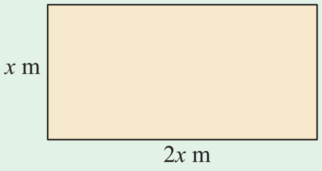
6 A carpet measuring 4 m by 3 m is placed in a room 5.2 m long and 4.8 m wide. What area of the floor is left uncarpeted?

7 A lounge room is 5.4 m long, 4.8 m wide, and 4.2 m high. It has a door 2 m by 1 m and a window 2 m by 1.5 m.

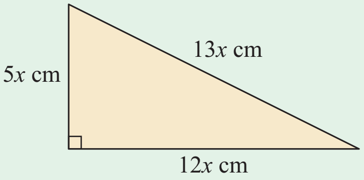
- **a** Draw a diagram to illustrate the room.
- **b** If wallpaper costs €5.50 per square metre, find the cost of wallpapering the four walls.

8 Find a formula for the **i** the perimeter P **ii** the area A of:

a



b



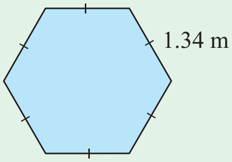
REVIEW SET 9B

1 Convert:

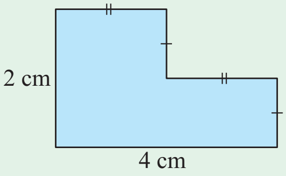
- **a** 323 mm^2 to cm^2
- **b** 23 462 m^2 to ha
- **c** 8.42 m^2 to cm^2
- **d** 2.8 mm^2 to cm^2
- **e** 25.3 cm^2 to mm^2
- **f** 2.92 km^2 to ha

2 Find the perimeter of:

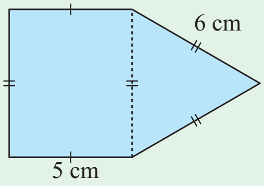
a



b

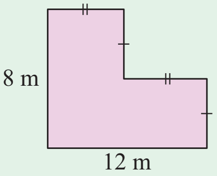


c

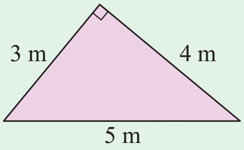


3 Find the area of:

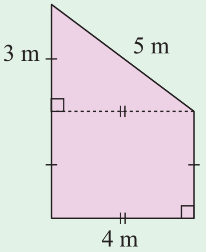
a



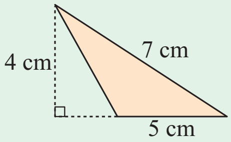
b



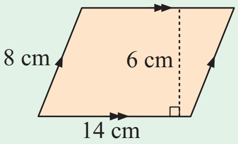
c



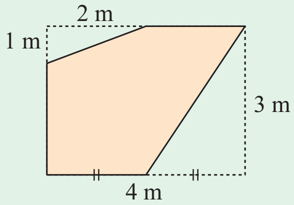
d



e



f



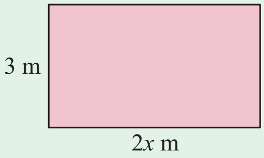
4 Chelsea measures the windows in her flat so she can calculate the length of curtain track that she needs to buy. Her measurements are 1.2 m, 1.2 m, 1.8 m, 90 cm and 45 cm. How many metres of track does Chelsea need?

5 A rug measuring 2.5 m by 3.5 m was placed in a room 6.4 m long and 8.2 m wide. What area of floor is not covered by the rug?

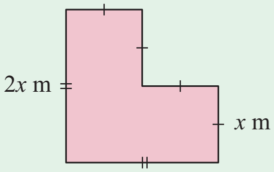
6 A brick wall will be built 5.6 m long and 10 bricks high. If each brick is 20 cm long, how many bricks will be needed?

7 Find a formula for **i** the perimeter P **ii** the area A of:

a



b



8 Answer the questions posed in the **Opening Problem** on page **175**.

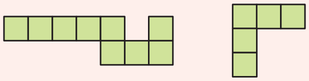
ACTIVITY

POLYOMINOES

Polyominoes are shapes built by connecting the same size squares *edge to edge*.

For example:

- these are polyominoes:



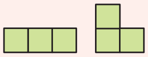
- these are *not* polyominoes:

There are many families of polyominoes. For example:

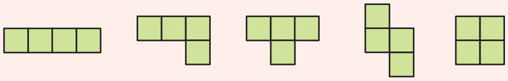
- Monomino

- Domino

- Trominoes (2 members)



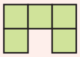
- Tetrominoes (5 members)

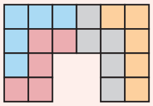


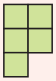
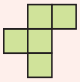
You will need: square paper, scissors

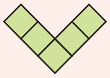
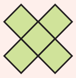
What to do:

1. Pentominoes are polyominoes made from 5 squares. Draw all 12 pentominoes on square paper and cut them out. They should all be different and not able to be transformed into each other by flipping or rotating.
2. Construct as many of the following rectangles as possible using some or all of one set of pentominoes.
   5 × 3, 5 × 4, 5 × 5, 5 × 6, 5 × 7, 5 × 8, 5 × 9, 5 × 10, 5 × 11,
   5 × 12, 10 × 2, 10 × 3, 10 × 4, 10 × 5, 10 × 6, 15 × 3, 15 × 4, 20 × 3.
   Note: The 5 × 10 has 6951 solutions whereas the 20 × 3 has only two possible solutions.
3. Construct an 8 × 8 square with all the pentominoes leaving four empty squares in a 2 × 2 hole. The hole can be anywhere.
4. Use any 4 pentominoes to make double scale models of 10 of the pentominoes.

   For example, a double scale model of  can be made like this:

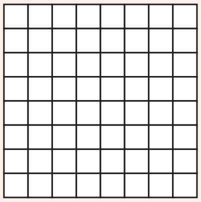
   

   These range in difficulty from  (the easiest) to  (the hardest).

    and  are impossible.
5. Play the games below with one set of pentominoes.

GAMES FOR TWO PLAYERS:

1. Set up an 8 × 8 square board.
2. Take turns to place the pentominoes on the board so they do not overlap.
3. The last player able to place a piece is the winner.
4. Modify the game by dividing the set of pentominoes into 2 sets of 6 pieces. Each player can now only use their own set.



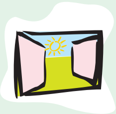
8×8 board

Chapter 10

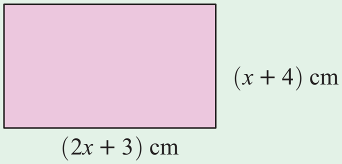
Algebra (Expansion and Factorisation)

Contents:

OPENING PROBLEM



A rectangle has length $(2x+3)$ cm and width $(x+4)$ cm. We let its perimeter be P and its area be A.



Things to think about:

1. How can we quickly write down formulae for P and A in terms of x?
2. How can we simplify the formulae so they do not contain brackets?

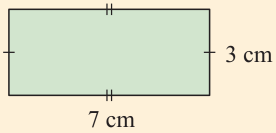
A THE DISTRIBUTIVE LAW

INVESTIGATION THE DISTRIBUTIVE LAW

The perimeter of a rectangle can be found by either:

- adding all four side lengths, or
- doubling the sum of length and width.

For example, the perimeter of the rectangle alongside is
$7+3+7+3=20$ cm or $2\times(7+3)=20$ cm.



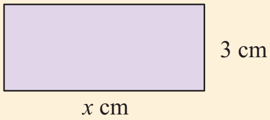
If we apply these methods to a rectangle with sides in terms of a variable or variables we discover a useful algebraic result.

What to do:

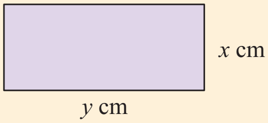
1. For the following rectangles, find the perimeter by:

   i adding all four side lengths **ii** doubling the sum of the length and width.

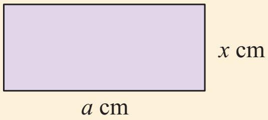
   a

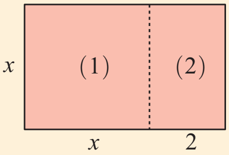
   b

   c

2. Use the results of **1** to suggest an expression equal to $2(a+b)$ which does not involve brackets.

3. 

   We can also find the area of a partitioned rectangle by two different ways.

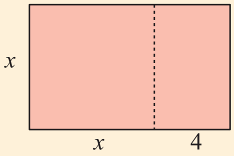
   a Write an expression for the area of the overall rectangle.

   b Write an expression for the area of (1) plus the area of (2).

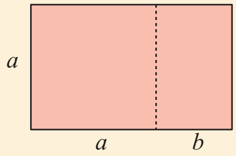
   c Copy and complete: $x(x+2)=\ldots\ldots$

4 Repeat **3** for:

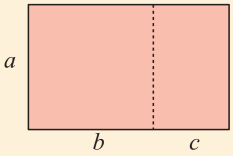
a



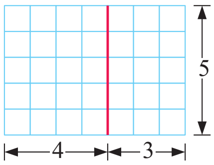
b



c



Consider the diagram alongside.



The total number of squares

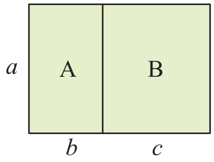
$= 5 \times (4+3)$

$= 5(4+3)$

However, it is also $5 \times 4 + 5 \times 3$ as there are 5×4 squares to the left of the red line and 5×3 squares to the right of it.

So, $5(4+3) = 5 \times 4 + 5 \times 3$.

Now consider the rectangle alongside, which has length $b+c$ and width a.



The area of the rectangle is $a(b+c)$ {length × width} but it is also area A + area B $= ab + ac$.

So, $a(b+c) = ab + ac$.

This is true for all values of a, b and c, and we call this result the **distributive law**.

$$\boldsymbol{a(b+c) = ab + ac}$$

So, to remove a set of brackets from an expression we multiply each term inside the brackets by the term in front of them. The resulting terms are then added.

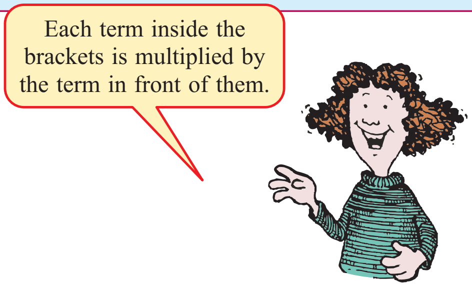
Example 1 **Self Tutor**

Expand: **a** $4(3x+1)$ **b** $5(7+2x)$ **c** $2(3y+4z)$

a $4(3x+1)$
$= 4 \times 3x + 4 \times 1$
$= 12x + 4$

b $5(7+2x)$
$= 5 \times 7 + 5 \times 2x$
$= 35 + 10x$

c $2(3y+4z)$
$= 2 \times 3y + 2 \times 4z$
$= 6y + 8z$



Each term inside the brackets is multiplied by the term in front of them.

EXERCISE 10A

1 Complete these expansions:

a $2(x+5) = 2x + ...$ **b** $5(y+3) = ... + 15$ **c** $6(3+a) = ... + 6a$

d $7(4+b) = 28 + ...$ **e** $3(z+4) = 3z + ...$ **f** $8(a+3) = ... + 24$

2 Expand these expressions:

a $3(a+2)$ **b** $2(x+5)$ **c** $5(a+4)$ **d** $7(2x+3)$

e $3(2y+1)$ **f** $4(4c+7)$ **g** $3(10+y)$ **h** $5(2+x)$

i $2(2+b)$ **j** $4(m+n)$ **k** $4(2a+b)$ **l** $3(2x+3y)$

Example 2 **Self Tutor**

Expand: **a** $2x(3x-2)$ **b** $(2a-1)b$

a
$$\begin{aligned} &2x(3x-2)\\ &= 2x(3x+-2)\\ &= 2x \times 3x + 2x \times -2\\ &= 6x^2 - 4x \end{aligned}$$

b
$$\begin{aligned} &(2a-1)b\\ &= b(2a-1)\\ &= b(2a+-1)\\ &= b \times 2a + b \times -1\\ &= 2ab - b \end{aligned}$$

3 Expand:

a $a(a-4)$ **b** $2a(a-3)$ **c** $a(a-6)$ **d** $y(4y-10)$

e $3p(2p-6)$ **f** $r(r-2)$ **g** $z(5-z)$ **h** $k(k-1)$

i $y(1-y)$ **j** $5x(3x-2)$ **k** $7p(2p-4)$ **l** $q(q-1)$

4 Expand:

a $(x+2)3$ **b** $(x+y)4$ **c** $(2+y)3$ **d** $(a+b)c$

e $(m+n)d$ **f** $(k+7)7$ **g** $(k+7)k$ **h** $(p+4)p$

5 Expand:

a $k(l+3)$ **b** $k(l-1)$ **c** $k(l+5)$

d $x(y-2)$ **e** $(a-2)b$ **f** $(x+6)y$

g $(k+7)l$ **h** $(z-1)p$ **i** $5x(2y-3)$

j $2a(a+c)$ **k** $4k(k-2l)$ **l** $2x(3x-4y)$

6 Use the distributive law to expand:

a $3(z+2)$ **b** $3(3z-2)$ **c** $10(2z-3y)$

d $7(x+3z+1)$ **e** $6(2-3a-5b)$ **f** $4(5z-2x+3y)$

g $2a(3x-4y+7)$ **h** $x(5-2x+3y)$ **i** $2p(3+x-2q)$

j $4(2x-5y-2)$ **k** $6(m+2n+8)$ **l** $7x(x+3y+4)$

m $5x(x+3y+7z)$ **n** $8x(a-3b+c)$ **o** $10x(x+5)+1$

p $9y(x-z+p)$ **q** $6a(a+5b+2c)$ **r** $3x(x^2+3x+9)$

B SIMPLIFYING ALGEBRAIC EXPRESSIONS

We have already seen that **like terms** are terms with exactly the same variable form. They contain the **same variables**, to the **same** powers or indices.

For example, xy and $3xy$ are like terms, and $2z^2y$ and $10yz^2$ are like terms,
but $5x$ and $3x^2$ are *not* like terms, and $5xy$ and $7yz$ are *not* like terms.

We can now simplify expressions involving brackets by expanding the brackets and then collecting like terms.

Example 3 **Self Tutor**

Expand the brackets and then simplify by collecting like terms:

a $6y + 2(y - 4)$ **b** $2(2x + 1) + 3(x - 2)$

a
$$\begin{aligned} &6y + 2(y - 4) \\ =\ &6y + 2y - 8 \\ =\ &8y - 8 \end{aligned}$$

b
$$\begin{aligned} &2(2x + 1) + 3(x - 2) \\ =\ &4x + 2 + 3x - 6 \\ =\ &7x - 4 \end{aligned}$$

Each term inside the brackets is multiplied by the term in front of them.

EXERCISE 10B

1 Expand and then simplify by collecting like terms:

a $2 + 3(x + 2)$
b $2 + 5(a + 7)$
c $3(n + 1) + 2(n + 3)$
d $3n + 2(n + 3)$
e $2(x - 6) + 5(x - 1)$
f $8(y - 2) + 3(y + 6)$
g $3(x + 4) + 6(5 + x)$
h $6(2 + y) + 8(y + 1)$
i $4(x + 7) + 11(2 + x)$
j $12(y + 3) + 3(3 + y)$
k $2(x - 4) + (x - 4)x$

Example 4 **Self Tutor**

Expand and then simplify by collecting like terms:
$2a(a + 5) + 3(a + 4)$

$$\begin{aligned} &2a(a + 5) + 3(a + 4) \\ =\ &2a \times a + 2a \times 5 + 3 \times a + 3 \times 4 \\ =\ &2a^2 + 10a + 3a + 12 \qquad \{10a \text{ and } 3a \text{ are like terms}\} \\ =\ &2a^2 + 13a + 12 \end{aligned}$$

Like terms have identical variables to the same powers.

2 Expand and then simplify by collecting like terms:

a $m(m + 2) + m(2m + 1)$
b $x(x + 2) - x^2$
c $3a(a + 2) - 2a^2$
d $5x(x + 2) - 2$
e $3a(a + 2) + 5a(a + 1)$
f $4(p + 3q) + 2(p + 2q)$
g $x(x + 3y) + 2x(x + y)$
h $4(3 + 2x) + 4x(x + 1)$

C BRACKETS WITH NEGATIVE COEFFICIENTS

When the number or term in front of a set of brackets is negative, we say it has a **negative coefficient**.

When we expand the brackets we use the distributive law as before. We place the negative coefficient in brackets to make sure we get the signs correct.

Example 5 **Self Tutor**

Expand: **a** $-3(x+4)$ **b** $-(5-x)$

a

$$\begin{aligned} & -3(x+4) \\ &= (-3)\times x + (-3)\times 4 \\ &= -3x + (-12) \\ &= -3x - 12 \end{aligned}$$

b

$$\begin{aligned} & -(5-x) \\ &= -1(5+-x) \\ &= (-1)\times 5 + (-1)\times(-x) \\ &= -5 + x \\ &= x - 5 \end{aligned}$$

With practice you should not need all the middle steps.

EXERCISE 10C

1 Complete the following expansions:

a $-2(x+5) = -2x - ....$ **b** $-2(x-5) = -2x + ...$

c $-3(y+2) = -3y - ....$ **d** $-3(y-2) = -3y + ...$

e $-(b+3) = -b - ....$ **f** $-(b-3) = -b + ...$

g $-4(2m+3) = ... - 12$ **h** $-4(2m-3) = ... + 12$

2 Expand:

a $-2(x+5)$ **b** $-3(2x+1)$ **c** $-3(4-x)$ **d** $-6(a+b)$

e $-(x+6)$ **f** $-(x-3)$ **g** $-(5+x)$ **h** $-(8-x)$

i $-5(x+1)$ **j** $-4(3+x)$ **k** $-(3b-2)$ **l** $-2(5-c)$

Example 6 **Self Tutor**

Expand and simplify:

a $3(x+2) - 5(3-x)$ **b** $x(3x-4) - 2x(x+1)$

a

$$\begin{aligned} & 3(x+2) - 5(3-x) \\ &= 3\times x + 3\times 2 + (-5)\times 3 + (-5)\times(-x) \\ &= 3x + 6 - 15 + 5x \\ &= 8x - 9 \end{aligned}$$

b

$$\begin{aligned} & x(3x-4) - 2x(x+1) \\ &= x\times 3x + x\times(-4) + (-2x)\times x + (-2x)\times 1 \\ &= 3x^2 - 4x - 2x^2 - 2x \\ &= x^2 - 6x \end{aligned}$$

3 Expand and simplify:

a $3(x+2)-2(x+1)$ **b** $4(x-7)-2(3-x)$ **c** $3(x-2)-2(x+2)$

d $3(y-4)-2(y+3)$ **e** $5(y+2)-2(y-3)$ **f** $6(b-3)-3(b-1)$

4 Expand and simplify:

a $x(x+4)-x(x+2)$ **b** $x(2x-1)-x(7-x)$ **c** $-(x+6)-2(x+1)$

d $-2(x-1)-3(5-x)$ **e** $-a(a+2)-2a(1-a)$ **f** $-(11-a)-2(a+6)$

D THE PRODUCT $(a+b)(c+d)$

The expression $(a+b)(c+d)$ can be expanded by using the distributive law *three times*.

$$\begin{aligned}(a+b)(c+d) &= (a+b)c+(a+b)d \qquad \{\text{compare } \blacksquare(c+d)=\blacksquare c+\blacksquare d\}\\ &= c(a+b)+d(a+b)\\ &= ac+bc+ad+bd\end{aligned}$$

Example 7 **Self Tutor**

Expand and simplify:

a $(x+y)(p+q)$ **b** $(x+2)(x+5)$

a
$$\begin{aligned}&(x+y)(p+q)\\ &=(x+y)p+(x+y)q\\ &=p(x+y)+q(x+y)\\ &=px+py+qx+qy\end{aligned}$$

b
$$\begin{aligned}&(x+2)(x+5)\\ &=(x+2)x+(x+2)5\\ &=x(x+2)+5(x+2)\\ &=x^2+2x+5x+10\\ &=x^2+7x+10\end{aligned}$$

EXERCISE 10D

1 Expand and simplify:

a $(a+b)(m+n)$ **b** $(p+q)(c+d)$ **c** $(x+y)(a+b)$

d $(a-b)(c+d)$ **e** $(c-d)(r+s)$ **f** $(x-y)(a+2)$

g $(a+b)(m-n)$ **h** $(c+d)(x-3)$ **i** $(r+s)(p-4)$

2 Expand and simplify:

a $(x+2)(x+3)$ **b** $(x+5)(x+3)$ **c** $(x+6)(x+8)$

d $(2x+1)(x+2)$ **e** $(3x+2)(x+6)$ **f** $(4x+1)(x+2)$

g $(x-1)(x+5)$ **h** $(x-4)(x+7)$ **i** $(x-6)(x+3)$

j $(x-y)(x+3)$ **k** $(x+2)(x-3)$ **l** $(x+3)(x-7)$

m $(x-4)(x-3)$ **n** $(x-5)(x-8)$ **o** $(x-6)(x-4)$

p $(a+2)^2$ **q** $(b+5)^2$ **r** $(c+7)^2$

s $(x-1)^2$ **t** $(x-4)^2$ **u** $(y-d)^2$

E GEOMETRIC APPLICATIONS

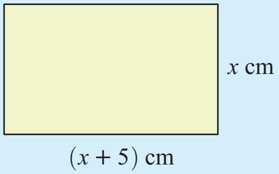
Now that we have the distributive law, we can often simplify expressions for the perimeter and area of geometric figures.

Example 8 Self Tutor

For the given rectangle, find in simplest form expressions for:

a its perimeter P

b its area A



a Perimeter $= 2(\text{length} + \text{width})$

$\therefore \quad P = 2[(x+5)+x]$ cm

$\therefore \quad P = 2(2x+5)$ cm

$\therefore \quad P = 4x+10$ cm

b Area $= \text{length} \times \text{width}$

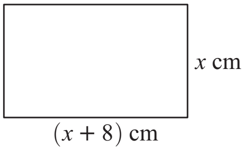
$\therefore \quad A = x(x+5)$ cm^2

$\therefore \quad A = x^2+5x$ cm^2

EXERCISE 10E

1 Find, in simplest form, an expression for the perimeter P of:

a



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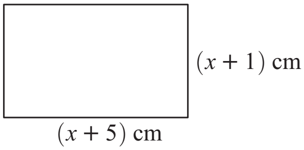
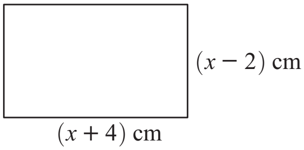
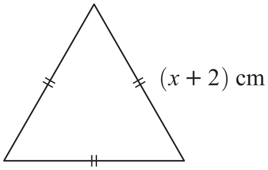


c

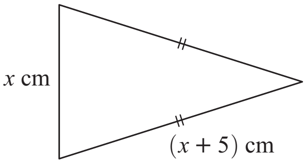


2 Find, in simplest form, an expression for the perimeter P of:

a



b



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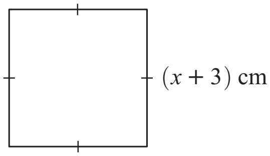
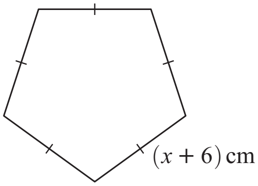


d



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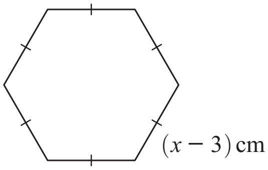
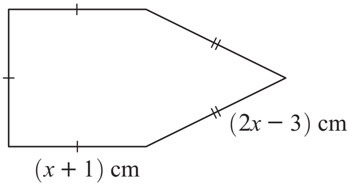
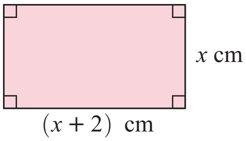


f



3 Find in simplest form, an expression for the area A of:

a



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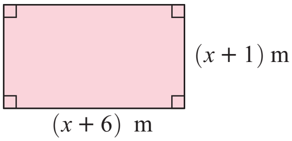

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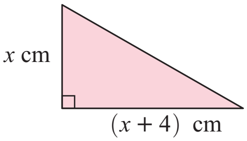

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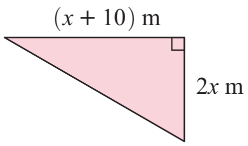

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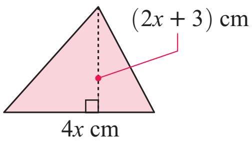

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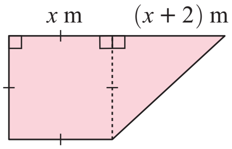


4 Find, in simplest form, expressions for the perimeter P and area A in the **Opening Problem** on page **200**.

F FACTORISATION OF ALGEBRAIC EXPRESSIONS

Factorisation is the reverse process of expansion.

For example: $3(x+2) = 3x+6$ is *expansion*
$(3x+6) = 3(x+2)$ is *factorisation.*

To factorise an algebraic expression we need to insert brackets.

We find the **HCF** (**highest common factor**) of all terms in the expression then place it before the brackets being inserted.

Example 9 Self Tutor

Find the HCF of:

a $3a$ and 9 **b** $4ab$ and $2b$ **c** $5x^2$ and $10x$

a
$3a = 3 \times a$
$9 = 3 \times 3$
$\therefore$ HCF $= 3$

b
$4ab = 2 \times 2 \times a \times b$
$2b = 2 \times b$
$\therefore$ HCF $= 2b$

c
$5x^2 = 5 \times x \times x$
$10x = 2 \times 5 \times x$
$\therefore$ HCF $= 5x$

Example 10 Self Tutor

Factorise: **a** $2a+6$ **b** $ab-bd$

a The HCF of $2a$ and 6 is 2.
$\therefore$ $2a+6$
$= 2 \times a + 2 \times 3$
$= 2(a+3)$

b The HCF of ab and $-bd$ is b.
$\therefore$ $ab-bd$
$= a \times b - b \times d$
$= b(a-d)$

EXERCISE 10F

1 Find the missing factor:

a $3 \times \square = 3x$ **b** $3 \times \square = 12b$ **c** $5 \times \square = 10xy$

d $\square \times 4x = 4x^2$ **e** $\square \times 5y = 10y^2$ **f** $\square \times 3a = 3a^2$

g $x \times \square = 2xy$ **h** $\square \times 2x = 6x^2$ **i** $6y \times \square = 12y^2$

2 Find the HCF of:

a $4x$ and 12 **b** $3x$ and 6 **c** $4y$ and 14

d $3ab$ and $6b$ **e** $4y$ and $4xy$ **f** $5ad$ and $10a$

g $6x^2$ and $2x$ **h** $3y$ and $9y^2$ **i** $12a$ and $3a^2$

j $2(x-1)$ and $3(x-1)$ **k** $4(x+2)$ and $x+2$ **l** $2(x+3)$ and $2x+6$

3 Factorise:

a $5a+10$ **b** $6a+8$ **c** $6a+12b$ **d** $4+8x$

e $11a+22b$ **f** $16x+8$ **g** $4a+8$ **h** $10+15y$

i $25x+20$ **j** $x+ax$ **k** $3x+mx$ **l** $ac+an$

4 Factorise:

a $2a-10$ **b** $4y-20$ **c** $3b-12$ **d** $6x-24$

e $6x-14$ **f** $14y-7$ **g** $5a-15$ **h** $10-15b$

i $20b-25$ **j** $16b-24$ **k** $x-xy$ **l** $ab-ac$

Example 11 **Self Tutor**

Factorise: **a** $3x^2+12x$ **b** $4y-2y^2$

a $3x^2+12x$

$= 3 \times x \times x + 4 \times 3 \times x$

$= 3x(x+4)$

b $4y-2y^2$

$= 2 \times 2 \times y - 2 \times y \times y$

$= 2y(2-y)$

5 Factorise:

a x^2+3x **b** $2x^2+8x$ **c** $3x^2-12x$ **d** $6x-x^2$

e $8x-4x^2$ **f** $15x-6x^2$ **g** $2x^3+4x^2$ **h** $2x^3+5x^2$

Example 12 **Self Tutor**

Factorise: $3(a+b)+x(a+b)$

$3(a+b)+x(a+b)$ has common factor $(a+b)$

$\therefore \quad 3(a+b)+x(a+b)$

$= (a+b)(3+x)$

6 Factorise:

a $2x^3 + 2x^2 + 4x$ **b** $x^4 + 2x^3 + 3x^2$ **c** $6x^3 - 3x^2 + 5x$

d $ax^2 + 2ax + a^2x$ **e** $3my^2 + 3my + 6m^2y$ **f** $4x^2a + 6x^2a^2 + x^4a^3$

7 Factorise:

a $2(x + a) + p(x + a)$ **b** $n(x - 2) + p(x - 2)$ **c** $r(y + 5) + 4(y + 5)$

d $3(x + 4) - x(x + 4)$ **e** $a(7 - x) - b(7 - x)$ **f** $4(x + 11) + y(x + 11)$

g $x(x + 2) + x + 2$ **h** $x(x + 2) - x - 2$ **i** $x(x + 3) + 2x + 6$

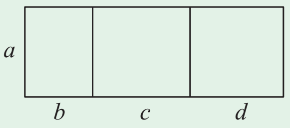
j $x(x - 1) + 2x - 2$ **k** $x(x + 5) + 3x + 15$ **l** $x(x - 4) - 2x + 8$

KEY WORDS USED IN THIS CHAPTER

- distributive law
- highest common factor
- expansion
- like terms
- factorisation
- negative coefficient

REVIEW SET 10A

1 Copy and complete:
Using the area of rectangles, the diagram alongside shows that
$a(.... + ... + ...) =$



2 Expand:

a $x(y + z)$ **b** $3(2x - 5)$ **c** $-x(3 - x)$ **d** $(x + 5)d$

3 Expand:

a $3(x^2 - 6x + 4)$ **b** $-2(x^2 - x + 1)$

4 Expand and simplify by collecting like terms:

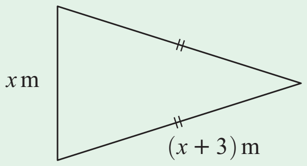
a $2(x + 5) + 3(2x + 1)$ **b** $3(x - 2) + 4(3 - x)$

c $3(x - 4) - 2(x + 3)$ **d** $x(x + 3) + 5(x + 6)$

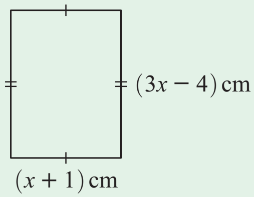
e $x(2 - x) - (x - 1)$ **f** $y(2 + y) - 3y(2 - y)$

5 Find, in simplest form, an expression for the perimeter P of:

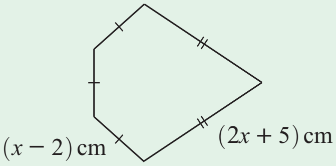
a



b

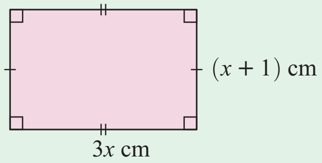


c



6 Find, in simplest form, an expression for the area A of:

a



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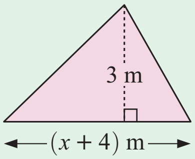

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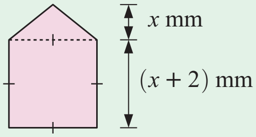


7 Expand and simplify:

a $(x+3)(x+4)$ **b** $(x-3)(2x+1)$ **c** $(2x-1)(x-7)$

8 Factorise:

a $3x+12$ **b** x^2-3x **c** $ab+bc-2b$

d $a(x-2)+3(x-2)$ **e** $x(x+3)+2x+6$

REVIEW SET 10B

1 Expand and simplify:

a $3(2-y)$ **b** $4(3t+2)$ **c** $-a(a+2)$ **d** $(x+6)n$

2 Expand and simplify:

a $x+2(x+1)$ **b** $x-2(x-4)$

3 Expand:

a $2x(x+y-3)$ **b** $-2x(3-x)$

4 Expand and simplify:

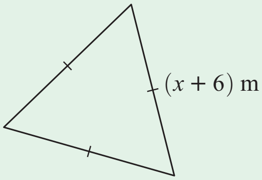
a $3(x+5)+2(x-3)$ **b** $4(y+5)+3(2+x)$

c $5(x-2)-2(x-1)$ **d** $2x(x+2)+x(x-3)$

e $3x(x+5)-(x-5)$ **f** $n(n+2)-2n(1-n)$

5 Find, in simplest form, an expression for the perimeter P of:

a



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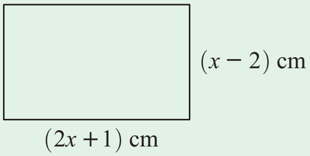

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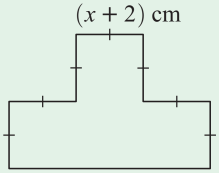
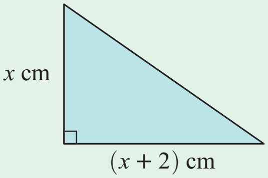


6 Find, in simplest form, an expression for the area A of:

a



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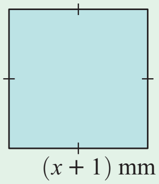

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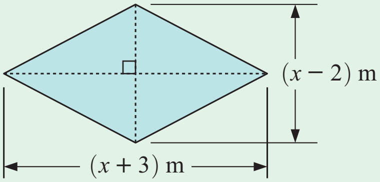


7 Expand and simplify:

a $(x+2)(x+9)$ **b** $(x+3)(x-2)$ **c** $(x-7)(x-4)$

8 Factorise:

a $4x+24y$ **b** $2x^2-8x$ **c** $3a+6ab+9a^2$

d $3(x-6)+d(x-6)$ **e** $2x(x+4)+3x+12$

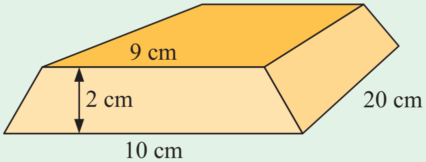
Chapter 11

Further measurement

Contents:

OPENING PROBLEM

Gold bars are made with a uniform cross-section that is a trapezium. The length of each bar is 20 cm.



Things to think about:

- How can we find the volume of each bar?
- How heavy is each bar if one cubic centimetre of gold weighs 19.3 grams?
- How can we find the capacity and shape of a rectangular box in which each bar would fit snugly?

A VOLUME

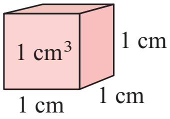
The **volume** of a three-dimensional object is the amount of space it occupies. This space is measured in **cubic units**.

The units we use for the measurement of volume are related to the units we use for length.

1 **cubic millimetre** (mm^3) is the volume of a cube with side length 1 mm.

The volume of a small jewellery box might be measured in mm^3.

1 **cubic centimetre** (cm^3) is the volume of a cube with side length 1 cm.



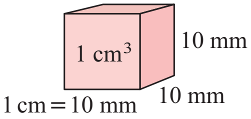
The volume of a petrol tank might be measured in cm^3.

1 **cubic metre** (m^3) is the volume of a cube with side length 1 m.

The volume of a rubbish dumpster might be measured in m^3.

VOLUME UNIT CONVERSIONS

We can use length unit conversions to convert from one unit of volume to another.



Consider a cube with a side of length 1 cm. We have just seen that this cube has a volume of 1 cm^3.

But we know that $1\text{ cm} = 10\text{ mm}$.

So,
$$\begin{aligned}1\text{ cm}^3 &= 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}\\ &= 10\text{ mm} \times 10\text{ mm} \times 10\text{ mm}\\ &= 1000\text{ mm}^3\end{aligned}$$

Likewise,
$$\begin{aligned}1\text{ m}^3 &= 1\text{ m} \times 1\text{ m} \times 1\text{ m}\\ &= 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm}\\ &= 1\,000\,000\text{ cm}^3\end{aligned}$$

We can use the following **conversion diagram** to help convert between units:

$$\begin{tikzcd}
\text{m}^3 \arrow[r, bend left, "\times 1\,000\,000"] & \text{cm}^3 \arrow[l, bend left, "\div 1\,000\,000"] \arrow[r, bend left, "\times 1000"] & \text{mm}^3 \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

Example 1 **Self Tutor**

Convert: **a** 4.56 cm^3 to mm^3 **b** $324\,000\text{ cm}^3$ to m^3

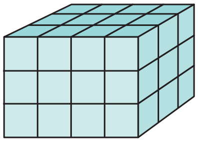
a
$$\begin{aligned}&4.56\text{ cm}^3\\ &= (4.56 \times 1000)\text{ mm}^3\\ &= 4560\text{ mm}^3\end{aligned}$$

b
$$\begin{aligned}&324\,000\text{ cm}^3 \text{ to m}^3\\ &= (324\,000 \div 1\,000\,000)\text{ m}^3\\ &= 0.324\text{ m}^3\end{aligned}$$

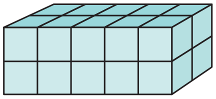
EXERCISE 11A

1 Find the number of cubic units in each of the following solids:

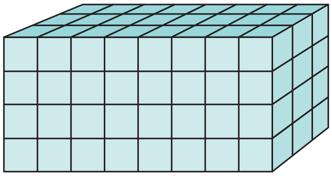
a



b



c



2 Give the units of volume that would be most suitable for measuring the amount of space occupied by:

a a chopping board **b** a nail **c** a cat
d a tractor **e** an orange **f** a mobile phone
g a grain of rice **h** a sand dune **i** a book

3 Convert:

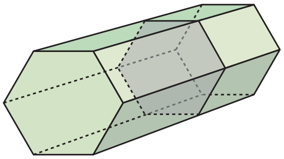
a 0.145 cm^3 to mm^3 **b** 0.003 m^3 to cm^3 **c** $14\,971\text{ mm}^3$ to cm^3
d $48\,500\text{ cm}^3$ to m^3 **e** 2.7 m^3 to cm^3 **f** $118\,000\text{ cm}^3$ to m^3
g 34.3 cm^3 to mm^3 **h** 1.694 mm^3 to cm^3 **i** $0.006\,25\text{ m}^3$ to mm^3

B VOLUME FORMULAE

As with the areas of plane figures, we have formulae for calculating the **volume** of common solids. In this chapter we will consider formulae for the group of solids called **prisms**.

A **prism** is a solid with a uniform cross-section that is a polygon.

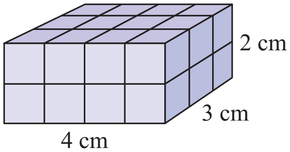
If we take any slice of a prism parallel to its end, then the exposed surface will be exactly the same shape and size as the end. This is what we mean by a **uniform cross-section**.



RECTANGULAR PRISMS

A simple example of a prism is the rectangular prism shown below.

Check that you agree with the following facts:



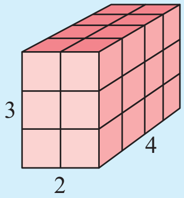
- There are $4 \times 3 = 12$ cubes in each layer.
- There are 2 layers.
- There are $12 \times 2 = 24$ cubes altogether.
- The volume of this rectangular prism is 24 cm^3.
- The volume can be found by the multiplication length × width × height.

Volume of rectangular prism = length × width × height = lwh

Example 2

Self Tutor

Find the number of cubic units in:



Volume $= lwh$

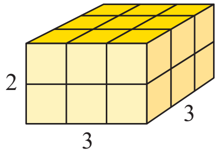
$\therefore \quad V = 4 \times 2 \times 3$

$\therefore \quad V = 24 \text{ units}^3$

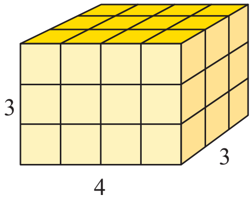
EXERCISE 11B.1

1 Find the number of cubic units in each of the following solids:

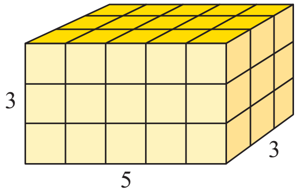
a



b



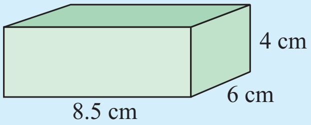
c



Example 3

Self Tutor

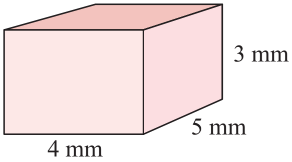
Find the volume of the rectangular prism:



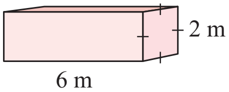
$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$
$$\therefore \quad V = 8.5 \text{ cm} \times 6 \text{ cm} \times 4 \text{ cm}$$
$$\therefore \quad V = 204 \text{ cm}^3$$

2 Find the volumes of the following prisms:

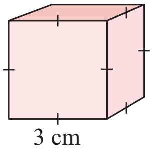
a



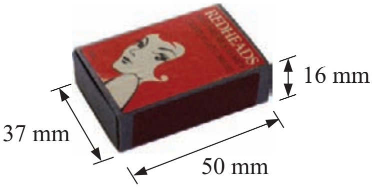
b



c

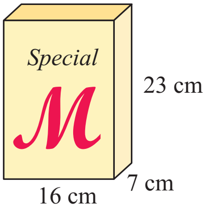


3 Find the volume of the match box with dimensions shown:

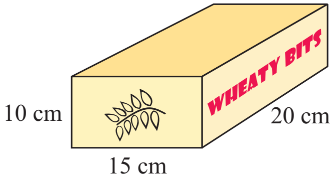


4 Which of the following cereal boxes has the larger volume?

A



B



5 Calculate the volume of the rubbish dumpster:

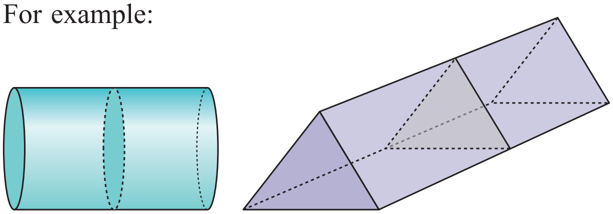
6 Find the volume of a rectangular tank 8 m by 4.5 m by 5 m.

7 Find the volume of air in a classroom 5 m long, 4 m wide and 3.2 m high.

SOLIDS OF UNIFORM CROSS-SECTION

If we can take any slice of a solid parallel to its end and find the exposed surface is exactly the same shape and size as the end, then the solid is said to have **uniform cross-section**.

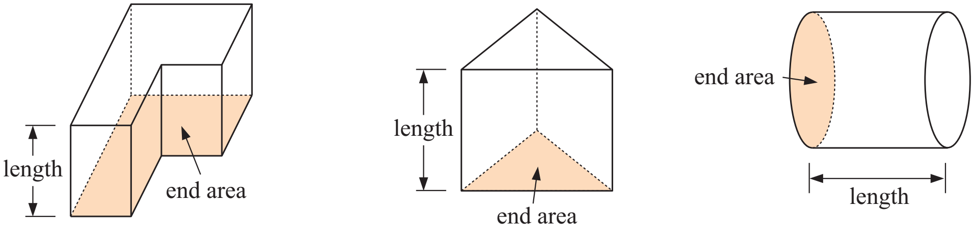
For example:



For any solid of uniform cross-section:

Volume = area of end × length

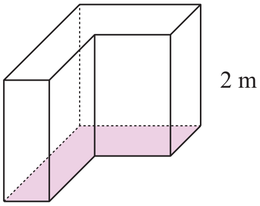
This formula also applies for these solids of uniform cross-section:



EXERCISE 11B.2

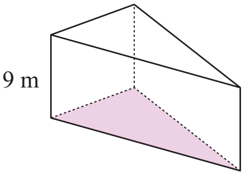
1 Find the volumes of the following solids:

a



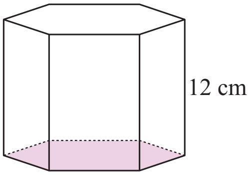
area of end = 9 m^2

b



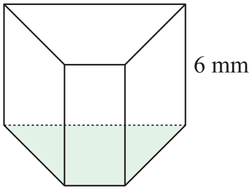
area of end = 28 m^2

c



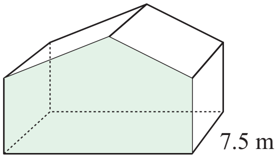
area of end = 50 cm^2

d



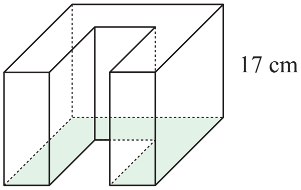
area of end = 6 mm^2

e



area of end = 12 m^2

f

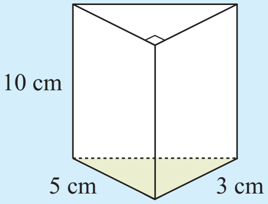


area of end = 30 cm^2

2 A solid of uniform cross-section has end area 42.5 cm^2 and volume 348.5 cm^3. How long is the solid?

Example 4 **Self Tutor**

Find the volume of this solid:



Volume

$= \text{area of end} \times \text{length}$

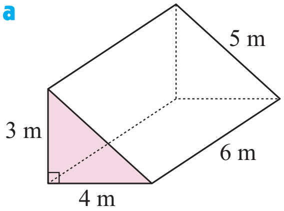
$= \frac{1}{2}bh \times \text{length}$ {area of triangle formula}

$= \frac{1}{2}(3 \times 5) \times 10 \text{ cm}^3$

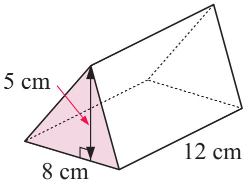
$= 75 \text{ cm}^3$

3 Find the volumes of the following solids:

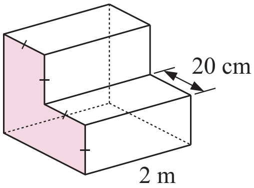
a



b



c



4 An empty garage has floor area 80 m^2 and a roof height of 4 m. Find the volume of air in the garage.

5 Each month a rectangular swimming pool 6 m by 5 m by 2 m deep costs $0.50 per cubic metre of water to maintain. How much will it cost to maintain the pool for one year?

6 Concrete costs €128 per cubic metre. What will it cost to concrete a driveway 20 m long and 3 m wide to a depth of 12 cm?

7 64 cartons of paper are delivered to your school. Each carton measures 40 cm by 30 cm by 25 cm. Is it possible to fit all the cartons into a storage cupboard 1 m by 1 m by 2 m? Explain your reasoning, using a diagram if you wish.

C CAPACITY

The **capacity** of a container is a measure of the volume it can hold. We can think of it as the space within the container.

We use the term **capacity** when we talk about fluids or gases.

For example, the **capacity** of a cup is the amount of liquid it can hold.

The **litre** (L) is the basic unit for the measurement of capacity.

Standard prefixes are added to measure smaller and larger capacities.

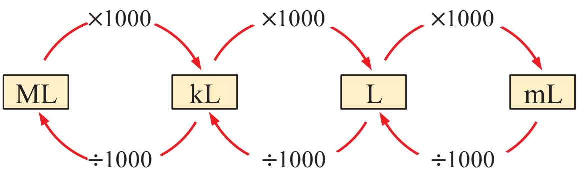
For example, other units include the millilitre (mL), kilolitre (kL), and megalitre (ML).

1 L = 1000 mL
1 kL = 1000 L
1 ML = 1000 kL

Here are some examples of familiar capacities:

Item	*Approx. capacity*	*Item*	*Approx. capacity*
Medicine glass	25 mL	Cup	250 mL
Milk carton	1 L	Petrol tank	65 L
Hot water system	170 L	50 m swimming pool	1500 kL
Dam	10 ML	Reservoir	1000 ML

We can convert between the units of capacity using the following **conversion diagram**:



Example 5

Self Tutor

Convert: **a** 4500 mL to L **b** 350 kL to L

a
$$\begin{aligned} & 4500 \text{ mL} \\ &= (4500 \div 1000) \text{ L} \\ &= 4.5 \text{ L} \end{aligned}$$

b
$$\begin{aligned} & 350 \text{ kL} \\ &= (350 \times 1000) \text{ L} \\ &= 350\,000 \text{ L} \end{aligned}$$

EXERCISE 11C.1

1 A small 'shot' glass for measuring spirits would most likely have a capacity of:

A 3 L **B** 30 mL **C** 3 mL **D** 3 kL **E** 30 L

2 A thermos used to keep hot drinks warm would most likely have a capacity of:

A 2 kL **B** 20 mL **C** 2 L **D** 20 L **E** 2 ML

3 A 25 m swimming pool would most likely have a capacity of:

A 400 mL **B** 400 ML **C** 400 L **D** 400 kL **E** 4 kL

4 A standard bucket would most likely have a capacity of:

A 900 mL **B** 90 L **C** 9 L **D** 9 mL **E** 90 kL

5 A bathtub would most likely have a capacity of:

A 65 L **B** 650 mL **C** 65 kL **D** 6500 mL **E** 650 L

6 Convert:

a 5 L to mL **b** 8.6 L to mL **c** 400 mL to L

d 5830 L to kL **e** 1 ML to mL **f** 3.25 kL to L

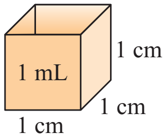
7 How many 350 mL bottles of ginger beer can be filled from a 20.3 L home brew kit?

VOLUME AND CAPACITY

The units for **capacity** and the units for **volume** are closely related.

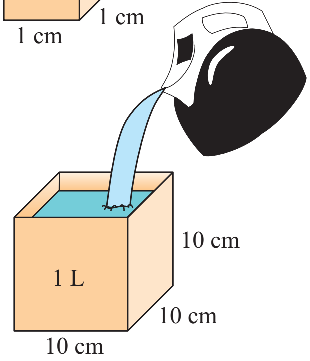
1 mL of fluid will fill a cube 1 cm × 1 cm × 1 cm.

1 cm^3 has capacity 1 mL.



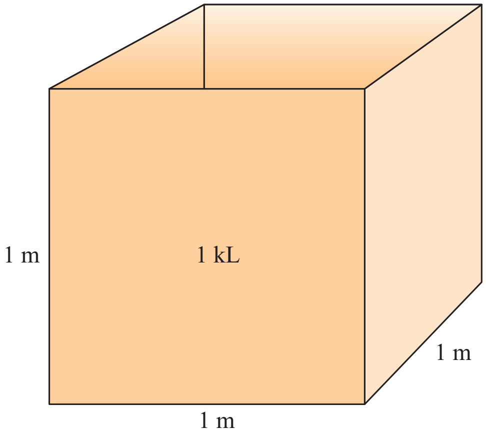
1 L of fluid will fill a cube 10 cm × 10 cm × 10 cm.

1000 cm^3 has capacity 1 L.



1 kL of fluid will fill a cube 1 m × 1 m × 1 m.

1 m^3 has capacity 1 kL.

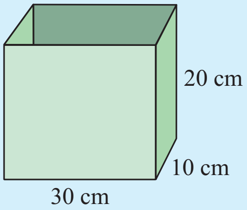


We can **summarise** the connection between volume units and capacity units in a table:

1 cm^3	has capacity	1 mL.
1000 cm^3	has capacity	1 L.
1 m^3	has capacity	1 kL.

Example 6 **Self Tutor**

Calculate the capacity of the container:



Volume

$= 30 \times 10 \times 20 \text{ cm}^3$

$= 6000 \text{ cm}^3$

$\therefore$ capacity

$= 6000 \text{ mL}$

$= (6000 \div 1000) \text{ L}$

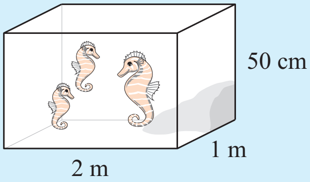
$= 6 \text{ L}$

We say that 1 cm^3 *is equivalent to* 1 mL and write 1 $\text{cm}^3 \equiv 1$ mL.

Example 7 **Self Tutor**

Find the capacity in litres of a fishtank 2 m by 1 m by 50 cm.

$$\begin{aligned} \text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 200 \text{ cm} \times 100 \text{ cm} \times 50 \text{ cm} \quad \{\text{units must all be the same}\} \\ &= 1\,000\,000 \text{ cm}^3 \\ \therefore \quad \text{capacity} &= 1\,000\,000 \text{ mL} \\ &= (1\,000\,000 \div 1000) \text{ L} \\ &= 1000 \text{ L} \end{aligned}$$



EXERCISE 11C.2

1 A container is 20 cm by 10 cm by 10 cm. Find:

- **a** the volume of space in the container in cm^3
- **b** the capacity of the container in mL
- **c** the capacity of the container in litres.

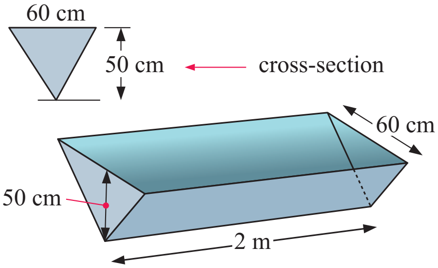
2 A rectangular container has dimensions 8 cm by 7 cm by 20 cm. Find its capacity in L.

3 A rectangular water tank has dimensions 4 m by 4 m by 2 m. Find its capacity in kL.

4 A rectangular petrol tank has dimensions 50 cm by 40 cm by 25 cm. How many litres of petrol are needed to fill it?

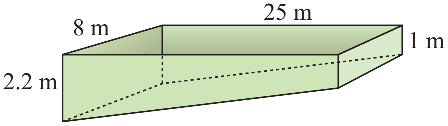
5 A water trough has triangular cross-section as shown. Its length is 2 m. Find:

- **a** the area of the triangle in cm^2
- **b** the volume of space in the trough in cm^3
- **c** the capacity of the trough in:
  - **i** litres
  - **ii** kilolitres.



6 A swimming pool has the dimensions shown. It has a cross-section in the shape of a trapezium. Find:

- **a** the area of the trapezium in m^2
- **b** the capacity of the swimming pool in ML.



Example 8 **Self Tutor**

Find the capacity, in megalitres, of a reservoir with a surface area of 1 hectare and an average depth of 2.5 metres.

$$\begin{aligned} \text{Volume} &= \text{surface area} \times \text{average depth} \\ &= 10\,000 \times 2.5 \text{ m}^3 \quad \{1 \text{ hectare} = 10\,000 \text{ m}^2\} \\ &= 25\,000 \text{ m}^3 \end{aligned}$$

$$\begin{aligned}\therefore \quad \text{capacity} &= 25\,000 \text{ kL} \qquad \{1 \text{ m}^3 \equiv 1 \text{ kL}\}\\ &= (25\,000 \div 1000) \text{ ML}\\ &= 25 \text{ ML}\end{aligned}$$

7 A kidney-shaped swimming pool has surface area 15 m^2 and a constant depth of 2 metres. Find the capacity of the pool in kilolitres.

8 A lake has an average depth of 6 m and a surface area of 35 ha. Find its capacity in ML.

D MASS

The **mass** of an object is the amount of matter it contains.

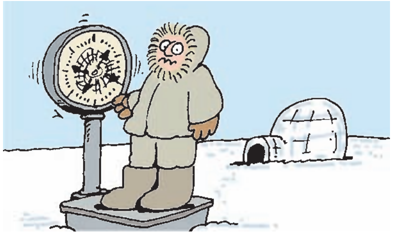
Be careful not to confuse **mass** with **weight**, although in everyday use they are often interchanged.

The **mass** of an object is constant. It is the same no matter where the object is.

The **weight** of an object is the force due to gravity which is exerted on the object. The weight of an object depends on its distance from the Earth's centre.

The Earth is not a perfect sphere. It is fatter around its equator than at its poles. This means that you would weigh more at the North Pole than you would at the equator.

The **kilogram** (kg) is the base unit of mass in the SI system. Other units of mass which are commonly used are the milligram (mg), gram (g), and tonne (t).

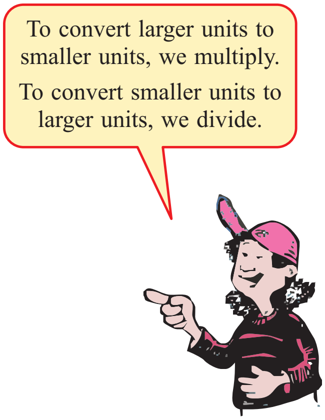


1 g = 1000 mg
1 kg = 1000 g
1 t = 1000 kg

The measurement standard of mass is a platinum-iridium cylinder with a mass of 1 kg. It is kept at the International Bureau of Weights and Measures at Sevres near Paris.

As we have done with other units of measurement, we can convert from one unit of mass to another.

$$\begin{tikzcd} \boxed{\text{t}} \arrow[r, bend left, "\times 1000"] & \boxed{\text{kg}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{g}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{mg}} \arrow[l, bend left, "\div 1000"] \end{tikzcd}$$



Example 9 Self Tutor

Convert the following to grams:

a 3.2 kg **b** 735 mg **c** 4.5 tonnes

a 3.2 kg
$= (3.2 \times 1000)$ g
$= 3200$ g

b 735 mg
$= (735 \div 1000)$ g
$= 0.735$ g

c 4.5 tonnes
$= (4.5 \times 1000 \times 1000)$ g
$= 4\,500\,000$ g

Example 10 Self Tutor

Express in kilograms:

a 8.6 t **b** 5860 g **c** 39 000 mg

a 8.6 t
$= (8.6 \times 1000)$ kg
$= 8600$ kg

b 5860 g
$= (5860 \div 1000)$ kg
$= 5.86$ kg

c 39 000 mg
$= (39\,000 \div 1000)$ g
$= 39$ g
$= (39 \div 1000)$ kg
$= 0.039$ kg

EXERCISE 11D.1

1 Give the units you would use to measure the mass of:

a the earth **b** a car **c** a vitamin pill
d this book **e** a gold ring **f** a chair

2 Convert the following to grams:

a 6 kg **b** 47 mg **c** 3750 mg **d** 7.3 t
e 0.45 kg **f** 24.5 kg **g** 0.32 t **h** 3642 mg

3 Express in kilograms:

a 5 t **b** 6000 g **c** 500 000 mg **d** 300 g
e 1847 mg **f** 386 g **g** 4.5 t **h** 5642 g

Example 11 Self Tutor

If one egg has a mass of 55 g, find the total mass of eggs in 50 cartons, each containing 12 eggs.

Total mass $= 55$ g $\times$ 12 eggs $\times$ 50 cartons
$= 33\,000$ g
$= 33$ kg {kg is more appropriate than g}

4 Find the total mass of 32 chocolates, each of mass 28 grams.

5 If a clothes peg has a mass of 6.5 g, how many pegs are there in a 13 kg box?

6 If a roof tile has mass 1.25 kilograms, how many tiles could a van with a load limit of 8 tonnes carry?

7 Find the total mass in tonnes of 3500 books, each with mass 800 grams.

8 If the mass of 8000 oranges is 1.04 tonnes, what is the average mass of one orange?

THE RELATIONSHIP BETWEEN UNITS

The units for volume, capacity and mass in the metric system are related in the following way:

1000 cm^3 or 1 L of pure water at 4^oC has a mass of 1 kg.
1 cm^3 or 1 mL of pure water at 4^oC has a mass of 1g.

Example 12 **Self Tutor**

a Find the mass of water which will fill a bucket with a capacity of 4 L.

b If the empty bucket has a mass of 250 g, what is the total mass of the bucket of water?

a 1 L of water has a mass of 1 kg
$\therefore$ 4 L of water has a mass of 4 kg
$\therefore$ 4 kg of water will fill the bucket.

b Mass of the water-filled bucket
$= 4$ kg $+ 250$ g
$= 4$ kg $+ 0.25$ kg
$= 4.25$ kg or 4250 g

EXERCISE 11D.2

1 What is the mass of 3 L of pure water at 4^oC?

2 What is the mass of 20 cm^3 of pure water at 4^oC?

3 A ceramic jug has a mass of 750 g. 2 litres of water are poured into the jug. What is the total mass of the water-filled jug?

4 A fishtank has internal measurements of 60 cm length, 25 cm width and 30 cm height.
  a What is the capacity of the fishtank?
  b What mass of water is needed to completely fill the fishtank?
  c The fishtank itself has a mass of 4 kg. If the fishtank is filled with water to a level 2 cm from the top of the tank, what is the total mass?

5 Paper is made in different thicknesses but it is graded in *grams per square metre* or *gsm.* A4 photocopy paper is usually 80 gsm and measures 29.7 cm by 21 cm.
  a What is the mass of 1 m^2 of 80 gsm photocopy paper?
  b Find the area in square metres of 16 A4 sheets of paper.
  c Find the mass of one sheet of A4 paper.

E TIME

The units of time we use today are based on the rotation of the Earth and its movement around the Sun.

The time for the Earth to complete one rotation about its axis is called a **day**. The day is divided into hours, minutes and seconds.

The time for the Earth to complete an orbit of the Sun is called a **year**.

The base unit of time in the International System of Units is the **second**, abbreviated **s**.

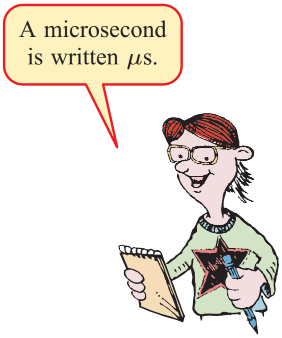
1 minute = 60 seconds
1 hour = 60 minutes = 3600 seconds
1 day = 24 hours
1 week = 7 days
1 year = 12 months = $365\frac{1}{4}$ days

For times **less than one second** we use base 10. Such small units of time are becoming more common with the advances in computer technology.

For example, some of the units of time used in the computer world are:

1 microsecond = $\frac{1}{1\,000\,000}$ second

1 nanosecond = $\frac{1}{1\,000\,000\,000}$ second



For times **greater than one year**, base 10 is also used.

1 decade = 10 years
1 century = 100 years
1 millennium = 1000 years

Example 13 **Self Tutor**

Convert: **a** 4 hours and 35 minutes to minutes **b** 90 000 seconds to hours.

a 4 hours and 35 minutes $= (4 \times 60)$ min + 35 min {60 min in 1 hour}
$= 275$ min

b 90 000 seconds $= (90\,000 \div 60)$ min {60 s in 1 min}
$= 1500$ min
$= (1500 \div 60)$ hours {60 min in 1 hour}
$= 25$ hours

Example 14 **Self Tutor**

Which time period is longer: 10 000 minutes or 8 days?

To compare the time periods, their units must be the same.

$$\begin{aligned} 8 \text{ days} &= (8 \times 24) \text{ hours} \qquad \{24 \text{ hours in 1 day}\} \\ &= 192 \text{ hours} \\ &= (192 \times 60) \text{ min} \qquad \{60 \text{ min in 1 hour}\} \\ &= 11\,520 \text{ min, which is more than } 10\,000 \text{ minutes.} \end{aligned}$$

$\therefore$ 8 days is longer than 10 000 minutes.

EXERCISE 11E.1

1 Convert to minutes:

- **a** 13 hours
- **b** 1560 seconds
- **c** 2 days
- **d** 6 hours 17 minutes
- **e** 3 days 5 hours 38 minutes

2 Convert to days:

- **a** 8 years
- **b** 4320 minutes
- **c** 864 hours
- **d** 864 000 s

3 Convert to seconds:

- **a** 3 hours
- **b** 47 minutes
- **c** 5 hours 7 minutes
- **d** 5 weeks

4 Which time period is longer:

- **a** 1000 seconds or 16 minutes
- **b** 6 hours or 20 000 seconds
- **c** 2 weeks or 20 000 minutes
- **d** 5 days 7 hours or 8000 minutes?

5 Simone has spent 10 minutes brushing her teeth every day for the last 10 years. Calculate the total time she has spent brushing her teeth in the last 10 years. Give your answer to the nearest day.

6 An English teacher marks 220 essays in 28 hours 36 minutes. On average, how long did it take to mark one essay?

Example 15 **Self Tutor**

What is the time difference between 8:45 am and 2:30 pm?

$$\begin{aligned} 8{:}45 \text{ am to } 9{:}00 \text{ am} &= 15 \text{ min} \\ 9{:}00 \text{ am to } 2{:}00 \text{ pm} &= 5 \text{ h} \\ 2{:}00 \text{ pm to } 2{:}30 \text{ pm} &= \underline{30 \text{ min}} \\ \therefore \text{ the time difference is } & \ 5 \text{ h } 45 \text{ min} \end{aligned}$$

7 Find the time difference between:

- **a** 3:20 am and 9:43 am
- **b** 11:17 am and 6:28 pm
- **c** 4:10 pm and 10:08 pm
- **d** 10:53 am and 3:29 pm
- **e** 9:46 pm and 3:15 am
- **f** 7:25 am and 9:45 am on the next day

8 A movie started at 8:40 pm and finished at 11:13 pm. How long was the movie?

9 Raj went to sleep at 9:45 pm and woke up at 6:30 the next morning. For how long did he sleep?

10 Study the train timetable for the route from Blue Creek to the City. How long does the train take to get from Blue Creek to the City:

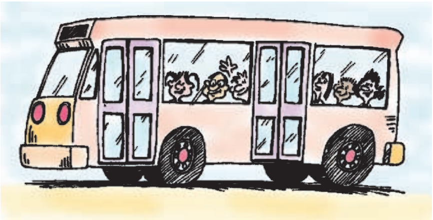
- **a** in the morning
- **b** in the afternoon
- **c** using the morning express?

	Blue Creek	*Orford*	*Jondy*	*Rufus*	*Willow*	*City*
am	6:02	6:06	6:11	6:14	6:17	6:24
pm	5:45	5:49	5:54	5:57	6:00	6:07
am exp.	7:56		8:03			8:12

11 Below is the summer timetable for a tourist bus service in Christchurch, New Zealand.

Departure Times	*Bus A*	*Bus B*	*Bus C*	*Bus D*	*Bus E*	*Bus F*
City depot	8:00	8:15	8:30	8:45	9:00	9:15
Science Alive	9:30	9:45	10:00	10:15	10:30	10:45
Christchurch Gondola	11:20	11:35	11:50	12:05	12:20	12:35
Lyttelton Harbour	11:45	12:00	12:15	12:30	12:45	1:00
Akaroa	1:00	1:15	1:30	1:45	2:00	2:15
Airforce World	2:30	2:45	3:00	3:15	3:30	3:45
Yaldhurst Transport Museum	3:15	3:30	3:45	4:00	4:15	4:30
International Antarctic Centre	4:30	4:45	5:00	5:15	5:30	5:45
Arrive at City depot	5:30	5:45	6:00	6:15	6:30	6:45

- **a** How many bus services are available?
- **b** What is the latest departure time?
- **c** What is the earliest arrival time back at the depot?
- **d** How long does it take to get from:
  - **i** Lyttelton Harbour to Airforce World
  - **ii** the Gondola to the International Antarctic Centre?



- **e** How long does a complete trip last?
- **f** If you wanted to arrive at Akaroa at 2:00, which bus should you take?
- **g** If you are to meet a friend at Yaldhurst Transport Museum at 3:25, which bus is it best to travel on?

Example 16 **Self Tutor**

What is the time: **a** $4\frac{1}{2}$ hours after 11:20 am **b** $2\frac{1}{4}$ hours before 5:10 pm?

a $4\frac{1}{2}$ hours after 11:20 am $= 11{:}20$ am $+ 4$ h $+ 30$ min
$= 3{:}20$ pm $+$ 30 min
$= 3{:}50$ pm

b $2\frac{1}{4}$ hours before 5:10 pm $= 5{:}10$ pm $- 2$ h $- 15$ min
$= 3{:}10$ pm $- 15$ min
$= 2{:}55$ pm

12 Calculate the time:

a 2 hours after 4:16 pm
b 4 hours before 10:15 am
c $7\frac{1}{2}$ hours after 9:20 am
d $2\frac{1}{2}$ hours after 11:00 am
e 5 hours after 10:15 pm
f $3\frac{1}{4}$ hours before 10:50 pm

13 Xian has bought a parking ticket which is valid for $6\frac{1}{2}$ hours. She parks her car at 10:40 am. At what time will her parking ticket expire?

14 There was a power failure at Ryan's house during the night. When power was restored, the time on his clock radio was reset to 12 midnight. When Ryan woke up it was 8:00 am, but his clock radio displayed 0:43. At what time was the power restored?

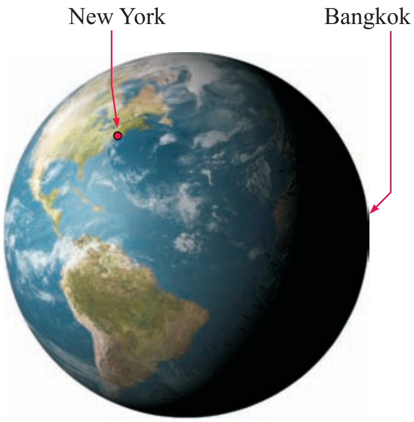
TIME ZONES

At any given time, different parts of the world are experiencing different phases of day and night.

For example, when it is the middle of the day in New York, Bangkok is in complete darkness.

This means the time of day varies depending on where you are.

Prior to the introduction of **time zones**, every city and town would calculate their own time by measuring the position of the Sun. This meant that cities that were only a short distance from each other would use slightly different times.



To solve this problem, the world was divided into **time zones**.

STANDARD TIME ZONES

The map shows lines that run between the North and South Poles. They are not straight lines like the lines of longitude, but rather follow the borders of countries, states or regions, and natural boundaries such as rivers and mountains.

The first line of longitude, 0^o, passes through Greenwich near London.

This first or **Prime Meridian** is the starting point for 12 time zones west of Greenwich and 12 time zones east of Greenwich.

Places which lie in the same time zone share the same **standard time**. Standard Time Zones are usually measured in 1 hour units, but there are also a few $\frac{1}{2}$ hour units around the world.

Time along the Prime Meridian is called **Greenwich Mean Time (GMT)**.

Places to the **east** of the Prime Meridian are **ahead** of GMT.

Places to the **west** of the Prime Meridian are **behind** GMT.

The map on page **229** shows the main time zones of the world. The numbers in the zones show how many hours have to be added or subtracted from Greenwich Mean Time to work out the standard time for that zone.

Example 17 **Self Tutor**

If it is 12 noon in Greenwich, what is the standard time in:

a Mumbai **b** Los Angeles?

a Mumbai is in a zone marked $+5\frac{1}{2}$

$\therefore$ the standard time in Mumbai is $5\frac{1}{2}$ hours ahead of GMT.

$\therefore$ the standard time in Mumbai is 5:30 pm.

b Los Angeles is in a zone marked -8

$\therefore$ the standard time in Los Angeles is 8 hours behind GMT.

$\therefore$ the standard time in Los Angeles is 4 am.

Example 18 **Self Tutor**

If it is 10 am in Moscow, find a city where the standard time is 5 pm.

5 pm is 7 hours ahead of 10 am.

Moscow is in a zone marked $+3$.

$\therefore$ we require the zone 7 hours ahead of $+3$, which is $+10$.

Sydney is a city in the zone $+10$.

$\therefore$ if it is 10 am in Moscow, it is 5 pm in Sydney.

EXERCISE 11E.2

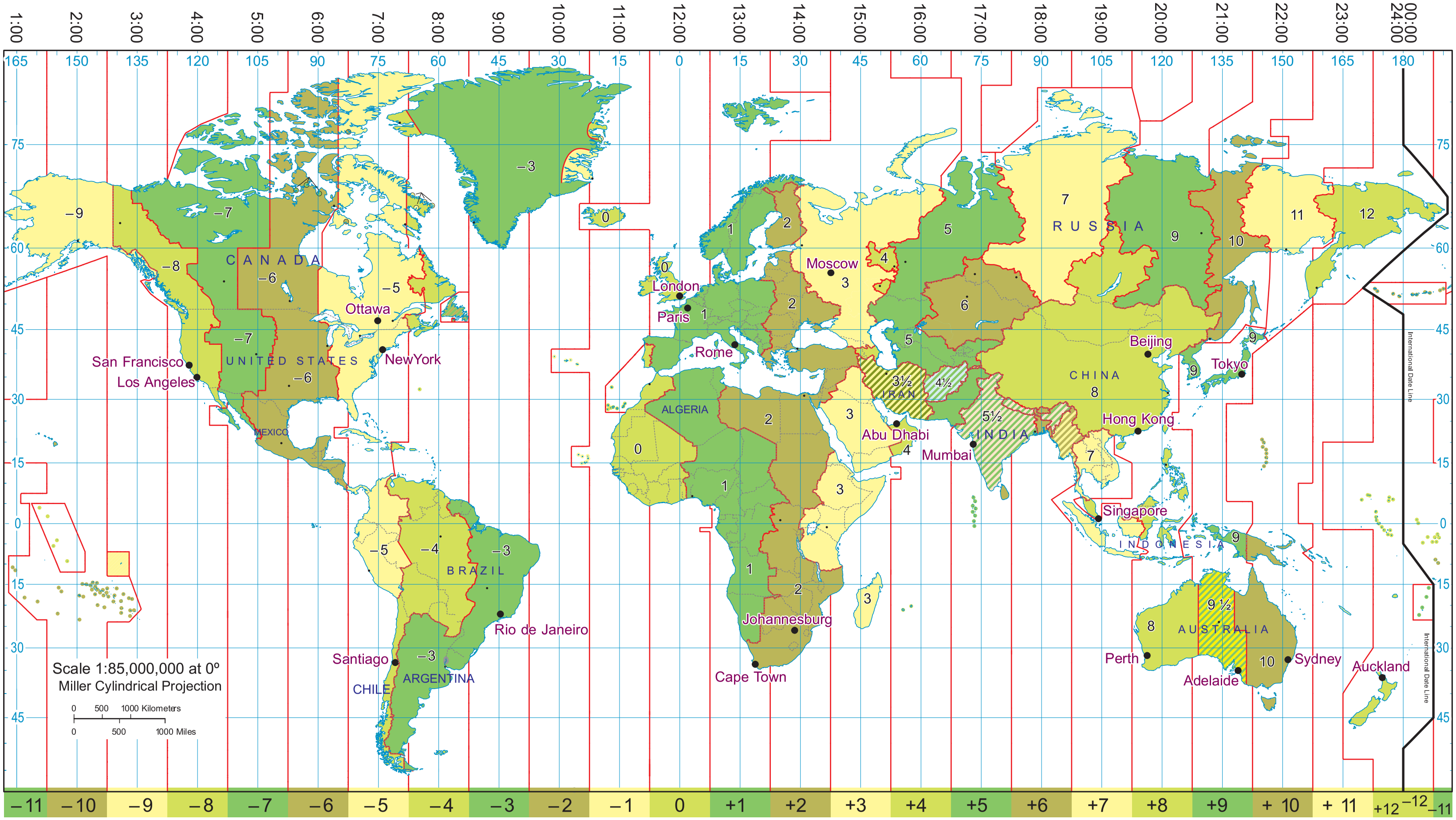
1 If it is 12 noon in Greenwich, what is the standard time in:

a Moscow **b** Beijing **c** Tokyo **d** Santiago?

2 If it is 11 pm on Tuesday in Greenwich, what is the standard time in:

a New York **b** San Francisco **c** Sydney **d** Johannesburg?

STANDARD TIME ZONES OF THE WORLD



https://www.cia.gov/library/publications/the-world-factbook/reference_maps/pdf/time_zones.pdf

3 If it is 6 pm on Friday in Moscow, what is the standard time in:

a Beijing **b** Sydney **c** London **d** Cape Town?

4 If it is 7 am in Johannesburg, find a city where the standard time is 9 am.

5 If it is 9 pm in London, find a city where the standard time is 4 pm.

6 Janice takes a 1:00 pm flight from Adelaide to Auckland. The flight takes $3\frac{1}{2}$ hours. What is the time in Auckland when she arrives?

KEY WORDS USED IN THIS CHAPTER

- capacity
- cubic units
- litre
- mass
- prism
- time zone
- volume

LINKS
click here

HOW MUCH WATER IS LOST WHEN A TAP IS LEFT DRIPPING?

Areas of interaction:
Environments, Community and service

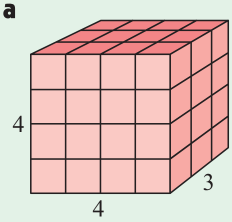
REVIEW SET 11A

1 Convert:

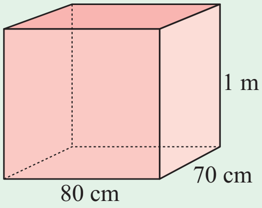
a 5 h 14 min to min
b 2.6 tonnes to kg
c 5683 g to kg
d 1 kL to cm^3
e 0.04 tonnes to mg
f 2634 minutes to days, hours, minutes
g 4.5 L to mL
h 26 150 L to ML.

2 Find the volume of:

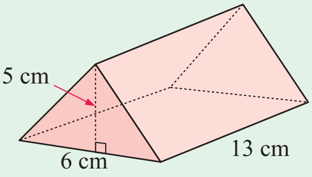
a



b



c

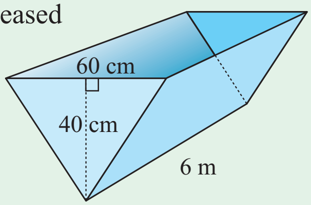


3 A petrol tank is in the shape of a rectangular prism 80 cm by 120 cm by 50 cm. Find:

a its volume in cubic centimetres **b** its capacity in litres.

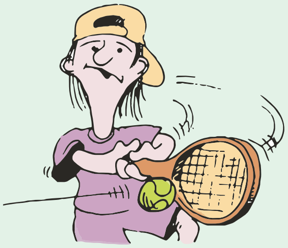
4 A rectangular tank with a base measuring 2.4 m by 1.2 m has water in it to a height of 1 m. If the level of water is increased by 20 cm, how many more litres are now in the tank?

5 A water trough 6 m long has a triangular cross-section as shown. Find the capacity of the trough in kilolitres.



6 Extra large eggs weigh 61 g. Find the mass in kg of 40 dozen extra large eggs.

7 A five set tennis match starts at 2:10 pm. The sets take the following times to complete: 16 minutes, 35 minutes, 28 minutes, 20 minutes, 24 minutes. There is an additional 20 minutes during the match for rests and injury time. At what time does the match finish?



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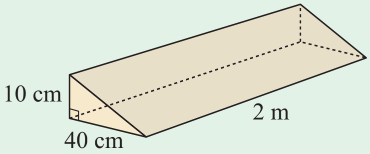


Alongside is a diagram of a wooden ramp. What is the volume of wood in the ramp?

REVIEW SET 11B

1 Convert:

a 1.6 tonnes to kilograms
b 2 days and 7 hours to min
c 5.3 L to cm^3
d 512 600 mg into kg
e 0.42 L to mL
f 2500 cm^3 to L
g 0.025 tonnes to g
h 46 L to kL.

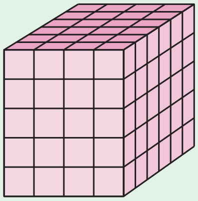
2 How long is it between 10:52 pm and 3:14 am the next day?

3 A rectangular box is 18 cm by 10 cm by 25 cm. Find its:

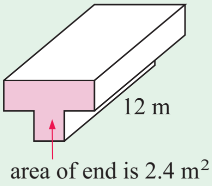
a volume in cm^3
b capacity in L.

4 Find the volume of:

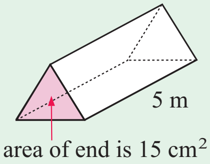
a



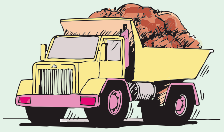
b



c



5 A truck driver charges £16.20 per cubic metre for delivering dirt fill. How much should the driver charge for filling a rectangular excavation 14 m long by 4 m wide by 2.5 m deep?



6 How many 15 cubic centimetre containers can be filled from a storage tank containing 2.6 litres of liquid?

7 If I fill a 20 m long by 10 m wide rectangular pool with 400 kL of water, what is its depth?

8 Tokyo is in a time zone marked +9 and Rio de Janeiro is in a time zone marked −3. If it is 12 noon in Greenwich, what is the standard time in:

a Tokyo
b Rio de Janeiro?

ACTIVITY FLY-BY-NIGHT AIRLINES

Airline companies frequently have restrictions on the number and size of packages which they allow passengers to carry with them onto an aircraft.

Fly-By-Night Airlines have the following package policy:

- *All packages must be rectangular.*
- *The sum of the length, width and depth of any package must not exceed 90 cm.*

Your task is to determine the rectangular package of largest volume which is allowed to be taken on the plane.

What to do:

1 Copy and complete the following table where the sum of the length, width and depth is always 90 cm. Add your own choices of dimensions for the second half of the table. You may need to extend the table.

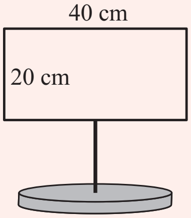
Length	*Width*	*Depth*	*Volume*	*Length*	*Width*	*Depth*	*Volume*
10	20	60	12 000				
10	30						
10	40						
20	20						
20	25						
20	30						
20	35						

2 What do you suspect are the dimensions of the package of greatest volume?

3 Fly-By-Night decides to introduce a further restriction to ensure all packages will fit in the overhead lockers:

- *All packages must pass through a 40 cm by 20 cm rectangle.*

Investigate the package of greatest volume given this new restriction.



Chapter 12

Ratio and proportion

Contents:

A **ratio** is a comparison of quantities.

We often hear statements about:

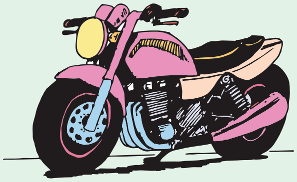
- a team's win-loss ratio
- the teacher-student ratio in a school
- the need to mix ingredients in a certain *ratio* or *proportion*.

These are all examples of ratios. Ratios are also commonly used to indicate the **scales** on **maps** and **scale diagrams**.

OPENING PROBLEMS

Problem 1:

Two-stroke fuel for a motor-bike is made by mixing 1 part oil with 7 parts petrol. How much oil and petrol is needed to fill a 20 litre tank?



Problem 2:

A map has a 1 : 1 000 000 scale. Jen measures the distance between two towns on the map to be 9.8 cm. How far are the towns actually apart?

A RATIO

A **ratio** is an **ordered** comparison of quantities of the **same kind**.

Carol bought some industrial strength disinfectant for use in her hospital ward.

The bottle instructs her to 'mix one part disinfectant to four parts water'.

Disinfectant and water are both liquids, so this statement can be written as a **ratio**.

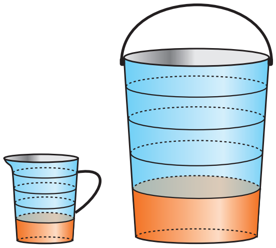
We say the ratio of disinfectant to water is 1 : 4 or "1 is to 4".

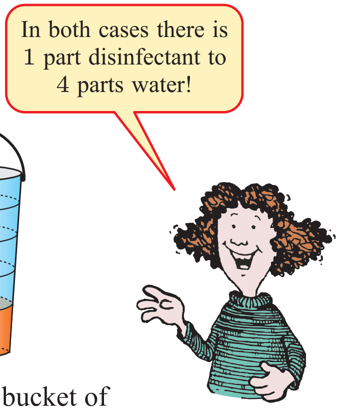
Note that the ratio of disinfectant to water is 1 : 4, but the ratio of water to disinfectant is 4 : 1. This is why **order** is important.

Also notice that the ratio is written *without units* such as mL or L.

It is important to mix the disinfectant and water in the correct **ratio** or **proportion** so that the disinfectant will kill germs but without wasting chemicals unnecessarily. Carol may make a jug or a bucket of disinfectant. As long as she mixes it in the correct ratio, it will be effective.

Although most ratios involve two quantities, they may involve more than two quantities.





For example, To make a certain strength of concrete you need 1 bucket of cement, 2 buckets of sand, and 4 buckets of gravel.

The ratio of cement to sand to gravel is 1 : 2 : 4, which we read as "1 is to 2 is to 4".

Example 1 **Self Tutor**

Express as a ratio: 3 km is to 5 km

"3 km is to 5 km" means 3 : 5

EXERCISE 12A

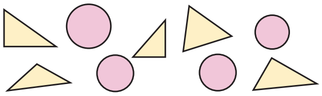
1 Express as a ratio:

a \$8 is to \$5 **b** 7 mL is to 13 mL **c** 5 kg is to 2 kg

d 2 tonne is to 7 tonne **e** €13.00 is to €1.00 **f** 8 mm is to 5 mm

2 Write a simple ratio for the following:

a triangles to circles



b cats to mice

c teachers to students

d trees to flowers

Example 2 **Self Tutor**

Express as a ratio: 7 minutes is to 2 hours

We must express both quantities in the *same units*.

2 hours is the same as 120 minutes, so "7 minutes is to 2 hours" is really "7 minutes is to 120 minutes", or 7 : 120.

3 Express as a ratio:

- **a** 65 g is to 1 kg
- **b** 87 pence is to £1.00
- **c** 5 months is to 2 years
- **d** 60 cents is to €2.40
- **e** $2.00 is to 80 cents
- **f** 200 kg is to 1 tonne.

Example 3 **Self Tutor**

Write as a ratio: Kirsty spends two hours watching TV and three hours doing homework.

We write TV : homework = 2 : 3.

4 Write as a ratio:

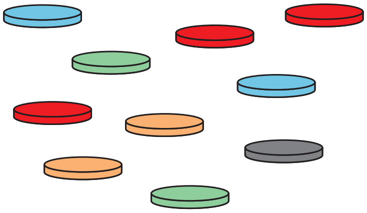
- **a** Peter has $11 and Jacki has $9.
- **b** In a theatre there are 3 girls for every boy.
- **c** The school spent €5 on volleyball equipment for every €1 on table tennis equipment.
- **d** There are 2 Japanese made cars for every 5 European made cars.
- **e** For every 15 km that you travel by car, I can travel 4 km by bicycle.
- **f** There are 2 blue-fin tuna for every 5 schnapper.
- **g** Mix 50 mL of liquid fertiliser with 2 litres of water.
- **h** Take 5 mg of medicine for every 10 kg of body weight.

B WRITING RATIOS AS FRACTIONS

When we compare the ratio of a *part* to its *total*, the ratio can be written as a *fraction*.

For example, in the diagram alongside there are 3 red discs out of a total of 10 discs.

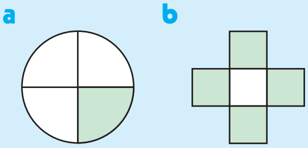
The ratio of red discs to the total number of discs is 3 : 10, and the fraction of discs which are red is $\frac{3}{10}$.



Example 4 **Self Tutor**

In the figures alongside, find:

- **i** the ratio of the shaded area to the unshaded area
- **ii** the ratio of the shaded area to the total area
- **iii** the fraction of the total area which is shaded.



a
- **i** shaded : unshaded = 1 : 3
- **ii** shaded : total = 1 : 4
- **iii** Fraction of total shaded is $\frac{1}{4}$.

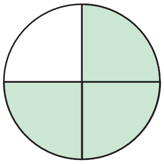
b
- **i** shaded : unshaded = 4 : 1
- **ii** shaded : total = 4 : 5
- **iii** Fraction of total shaded is $\frac{4}{5}$.

EXERCISE 12B

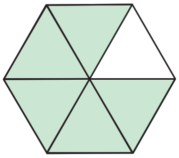
1 In each of the following diagrams:

- **i** Find the ratio of the shaded area to the unshaded area.
- **ii** Find the ratio of the shaded area to the total area.
- **iii** Find the fraction of the total area which is shaded.

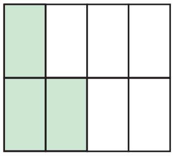
a



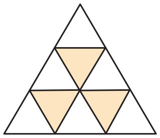
b



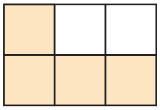
c



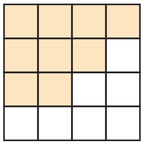
d



e

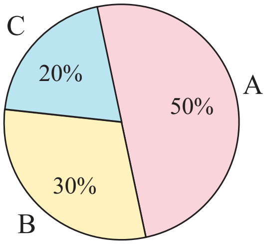


f



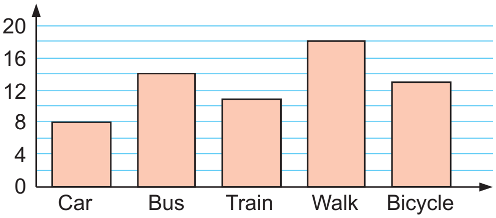
2 The pie chart represents the sales of three different types of soap powders, A, B and C.

- **a** Write as a ratio:
  - **i** the sales of A to the sales of B
  - **ii** the sales of A to the total sales.
- **b** What fraction of the total sales made is the sales of brand A?



3 The column graph represents the results of a survey to determine the method by which students travel to school.

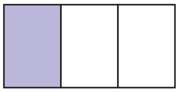
- **a** Find the total number of students surveyed.
- **b** Write as a ratio:
  - **i** students arriving by car : students who walk
  - **ii** students arriving by bus : total number of students surveyed.
- **c** What fraction of the students surveyed travel to school by bus?



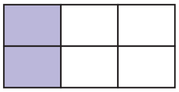
C EQUAL RATIOS

Consider the diagrams:

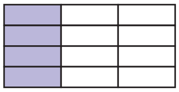
A



B



C



The ratio of shaded area : unshaded area in each case is **A** 1 : 2 **B** 2 : 4 **C** 4 : 8.

However, by looking carefully at the three diagrams we can see that the fraction of the total area which is shaded is the same in each case.

The ratio of the shaded area to the unshaded area is also the same or **equal** in each case.

We can write that $1 : 2 = 2 : 4 = 4 : 8$.

We can see whether ratios are equal in the same way we see if fractions are equal.

Just as $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$, $1 : 2 = 2 : 4 = 4 : 8$.

If we multiply or divide both parts of a ratio by the same non-zero number, we obtain an **equal ratio**.

A ratio is in **simplest form** when it is written in terms of whole numbers with no common factors.

Example 5 **Self Tutor**

Express the following ratios in simplest form:

a 8 : 16 **b** 35 : 20

a $8 : 16$
$= 8 \div 8 : 16 \div 8$
$= 1 : 2$

b $35 : 20$
$= 35 \div 5 : 20 \div 5$
$= 7 : 4$

To convert a ratio to simplest form, divide by the **highest common factor**.

Two ratios are **equal** if they can be written in the same simplest form.

EXERCISE 12C

1 Express the following ratios in simplest form:

a 2 : 4 **b** 10 : 5 **c** 6 : 14 **d** 6 : 18 **e** 20 : 50
f 15 : 25 **g** 9 : 15 **h** 21 : 49 **i** 56 : 14

Example 6 **Self Tutor**

Express the following ratios in simplest form: **a** $\frac{2}{5} : \frac{3}{5}$ **b** $1\frac{1}{2} : 5$

a $\frac{2}{5} : \frac{3}{5}$
$= \frac{2}{5} \times 5 : \frac{3}{5} \times 5$
$= 2 : 3$

b $1\frac{1}{2} : 5$
$= \frac{3}{2} : 5$ {convert to an improper fraction}
$= \frac{3}{2} \times 2 : 5 \times 2$
$= 3 : 10$

2 Express the following ratios in simplest form:

a $\frac{1}{5} : \frac{4}{5}$ **b** $\frac{5}{4} : \frac{1}{4}$ **c** $\frac{2}{3} : \frac{1}{3}$ **d** $1 : \frac{1}{2}$ **e** $3 : \frac{3}{4}$

f $\frac{1}{3} : 2$ **g** $3 : \frac{1}{2}$ **h** $3\frac{1}{2} : 8$ **i** $2\frac{1}{2} : 1\frac{1}{2}$ **j** $\frac{1}{2} : \frac{1}{3}$

Example 7

Self Tutor

Express the following ratios in simplest form:

a $0.3 : 1.7$ **b** $0.05 : 0.15$

a
$$\begin{aligned} &0.3 : 1.7 \\ &= 0.3 \times 10 : 1.7 \times 10 \\ &= 3 : 17 \end{aligned}$$

b
$$\begin{aligned} &0.05 : 0.15 \\ &= 0.05 \times 100 : 0.15 \times 100 \\ &= 5 : 15 \\ &= 5 \div 5 : 15 \div 5 \\ &= 1 : 3 \end{aligned}$$

We multiply or divide both parts of a ratio by the same non-zero number to get an equal ratio.

3 Express the following ratios in simplest form:

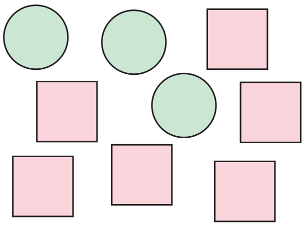
a $0.4 : 0.5$ **b** $1.3 : 1.8$ **c** $0.1 : 0.9$

d $0.2 : 0.6$ **e** $0.4 : 0.2$ **f** $1.4 : 0.7$

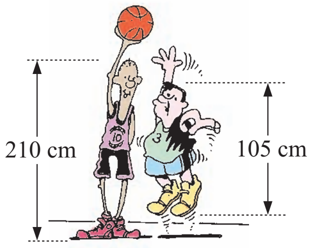
g $0.03 : 0.15$ **h** $0.02 : 0.12$ **i** $0.18 : 0.06$

4 Express as a ratio in simplest form:

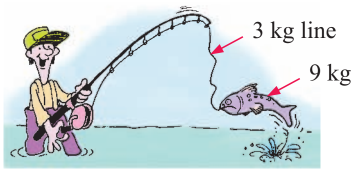
a the number of circles to squares



b the height of the tall player to the height of the short player

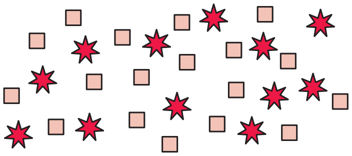


c the weight of the fish to the breaking strain of the fishing line

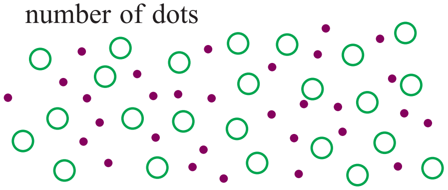


d the capacity of the small cola to the capacity of the large cola

e the number of squares to the number of stars



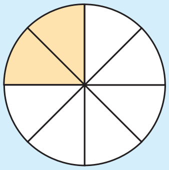
f the number of circles to the number of dots



Example 8

Self Tutor

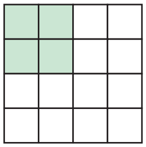
Express the ratio
shaded area : unshaded area
in simplest form.



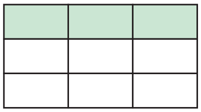
$$\begin{aligned}\text{shaded area} : \text{unshaded area} &= 2 : 6\\ &= 2 \div 2 : 6 \div 2 \qquad \{\text{HCF} = 2\}\\ &= 1 : 3\end{aligned}$$

5 Express in simplest form the ratio of shaded area : unshaded area.

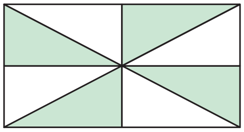
a



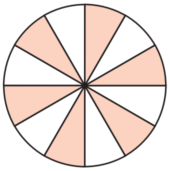
b



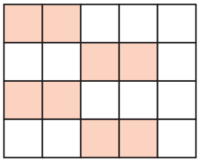
c



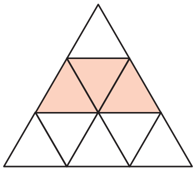
d



e



f



Example 9

Self Tutor

Express as a ratio in simplest form:

a 2 hours to 4 minutes

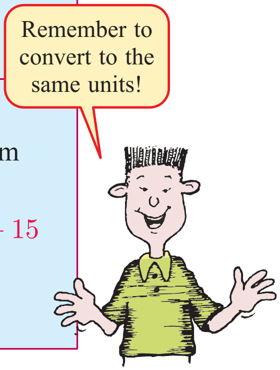
b 45 cm to 3 m

a

$$\begin{aligned}&\ 2 \text{ hours to } 4 \text{ minutes}\\ &= 2 \times 60 \text{ min to } 4 \text{ min}\\ &= 120 \text{ min to } 4 \text{ min}\\ &= 120 : 4\\ &= 120 \div 4 : 4 \div 4 \qquad \{\text{HCF} = 4\}\\ &= 30 : 1\end{aligned}$$

b

$$\begin{aligned}&\ 45 \text{ cm to } 3 \text{ m}\\ &= 45 \text{ cm to } 300 \text{ cm}\\ &= 45 : 300\\ &= 45 \div 15 : 300 \div 15\\ &= 3 : 20\end{aligned}$$



6 Express as a ratio in simplest form:

a 20 cents to \$1
b £3 to 60 pence
c 15 kg to 30 kg
d 13 cm to 26 cm
e 27 mm to 81 mm
f 9 mL to 63 mL
g 800 g to 1 kg
h 1 hour to 30 min
i 40 mins to $1\frac{1}{2}$ hours
j 34 mm to 1 cm
k 2 m to 125 cm
l 24 seconds to $1\frac{1}{2}$ min
m 2 L to 750 mL
n 1 week to 3 days
o 4 weeks to 1 year
p 250 g to 1 kg
q 1.5 kg to 125 g
r 3 min to 45 seconds

Example 10 **Self Tutor**

Show that the ratio 4 : 6 is equal to 20 : 30.

$$\begin{aligned} & 4 : 6 && \text{and} && 20 : 30 \\ &= 4 \div 2 : 6 \div 2 && && = 20 \div 10 : 30 \div 10 \\ &= 2 : 3 && && = 2 : 3 \qquad \therefore \quad 4 : 6 = 20 : 30 \end{aligned}$$

7 Which of the following pairs of ratios are equal?

a 16 : 20, 4 : 5
b 3 : 5, 21 : 35
c 2 : 7, 8 : 21
d 5 : 6, 30 : 36
e 12 : 16, 18 : 24
f 15 : 35, 21 : 56
g 18 : 27, 6 : 4
h $2 : 2\frac{1}{2}$, 32 : 40
i $1 : 1\frac{1}{3}$, 15 : 20

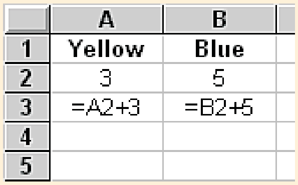
INVESTIGATION 1 EQUAL RATIOS BY SPREADSHEET

A painter needs to obtain a particular shade of green paint by mixing yellow paint with blue paint in the ratio 3 : 5. If he buys 45 L of yellow paint, how many litres of blue paint are required?

SPREADSHEET

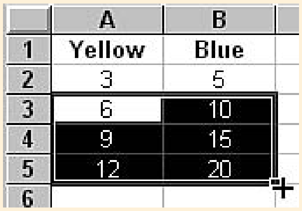
What to do:

1 Open a new spreadsheet and enter the details shown alongside.



	A	B
1	Yellow	Blue
2	3	5
3	=A2+3	=B2+5
4		
5		

2 Highlight the formulae in row 3. Fill these down to row 5. Your spreadsheet should look like this:
Notice that in each row, the ratio of yellow paint to blue paint is still 3 : 5.



	A	B
1	Yellow	Blue
2	3	5
3	6	10
4	9	15
5	12	20
6		

3 Now fill down the formulae until your spreadsheet shows 45 L of yellow paint. How many litres of blue paint are required?

4 The painter also needs to obtain a particular shade of purple by mixing red paint with blue paint in the ratio 4 : 7.
In A1 type 'Red'.
In A2 enter 4 and in B2 enter 7. In A3 enter =A2+4 and in B3 enter =B2+7.
Fill the formulae down to find out how many litres of red paint are required to mix with 42 L of blue paint.

5 White paint and black paint are to be mixed in the ratio 2 : 9 to make a shade of grey paint. How much of each type of paint is required to make 55 L of grey paint?

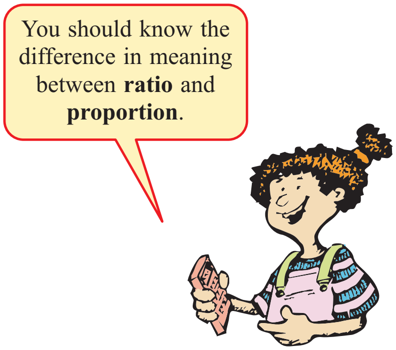
D PROPORTIONS

A **proportion** is a statement that two ratios are equal.

For example, the statement that $4 : 6 = 20 : 30$ is a proportion, as both ratios simplify to $2 : 3$.

Suppose we are given the proportion $2 : 5 = 6 : \square$.

If we know that two ratios are equal and we know three of the numbers then we can always find the fourth number.



Example 11 Self Tutor

Find $\square$ to complete the proportion:

a $2 : 5 = 6 : \square$ **b** $16 : 20 = \square : 35$

a $2 : 5 = 6 : \square$

$\times 3$

$2 : 5 = 6 : \square$

$\times 3$

$\therefore \quad \square = 5 \times 3 = 15$

b We begin by reducing the LHS to simplest form.

$16 : 20 = 16 \div 4 : 20 \div 4$

$= 4 : 5$

$\times 7$

$\therefore \quad 4 : 5 = \square : 35$

$\times 7$

$\therefore \quad \square = 4 \times 7 = 28$

Example 12 Self Tutor

The student to leader ratio at a youth camp must be $9 : 2$. How many leaders are required if there are 63 students enrolled?

students : leaders $= 9 : 2$

$\therefore \quad 63 : \square = 9 : 2$

$\div 7$

$\therefore \quad 63 : \square = 9 : 2$

$\div 7$

$\therefore \quad \square \div 7 = 2$

$\therefore \quad \square = 14$

So, 14 leaders are needed.

or 9 parts is 63

$\therefore \quad$ 1 part is $\frac{63}{9} = 7$

$\therefore \quad$ 2 parts is $7 \times 2 = 14$

EXERCISE 12D

1 Find the missing numbers in the following proportions:

a $3:4=6:\square$ **b** $3:6=12:\square$ **c** $2:5=\square:15$

d $5:8=\square:40$ **e** $1:3=\square:27$ **f** $4:1=24:\square$

g $7:21=\square:33$ **h** $15:25=30:\square$ **i** $\square:18=32:48$

j $5:100=\square:40$ **k** $18:30=24:\square$ **l** $\square:12=33:44$

2 A disaster relief team consists of engineers and doctors in the ratio of 2 : 5.

a If there are 18 engineers, find the number of doctors.

b If there are 65 doctors, find the number of engineers.

3 The ratio of two angles in a triangle is 3 : 1. Find the:

a larger angle if the smaller is 18^o **b** smaller angle if the larger is 63^o.

4 The ratio of teachers to students in a school is 1 : 15. If there are 675 students, how many teachers are there?

5 An MP3 player is bought for €240 and sold for €270. Find the ratio of the cost price to the selling price.

6 The maximum speeds of a boat and a car are in the ratio 2 : 7. If the maximum speed of the boat is 30 km per hour, find the maximum speed of the car.

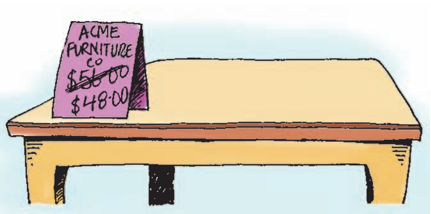
7 A farmer has sheep and cattle in the ratio 8 : 3.

a How many sheep has the farmer if he has 180 cattle?

b Find the ratio of the number of sheep to the total number of animals.

c Find the ratio of the total number of animals to the number of cattle.

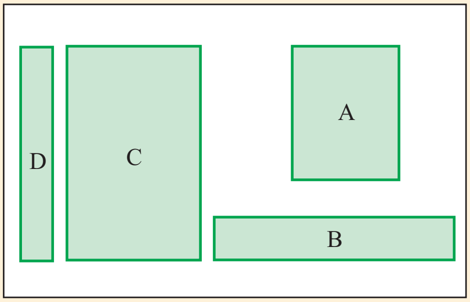
8 The price of a table is reduced from \$56 to \$48. The set of chairs which go with the table was originally priced at \$140. If the price of the chairs is reduced in the same ratio as that of the table, find the new price of the chairs.



9 Sue invested money in stocks, shares, and property in the ratio 6 : 4 : 5. If she invested €36 000 in property, how much did she invest in the other two areas?

INVESTIGATION 2 THE GOLDEN RATIO AND THE HUMAN BODY

Look at the four given rectangles and choose the one which you find most appealing.



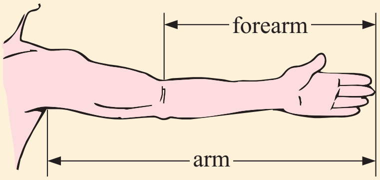
If you said Rectangle C you would agree with many artists and architects from the past. Rectangle C is called a **Golden Rectangle** and is said to be one of the most visually appealing geometric shapes. The ratio of length to width in a Golden Rectangle is the ratio $1.618\,033\,98... : 1$, but $1.6 : 1$ is close enough for our purposes here.

The ratio $1.6 : 1$ is known as the **Golden Ratio**. The Greeks used the Golden Ratio extensively in paintings, architecture, sculpture, and designs on pottery.

Leonardo Da Vinci suggested that the ratio of certain body measurements is close to the Golden Ratio. Keeping in mind that we are all different shapes and sizes, let us see if we can find the Golden Ratio using our body measurements.

You will need: a tape measure and a partner.

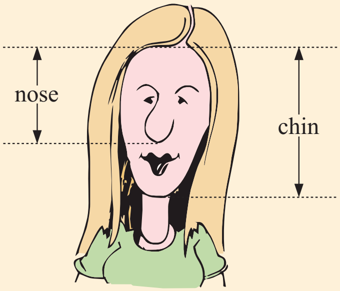
What to do:



1 Ask your partner to carefully take your body measurements as indicated in the following diagrams. Record these measurements and calculate the required ratios.

- length of arm from fingertip to armpit = ... mm
  length of forearm from fingertip to elbow = ... mm
  So, arm : forearm = ... : ......
  = ... : 1

- hairline to chin = ... mm.
  hairline to bottom of nose = ... mm.
  hairline to chin : hairline to nose = ... : ......
  = ... : 1



2 How do your ratios compare to the Golden Ratio?

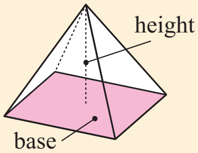
3 Compile a class list of these ratios. Are they all close to the Golden Ratio? Check any ratios which are a long way away from the Golden Ratio to make sure they were measured accurately.

4 Can you find any other ratios of body measurements which are close to the Golden Ratio?

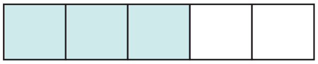
Further Research:

5 Find out more about where the Golden Ratio occurs in nature.

6 Research the dimensions of the Great Pyramids of Ancient Egypt. Calculate the ratio base length : height for each pyramid.



E USING RATIOS TO DIVIDE QUANTITIES



In the diagram alongside we see that the ratio of the shaded area to the unshaded area is $3:2$.

There are a total of $3+2=5$ parts, so $\frac{3}{5}$ is shaded and $\frac{2}{5}$ is unshaded.

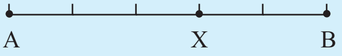
When a quantity is divided in the ratio $3:2$, the larger part is $\frac{3}{5}$ of the total and the smaller part is $\frac{2}{5}$ of the total.

Example 13 **Self Tutor**

Line segment [AB] is divided into five equal intervals.

a In what ratio does X divide line segment AB?

b What fraction of [AB] is [AX]?



a X divides [AB] in the ratio $3:2$.

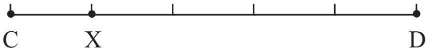
b [AX] has length 3 units
[AB] has length 5 units
$\therefore$ [AX] is $\frac{3}{5}$ of [AB].

EXERCISE 12E

1 [CD] is divided into equal intervals.

i In what ratio does X divide [CD]? **ii** What fraction of [CD] is [CX]?

a



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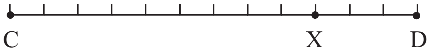

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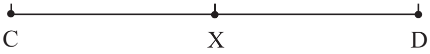

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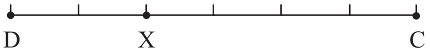
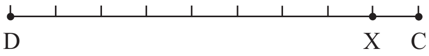


e

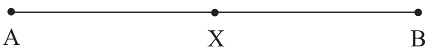


f

2 Estimate the ratio in which X divides [AB], then check your estimate by measuring:

a

b



c

d

3 The line segment [AM] is divided into equal intervals. Which point divides [AM] in the ratio:

a $5:7$ **b** $11:1$ **c** $4:8$ **d** $1:2$

e $2:1$ **f** $6:6$ **g** $1:1$ **h** $1:3$?

Example 14

Self Tutor

I wish to divide \$12 000 in the ratio 2 : 3 to give to my children Pam and Sam. How much does each one receive?

There are $2 + 3 = 5$ parts.

$\therefore$ Pam gets $\frac{2}{5}$ of \$12 000 $= \frac{2}{5} \times \$12\,000 = \frac{\$24\,000}{5} = \$4800$

and Sam gets $\frac{3}{5}$ of \$12 000 $= \frac{3}{5} \times \$12\,000 = \frac{\$36\,000}{5} = \$7200$

or $\$12\,000 - \$4800 = \$7200$

4 The recommended cordial to water ratio is 1 : 4. Calculate how many mL of cordial are needed to make:

a a 300 mL glass of mixture **b** a 2L container of mixture.

5 The recommended disinfectant to water ratio is 1 : 20. How many mL of concentrated disinfectant are required to make a 9 L bucket of mixture?

6 Answer *Problem 1* of the **Opening Problem** on page **234**.

7

A bag of 25 marbles is divided between Jill and John in the ratio 2 : 3.

a What fraction does Jill receive?

b How many marbles does Jill receive?

c How many marbles does John receive?

8 Divide: **a** \$20 in the ratio 1 : 4 **b** €49 in the ratio 5 : 2.

9 £400 is divided in the ratio 3 : 2. What is the larger share?

10 ¥160 000 is divided in the ratio 3 : 5. What is the smaller share?

Example 15

Self Tutor

To make standard concrete, gravel, sand and cement are mixed in the ratio 5 : 3 : 1. I wish to make 18 tonnes of concrete. How much gravel, sand and cement must I purchase?

There are $5 + 3 + 1 = 9$ parts.

$\therefore$ I need $\frac{5}{9} \times 18$ tonnes $= 10$ tonnes of gravel

$\frac{3}{9} \times 18$ tonnes $= 6$ tonnes of sand

and $\frac{1}{9} \times 18$ tonnes $= 2$ tonnes of cement.

11 My fortune of $810 000 is to be divided in the ratio 4 : 3 : 2. How much does each person receive?

12 An alloy is made from copper, zinc and tin in the ratio 17 : 2 : 1. How much zinc is required to make 10 tonnes of the alloy?

13 When Michael makes pici pasta, he mixes semolina, "00" flour, and water in the ratio 6 : 3 : 2. If he uses 150 g of "00" flour, what mass does he require of:

a semolina **b** water?

14 Joe and Bob share the cost of a video game in the ratio 3 : 7.

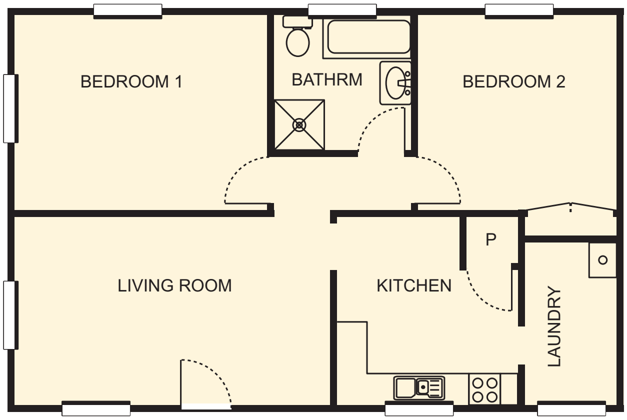
- **a** What fraction does each pay?
- **b** If the game costs £35, how much does each pay?
- **c** If Joe pays £12, how much does Bob pay?
- **d** If Bob pays £42, what is the price of the video game?

F SCALE DIAGRAMS

When designing a house it would be ridiculous for an architect to draw a full-size plan.

Instead the architect draws a smaller diagram in which all measurements have been divided by the same number or **scale factor**.

For house plans a scale factor of 100 would be suitable.



Similarly, a map of Brazil must preserve the shape of the country. All distances are therefore divided by the same scale factor. In this case the scale factor is 80 000 000.

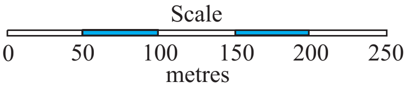
These diagrams are called **scale diagrams**.

In **scale diagrams**:

- All lengths have been changed by the same **scale factor**.
- All angles are unaltered.

To properly use a scale diagram we need to know the scale used.
Scales are commonly given in the following ways:

- *Scale:* 1 cm represents 50 m.
  This tells us that 1 cm on the scale diagram represents 50 m on the real thing.
- A divided bar can be used to show the scale.
  This scale tells us that 1 cm on the scale diagram represents 50 m on the real thing.



- *Scale:* 1 : 5000
  This ratio tells us that 1 unit on the scale diagram represents 5000 of the same units on the real thing.
  For example, 1 cm would represent 5000 cm or 50 m,
  1 mm would represent 5000 mm or 5 m.

Scales are written in ratio form as drawn length : actual length.
We usually simplify the scale to an equal ratio of the form 1 : the scale factor.

Example 16 **Self Tutor**

On a scale diagram 1 cm represents 20 m.

a Write the scale as a ratio. **b** What is the scale factor?

a 1 cm to 20 m
$= 1$ cm to (20×100) cm
$= 1$ cm to 2000 cm
$= 1 : 2000$

b The ratio simplifies to 1 : 2000 so the scale factor is 2000.

Example 17 **Self Tutor**

Interpret the ratio 1 : 5000 as a scale.

1 : 5000 means 1 cm represents 5000 cm
$\therefore$ 1 cm represents $(5000 \div 100)$ m {100 cm = 1 m}
$\therefore$ 1 cm represents 50 m

EXERCISE 12F.1

1 Write the following scales as ratios and state the corresponding scale factors:

a 1 cm represents 10 m
b 1 cm represents 50 km
c 1 mm represents 2 m
d 1 cm represents 250 km
e 1 mm represents 5 m
f 1 cm represents 200 km.

2 Interpret the following ratios as scales, explaining what 1 cm represents:

a 1 : 200
b 1 : 3000
c 1 : 500
d 1 : 20 000
e 1 : 100 000
f 1 : 5 000 000

USING SCALES AND RATIOS

Suppose we need a scale diagram of a rectangular area to give to a landscape gardener. The area is 12 m by 5 m and the landscaper wants us to use a scale of 1 : 100.

What lengths will we draw on the scale diagram? They must be much less than the actual lengths so that we can fit them on paper. We **divide** by the scale factor, as division by larger numbers produces a smaller answer.

drawn length = actual length ÷ scale factor

Example 18 Self Tutor

An object 12 m long is drawn with the scale 1 : 100. Find the drawn length of the object.

$$\begin{aligned} 12\text{ m} &= 1200\text{ cm} \\ \text{drawn length} &= \text{actual length} \div 100 \quad \{\text{scale factor } 100\} \\ \therefore\ \text{drawn length} &= (1200 \div 100)\text{ cm} \\ \therefore\ \text{drawn length} &= 12\text{ cm} \end{aligned}$$

Now suppose we have a map with a scale of 1 : 500 000 marked. We measure the distance between towns A and B to be 15 cm. We can calculate the actual distance between towns A and B using the formula:

actual length = drawn length × scale factor

Example 19 Self Tutor

For a scale of 1 : 500 000, find the actual length represented by a drawn length of 15 cm.

$$\begin{aligned} \text{actual length} &= \text{drawn length} \times 500\,000 \quad \{\text{scale factor } 500\,000\} \\ \therefore\ \text{actual length} &= 15\text{ cm} \times 500\,000 \\ &= 7\,500\,000\text{ cm} \\ &= (7\,500\,000 \div 100)\text{ m} \quad \{1\text{ m} = 100\text{ cm}\} \\ &= 75\,000\text{ m} \\ &= (75\,000 \div 1000)\text{ km} \quad \{1\text{ km} = 1000\text{ m}\} \\ \therefore\ \text{actual length} &= 75\text{ km} \end{aligned}$$

EXERCISE 12F.2

1. If the scale is 1 : 50, find the length drawn to represent an actual length of:
   - **a** 20 m
   - **b** 4.6 m
   - **c** 340 cm
   - **d** 7.2 m
2. If the scale is 1 : 1000, find the actual length represented by a drawn length of:
   - **a** 2.5 mm
   - **b** 12 cm
   - **c** 4.6 cm
   - **d** 7.4 mm

3 Make a scale drawing of:

a	a square with sides 200 m	*scale:*	1 cm represents 50 m
b	a triangle with sides 16 m, 12 m and 8 m	*scale:*	1 : 400
c	a rectangle 8 km by 6 km	*scale:*	1 cm represents 2 km
d	a circle of radius 16 km	*scale:*	1 : 1 000 000

4 Select an appropriate scale and make a scale diagram of:

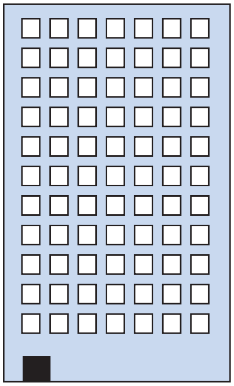
- **a** a rectangular house block 42 m by 36 m
- **b** a kitchen door 2.2 m by 1.2 m
- **c** a triangular paddock with sides 84 m, 76 m and 60 m
- **d** the front of an A-framed house with base 12 m and height 9.6 m.

5 Consider the scale diagram of a rectangle. Use your ruler to find the actual dimensions given that the scale is 1 : 200. Which of the following could it represent:

- **A** a book
- **B** a postage stamp
- **C** a basketball court
- **D** a garage for a car?

6 A scale diagram of a building is shown with scale 1 : 1000.

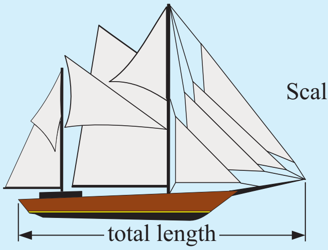
- **a** If the height is 5 cm and width is 3 cm on the drawing, find the actual height and width of the building in metres.
- **b** If the height of the windows on the drawing are 2.5 mm, how high are the actual windows?
- **c** If the actual height of the entrance door is 3.2 m, what is its height on the scale drawing?



Example 20 **Self Tutor**

This is a scale diagram of a ship. Use your ruler and the given scale to determine:

- **a** the total length of the ship
- **b** the height of the taller mast
- **c** the distance between the masts.



Scale 1 : 1000

a The measured length of the ship is 3.8 cm.

So, the actual length is 3.8 cm × 1000 = 3800 cm = 38 m.

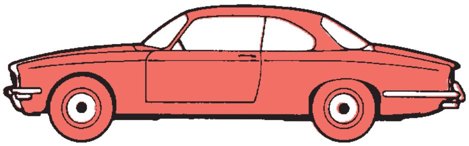
b The measured height of the taller mast is 2.5 cm

So, the actual height is 2.5 cm × 1000 = 2500 cm = 25 m.

c The measured distance between the masts = 1.4 cm.

So, the actual distance is 1.4 cm × 1000 = 1400 cm = 14 m.

7



Scale 1 : 80

Consider the diagram alongside. Find:

a the length of the vehicle

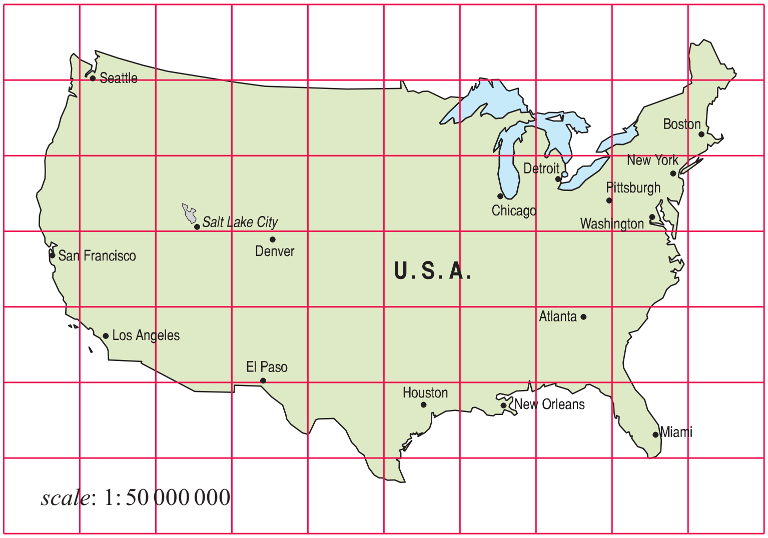
b the diameter of a tyre

c the height of the top of the vehicle above ground level

d the width of the bottom of the door.

8 For this part map of the USA, find the actual distance in kilometres in a straight line between:

a New York and New Orleans **b** El Paso and Miami **c** Seattle and Denver.



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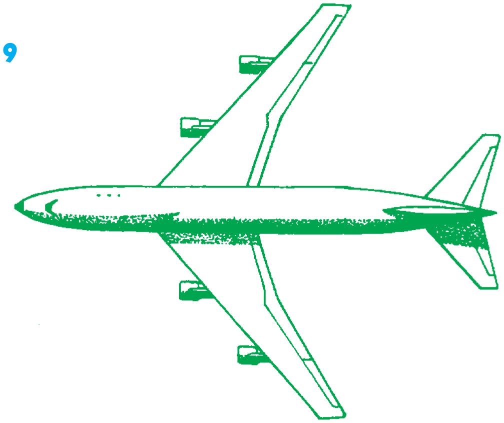


The actual length of the aeroplane shown in the scale drawing is 64 m.

Find:

a the scale used in the drawing

b the actual wingspan of the aeroplane

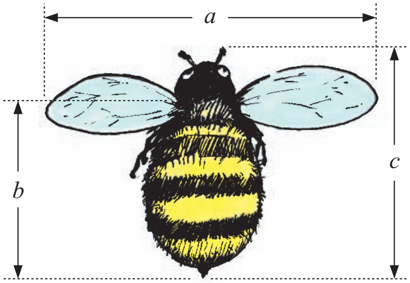
c the actual width of the fuselage.

10 The diagram given is of an *enlarged* bee, drawn to a scale of 1 : 0.25. Find the actual length of the dimensions marked:

a wing span, *a*

b body length, *b*

c total length, *c*.

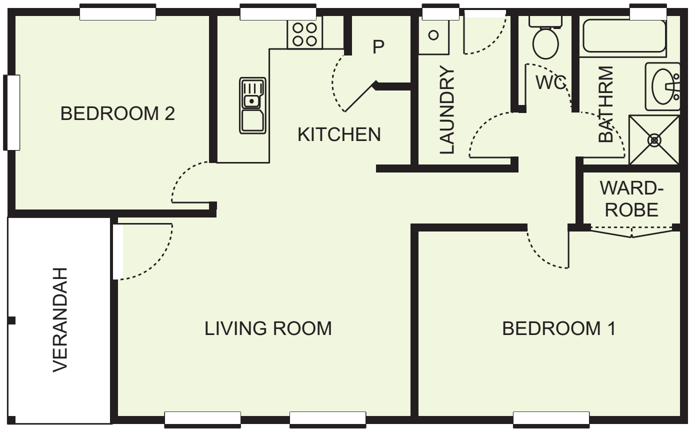


11 The floor plan of this house has been drawn to a scale 1 : 120. Find:

a the external dimensions of the house including verandah

b the dimensions of the verandah

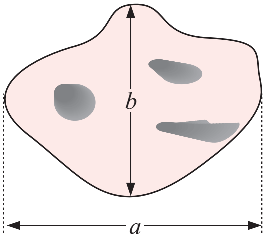
c the cost of covering each of the bedroom floors with carpet tiles if carpet tiles cost $45.50 per square metre laid.



12 The diagram given shows a microscopic organism *enlarged* using the scale 1 : 0.001. Find the actual length of the dimensions marked:

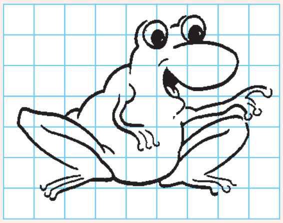
a width, a

b height, b.



ACTIVITY 1 SCALE DIAGRAMS

One way to make a scale drawing is to draw a grid over the picture to be enlarged or reduced. We then copy the picture onto corresponding positions on a larger or smaller grid. Grid paper is available on the worksheet. You could use a photocopier to further enlarge or reduce it.



GRID PAPER

ACTIVITY 2 DISTANCES IN EUROPE

You will need: a ruler marked in mm, an atlas or map showing some major cities in Europe (available on the CD).

MAP

What to do:

1 Use your ruler and the scale given on the map to estimate the distance in a straight line from:

a Paris to Rome

b Vienna to Madrid

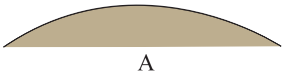
c London to Warsaw

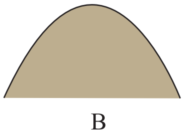
d Belgrade to Amsterdam.

G GRADIENT OR SLOPE

Have you ever seen a sign like the one pictured here? Watch out for one next time you are travelling through mountains. It is used to warn drivers that the road has a steep slope or a steep gradient.

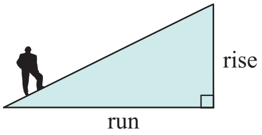
Consider yourself walking up the two hills pictured below.





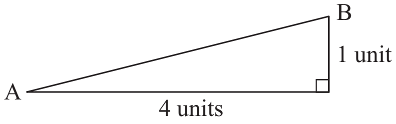
You can quite easily see that hill B is steeper than hill A, but in addition to **comparing** gradient or slope, we often need to **measure** it.

One way to measure the gradient is to use ratios.



Picture yourself walking up the sloping side of a right angled triangle. Each step you take, you are moving horizontally to the right, and also vertically upwards.

We can measure the gradient by using these vertical and horizontal distances. They are the height and base of the triangle. We sometimes call the vertical distance the **rise** and the horizontal distance the **run**.



For example, a gradient of 1 in 4 means we move up 1 unit for every 4 units we move across.

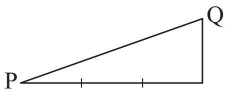
So, the line segment [AB] shown has gradient 1 in 4 *or* 1 : 4.

Gradient is the **rise : run** ratio.

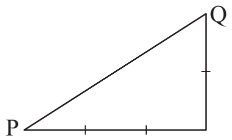
EXERCISE 12G

1 Judge by sight which line segment [PQ] is the steepest:

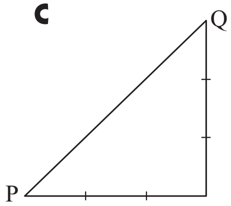
A



B



C



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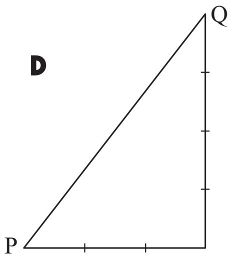


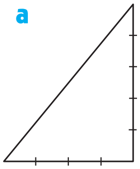
2 Copy and complete the following table for the triangles of question **1**:

Line [PQ]	*rise*	*run*	*rise* : *run*
A	1	3	1 : 3
B			1 :
C			1 :
D			1 :

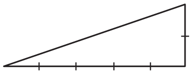
3 By comparing the ratios in the last column of question **2**, what happens to the *rise* : *run* ratio as the steepness of the line increases?

4 Write down the *rise* : *run* ratios of the following triangles in the form 1 : x.

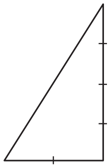
a



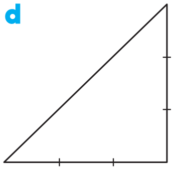
b



c



d



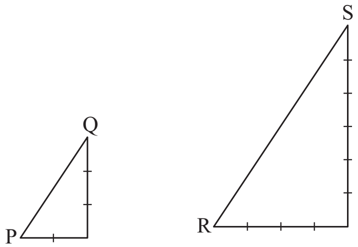
5 Draw a triangle which has a *rise* : *run* ratio of:

a 1 in 5 **b** 3 : 1 **c** 1 in 4 **d** 2 in 3 **e** 3 : 5 **f** 7 in 2

6 **a** Express the *rise* : *run* ratio of the line segments [PQ] and [RS] in simplest form.

b What do you notice about the slopes of [PQ] and [RS]?

c Copy and complete the following statement: [PQ] *and* [RS] *are ...... line segments.*



KEY WORDS USED IN THIS CHAPTER

- actual length
- gradient
- scale factor
- drawn length
- proportion
- simplest form
- equal ratio
- ratio
- Golden Ratio
- scale diagram

REVIEW SET 12A

1 **a** Express 72 cents to \$1.80 as a ratio in simplest form.

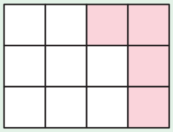
b Express 10 : 15 as a ratio in simplest form.

c 5 : 12 = ☐ : 96. Find the missing number.

d Express $1\frac{1}{5} : \frac{4}{5}$ in simplest form.

e Write "In a class there are 4 girls for every 3 boys" as a ratio.

f Find the ratio of the shaded area to the unshaded area in the figure shown:



2 Divide £400 in the ratio 3 : 7.

3 A watch is bought for €120 and sold for a profit of €40. Find the ratio of the cost price to the selling price.

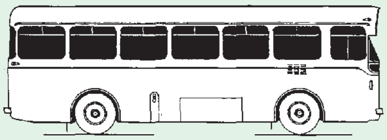
4 In a school the ratio of students playing football, basketball, and hockey is 12 : 9 : 8. If 144 students play football, how many students play hockey?

5 The actual length of the bus shown alongside is 10 m. Find:

a the scale used for this diagram

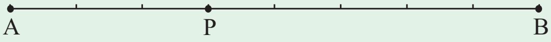
b the actual height of the windows

c the actual height of the bus.



6 A fruit grower plants apricot trees and peach trees in the ratio $4:5$. If he plants a total of 3600 trees, how many of each type did he plant?

7 [AB] is divided into equal intervals.
In what ratio does P divide [AB]?



8 **a** Divide \$200 in the ratio $2:3$.

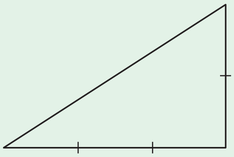
b George specified that his bank balance be divided amongst his 3 daughters in the ratio $2:3:4$. If his bank balance was €7200, how much did each daughter receive?

9 Make a scale drawing of a rectangular room 4 m by 6 m. Use a scale of $1:500$.

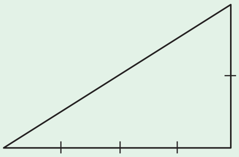
10 A law firm has lawyers and secretaries in the ratio $5:2$. If the law firm has 30 lawyers, how many secretaries are there?

11 Express the *rise* : *run* ratio of the following triangles in simplest form:

a



b



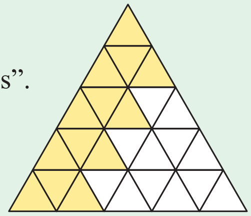
REVIEW SET 12B

1 **a** Express $96:72$ as a ratio in simplest form.

b Write 750 mL is to 2 litres as a ratio in simplest form.

c Write as a ratio: "I saw five Audis for every six BMWs".

d Find the ratio of the shaded area : total area in the figure shown.



e Express $0.4:0.9$ as a ratio in simplest form.

f Express 2 hours 20 minutes : 4 hours in simplest form.

g Express the ratio $3:2\frac{1}{2}$ in simplest form.

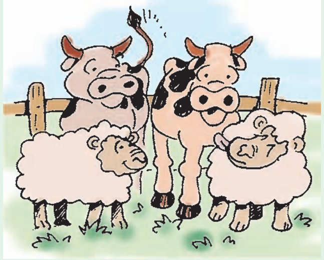
2 Find the missing numbers in the following proportions:

a $2:3=8:\square$ **b** $5:8=\square:24$

3 A farm has 6000 animals. 2500 are sheep and the rest are cattle. Find the ratio of:

a number of sheep : number of cattle

b number of sheep : total number of animals.



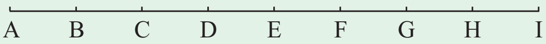
4 Two families share the cost of buying 75 kg of meat in the ratio $7:8$. How much meat should each family receive?

5 During the football season the school team's win-loss ratio was 2 : 3. If the team lost twelve matches, how many did it win?

6 A man owning 7000 hectares of land divided it between his three sons in the ratio 6 : 9 : 5. How much land did each son receive?

7 Draw a scale diagram of a rectangular window which is 2 m long and 1 m high. Use the scale 1 : 250.

8



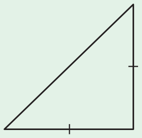
The line segment [AI] is divided into equal intervals. Which point divides [AI] in the ratio:

a 3 : 5 **b** 1 : 3 **c** 1 : 1 ?

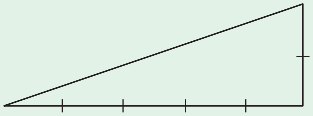
9 A matchbox model Jaguar XJ6 is 75 mm long. The scale stamped on the bottom of the car is 1 : 64. What is the length of the real car?

10 Express the *rise* : *run* ratio of the following triangles in simplest form:

a



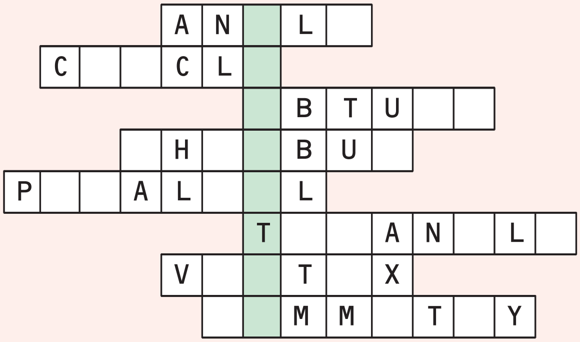
b



11 The ratio of girls to boys in a sports club is 4 : 5. If there are 20 girls, what is the total number of people in the sports club?

PUZZLE WORD PUZZLE

Complete the missing words in the puzzle below to discover the shaded word.



Chapter 13

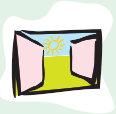
Equations

Contents:

Algebraic equations are an extremely powerful and important tool used in problem solving. We convert the information given into equations which we then solve.

In this chapter we will be examining algebraic equations and methods for solving them. Some applications to problem solving follow later in the course.

OPENING PROBLEM



If x is an unknown number, how do we solve the equations:

$3x - 5 = x + 11$ and $3(2x - 1) - x + 7 = 5 - 3x$?

A EQUATIONS

Algebraic equations are equations which contain at least one unknown or variable.

$4x - 7 = x + 5$ is an algebraic equation.

It has a **left hand side (LHS)** and a **right hand side (RHS)** separated by an **equal** sign.

$$\underbrace{4x - 7}_{\text{LHS}} = \underbrace{x + 5}_{\text{RHS}}$$

The **solution** of an equation is the value or values of the variable which make the equation true.

For the equation $4x - 7 = x + 5$,

$$\text{when } x = 4 \quad \text{the LHS } \begin{aligned}[t] &= 4(4) - 7 \\ &= 16 - 7 \\ &= 9 \end{aligned} \quad \text{and the RHS } \begin{aligned}[t] &= (4) + 5 \\ &= 9 \quad \text{also.} \end{aligned}$$

So, the solution of the equation is $x = 4$.

SOLUTION BY INSPECTION

Some simple equations are easily solved by **inspection**.

For example, consider the equation $x + 7 = 20$.

We know that $13 + 7 = 20$, so $x = 13$ must be a solution.

SOLUTION BY TRIAL AND ERROR

Another method for solving equations is to make some guesses. From the results we obtain, we can *refine* our guess or make it better until we have found the actual solution.

For example, consider the equation $2x + 7 = 23$.

If we guess that $x = 2$, the LHS $= 2 \times 2 + 7 = 11$.

If we guess that $x = 6$, the LHS $= 2 \times 6 + 7 = 19$.

Although we don't yet have the solution, our guess is closer.

If we guess that $x = 8$, the LHS $= 2 \times 8 + 7 = 23$, so $x = 8$ is the solution.

THE NUMBER OF SOLUTIONS

Some equations have exactly one solution. Others have no solutions, two solutions, three solutions, and some have infinitely many solutions.

Equations which are always true no matter what value the variable takes are called **identities**.

For example,

- $x + 1 = x$ has no solutions
- $x^2 = x$ has two solutions: $x = 0$ and $x = 1$
- $x^3 = 4x$ has three solutions: $x = 0$, $x = 2$, and $x = -2$
- $3x - x = 2x$ is true for all values of the variable x, so this equation is an identity.

EXERCISE 13A

1 Solve by inspection:

a $x + 5 = 8$
b $7 - x = 4$
c $11 + x = 25$
d $8 - x = -2$
e $x - 3 = -4$
f $6x = 24$
g $-3x = 9$
h $7x = 91$
i $\frac{x}{3} = 9$
j $\frac{x}{-3} = 27$
k $\frac{3}{x} = -1$
l $\frac{-6}{x} = -2$

2 One of the numbers in brackets is the correct solution to the given equation. Find the correct solution using trial and error.

a $4x + 7 = 11$ $\{0, 1, 2, 3\}$
b $5 - 2x = 7$ $\{-2, -1, 0, 1, 2\}$
c $3x + 4 = -2$ $\{-3, -4, -2, -1\}$
d $4x - 9 = x$ $\{0, -2, 3, 1\}$
e $\frac{p}{2} = p + 4$ $\{-10, -9, -8, -7\}$
f $2 + 2a = a + 7$ $\{2, -1, 0, 5, 4\}$

3 For each of the following equations, state if they are:

A true for *exactly one value* of x
B true for *two values* of x
C true for *all values* of x
D *never* true.

a $x + 8 = 5$
b $x - x = 0$
c $2x = 6$
d $x + 4 = 4 + x$
e $8 - x = 3$
f $x^2 = 0$
g $x^2 = x$
h $3x - x = 2x$
i $x = 6 + x$
j $x \times 0 = 0$
k $x + 3 = x - 3$
l $x \times 1 = x$

4 What integers, if any, make the following true?

a $q - q = 0$
b $3 + s = s + 3$
c $a \times a = a$
d $x + 2 = x$
e $* + 8 = 8 + *$
f $k - k = 0$
g $x \times x = -4$
h $k \times 1 = k$
i $k - k = k$
j $4 - y = y - 4$
k $t + t = t$
l $n \times 1 = n$

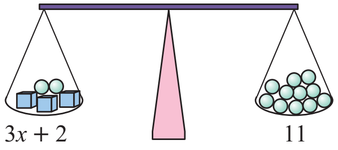
5 Which of the following are identities?

a $x + 7 = 7 + x$
b $4x - x = 3x$
c $5 - x = x - 5$
d $x = 1 \times x$
e $2 - x = 7$
f $2x + x = 3x$

B MAINTAINING BALANCE

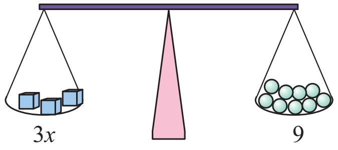
Equations can be compared to a set of scales. The left hand side is equal to the right hand side, so the scales are in balance.

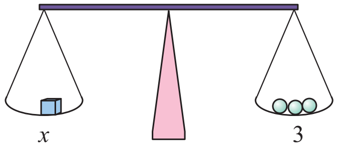
For example, the equation $3x + 2 = 11$ can be compared with the balance set up on the scales alongside.



Each sphere ○ is one unit and in each small box □ there are x spheres.

The balance is maintained if we take 2 spheres from both sides. We are left with $3x = 9$.





Dividing by 3 removes 2 out of every 3 objects from both sides. We are left with $x = 3$, and this is the solution to the equation.

DEMO

The **balance** of an equation will be maintained if we:

- add the same amount to both sides
- subtract the same amount from both sides
- multiply both sides by the same amount
- divide both sides by the same amount.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To maintain the balance, whatever operation we perform on one side of the equation we must also perform on the other.

Example 1 **Self Tutor**

What equation results when:

a 3 is added to both sides of $x - 3 = 8$
b 4 is taken from both sides of $2x + 4 = 18$
c both sides of $5x = 15$ are divided by 5
d both sides of $\frac{x}{4} = -7$ are multiplied by 4?

a
$$\begin{aligned} x-3&=8\\ \therefore\quad x-3+3&=8+3\\ \therefore\quad x&=11 \end{aligned}$$

b
$$\begin{aligned} 2x+4&=18\\ \therefore\quad 2x+4-4&=18-4\\ \therefore\quad 2x&=14 \end{aligned}$$

c
$$\begin{aligned} 5x&=15\\ \therefore\quad \frac{5x}{5}&=\frac{15}{5}\\ \therefore\quad x&=3 \end{aligned}$$

d
$$\begin{aligned} \frac{x}{4}&=-7\\ \therefore\quad \frac{x}{4}\times 4&=-7\times 4\\ \therefore\quad x&=-28 \end{aligned}$$

EXERCISE 13B

1 What equation results when:

- **a** 5 is added to both sides of $x-5=2$
- **b** 7 is added to both sides of $x-7=0$
- **c** 3 is added to both sides of $4x+1=x-3$
- **d** 4 is added to both sides of $7x-4=x+2$?

2 What equation results when:

- **a** 3 is subtracted from both sides of $x+3=2$
- **b** 4 is subtracted from both sides of $3x+4=-2$
- **c** 5 is subtracted from both sides of $2x+7=x+5$
- **d** 7 is taken from both sides of $4x+6=x+7$?

3 What equation results when:

- **a** both sides of $\dfrac{x}{2}=13$ are multiplied by 2
- **b** both sides of $\dfrac{x-1}{3}=8$ are multiplied by 3
- **c** both sides of $\dfrac{2x}{5}=6$ are multiplied by 5
- **d** both sides of $\dfrac{2x-1}{4}=-1$ are multiplied by 4?

4 What equation results when:

- **a** both sides of $3x=-33$ are divided by 3
- **b** both sides of $-4x=12$ are divided by -4
- **c** both sides of $5(2+x)=-25$ are divided by 5
- **d** both sides of $-6(2x-1)=-42$ are divided by -6?

C INVERSE OPERATIONS

When I wake up this morning there are 4 eggs in my refrigerator.

I find my chickens have already laid 2 eggs, so I then have $4+2=6$ eggs in total.

I fry 2 eggs to have with my breakfast, so I now have $6-2=4$ eggs left.

I now have the same number of eggs that I started with, so adding 2 eggs and subtracting 2 eggs cancel each other out.

Adding and subtracting are **inverse operations**.

Over the next week my chickens lay very well. The number of eggs in my fridge trebles, so I now have $4 \times 3 = 12$ eggs.

On the weekend I bake three cakes. I use a third of my eggs in each cake, and this is $12 \div 3 = 4$ eggs.

The operations of multiplying by 3 and dividing by 3 cancel each other out.

Multiplying and dividing are **inverse operations**.

Example 2 **Self Tutor**

State the inverse of: **a** $\times 4$ **b** $\div 7$ **c** $+6$ **d** -3

a The inverse of $\times 4$ is $\div 4$. **b** The inverse of $\div 7$ is $\times 7$.

c The inverse of $+6$ is -6. **d** The inverse of -3 is $+3$.

Example 3 **Self Tutor**

Solve for x using a suitable inverse operation:

a $x + 6 = 13$ **b** $y - 4 = -1$ **c** $4g = 20$ **d** $\dfrac{h}{7} = -6$

a
$$\begin{aligned} x + 6 &= 13 \\ \therefore\ x + 6 - 6 &= 13 - 6 \\ \therefore\ x &= 7 \end{aligned}$$

b
$$\begin{aligned} y - 4 &= -1 \\ \therefore\ y - 4 + 4 &= -1 + 4 \\ \therefore\ y &= 3 \end{aligned}$$

c
$$\begin{aligned} 4g &= 20 \\ \therefore\ \frac{4g}{4} &= \frac{20}{4} \\ \therefore\ g &= 5 \end{aligned}$$

d
$$\begin{aligned} \frac{h}{7} &= -6 \\ \therefore\ \frac{h}{7} \times 7 &= -6 \times 7 \\ \therefore\ h &= -42 \end{aligned}$$

EXERCISE 13C

1 State the inverse of:

a $\times 6$ **b** -4 **c** $\div 2$ **d** $+10$ **e** -6 **f** $\div 8$ **g** $\times \frac{1}{2}$ **h** $+11$

2 Simplify:

a $x + 9 - 9$ **b** $x - 7 + 7$ **c** $x \div 2 \times 2$

d $x \times 3 \div 3$ **e** $x \div 5 \times 5$ **f** $4x \div 4$

g $\dfrac{x}{8} \times 8$ **h** $\dfrac{3x}{4} \div \dfrac{3}{4}$ **i** $\dfrac{4x}{3} \times 3$

3 Solve for x using a suitable inverse operation:

a $x+2=8$
b $x-4=7$
c $3x=15$
d $\frac{x}{2}=4$
e $5x=35$
f $x+7=3$
g $\frac{x}{3}=-2$
h $x-8=-2$
i $x-6=-9$
j $-3x=12$
k $x+11=43$
l $\frac{x}{4}=9$
m $x+19=11$
n $x-11=11$
o $-7x=-49$
p $\frac{x}{-2}=7$
q $25x=125$
r $\frac{x}{-3}=-2$
s $x-8=-15$
t $x+7=-7$
u $x-21=-14$
v $\frac{x}{21}=-4$
w $-9x=-63$
x $x+9=0$

4 Solve using a suitable inverse operation:

a $-a=3$
b $7b=2$
c $\frac{c}{5}=-1$
d $4+d=0$
e $e-2=0$
f $\frac{1}{2}f=1$
g $g-4=-3$
h $\frac{h}{12}=-4$

D BUILDING AND UNDOING EXPRESSIONS

We have now solved simple equations by:
- inspection
- trial and error
- using one inverse operation.

To solve harder equations we need to know how algebraic expressions are built up. We can then use appropriate inverse operations to 'undo' the expression and isolate the variable.

A **flowchart** is useful to help us do this.

For example, to build up the expression $3x+2$, we start with x, multiply it by 3, then add on 2.

$$\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+2} \boxed{3x+2}$$

To 'undo' the expression $3x+2$, we perform **inverse operations** in the **reverse order**.

$$\boxed{3x+2} \xrightarrow{-2} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$$

Example 4 **Self Tutor**

Use a flowchart to show how the expression $4x-7$ is 'built up'.

Perform inverse operations in the reverse order to 'undo' the expression.

Build up: $\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{-7} \boxed{4x-7}$

Undoing: $\boxed{4x-7} \xrightarrow{+7} \boxed{4x} \xrightarrow{\div 4} \boxed{x}$

Example 5 **Self Tutor**

Use a flowchart to show how the expression $4(x-7)$ is 'built up'.

Perform inverse operations in the reverse order to 'undo' the expression.

Build up: $\boxed{x} \xrightarrow{-7} \boxed{x-7} \xrightarrow{\times 4} \boxed{4(x-7)}$

Undoing: $\boxed{4(x-7)} \xrightarrow{\div 4} \boxed{x-7} \xrightarrow{+7} \boxed{x}$

EXERCISE 13D

1 Use flowcharts to show how to 'build up' and 'undo':

- **a** $4x+1$
- **b** $3x+8$
- **c** $4x-2$
- **d** $7x-9$
- **e** $\dfrac{x}{2}+6$
- **f** $\dfrac{x+6}{2}$
- **g** $\dfrac{x}{5}-4$
- **h** $\dfrac{x-4}{5}$
- **i** $3x-5$
- **j** $3(x-5)$
- **k** $\dfrac{x+8}{3}$
- **l** $\dfrac{x}{3}+8$
- **m** $4x+1$
- **n** $4(x+1)$
- **o** $\dfrac{x}{-7}+9$
- **p** $\dfrac{x+9}{-7}$

Example 6 **Self Tutor**

Use flowcharts to show how to 'build up' and 'undo' $\dfrac{3x+4}{7}$.

Build up: $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+4} \boxed{3x+4} \xrightarrow{\div 7} \boxed{\dfrac{3x+4}{7}}$

Undoing: $\boxed{\dfrac{3x+4}{7}} \xrightarrow{\times 7} \boxed{3x+4} \xrightarrow{-4} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

2 Use flowcharts to show how to 'build up' and 'undo':

- **a** $\dfrac{2x+1}{3}$
- **b** $\dfrac{2x}{3}+1$
- **c** $\dfrac{2(x+1)}{3}$
- **d** $\dfrac{3x-2}{4}$
- **e** $\dfrac{3x}{4}-2$
- **f** $\dfrac{3(x-2)}{4}$
- **g** $\dfrac{4x+7}{5}$
- **h** $\dfrac{4(x+7)}{5}$
- **i** $\dfrac{4x}{5}+7$
- **j** $\dfrac{1-3x}{2}$
- **k** $1-\dfrac{3x}{2}$
- **l** $\dfrac{3(1-x)}{2}$
- **m** $\dfrac{2x+1}{3}+4$
- **n** $\dfrac{3x-2}{5}-4$
- **o** $\dfrac{2-x}{4}+6$

E SOLVING EQUATIONS

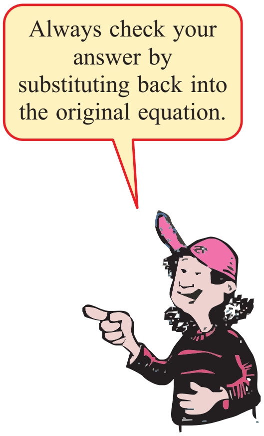
If the variable occurs only once in an equation, we use inverse operations to 'undo' the expression and hence isolate the variable on one side of the equation.

Example 7 **Self Tutor**

Solve for x: $4x - 5 = 25$

$$\begin{aligned} 4x - 5 &= 25 \\ \therefore\ 4x - 5 + 5 &= 25 + 5 && \{\text{adding 5 to both sides}\} \\ \therefore\ 4x &= 30 && \{\text{simplifying}\} \\ \therefore\ \frac{4x}{4} &= \frac{30}{4} && \{\text{dividing both sides by 4}\} \\ \therefore\ x &= 7\tfrac{1}{2} && \{\text{simplifying}\} \end{aligned}$$

Check: LHS $= 4(7\frac{1}{2}) - 5 = 30 - 5 = 25$ ✓



EXERCISE 13E

1 Solve for x:

a $3x - 7 = 14$ **b** $4x + 9 = 25$ **c** $2x + 7 = 17$ **d** $5x - 4 = 11$

e $6x - 7 = -2$ **f** $7x + 4 = 0$ **g** $8x - 7 = 25$ **h** $2x + 18 = 39$

i $5 + 6x = 17$ **j** $9 + 12x = -1$ **k** $7 + 13x = 7$ **l** $11 + 4x = -6$

Example 8 **Self Tutor**

Solve for x: $\frac{x}{4} + 5 = -8$

$$\begin{aligned} \frac{x}{4} + 5 &= -8 \\ \therefore\ \frac{x}{4} + 5 - 5 &= -8 - 5 && \{\text{subtracting 5 from both sides}\} \\ \therefore\ \frac{x}{4} &= -13 && \{\text{simplifying}\} \\ \therefore\ \frac{x}{4} \times 4 &= -13 \times 4 && \{\text{multiplying both sides by 4}\} \\ \therefore\ x &= -52 && \{\text{simplifying}\} \end{aligned}$$

Check: LHS $= \frac{-52}{4} + 5 = -13 + 5 = -8$ ✓

2 Solve for x:

a $\frac{x}{2} + 4 = 11$ **b** $\frac{x}{2} - 1 = 8$ **c** $\frac{x}{3} + 4 = 7$

d $\frac{x}{4} - 2 = -3$ **e** $\frac{x}{5} + 2 = -1$ **f** $\frac{x}{6} - 5 = 8$

g $\dfrac{x}{8}+6=-2$ **h** $\dfrac{x}{9}-5=4$ **i** $\dfrac{x}{11}+7=-3$

Example 9 **Self Tutor**

Solve for x: $32-5x=8$

$$\begin{aligned} 32-5x&=8 \\ \therefore\ 32-5x-32&=8-32 \quad \{\text{subtracting 32 from both sides}\} \\ \therefore\ -5x&=-24 \\ \therefore\ \frac{-5x}{-5}&=\frac{-24}{-5} \quad \{\text{dividing both sides by } -5\} \\ \therefore\ x&=4\tfrac{4}{5} \end{aligned}$$

Check: $\text{LHS}=32-5(4\frac{4}{5})=32-5(\frac{24}{5})=32-24=8=\text{RHS}$ ✓

3 Solve for x:

a $25-3x=16$ **b** $3-4x=11$ **c** $1-5x=6$

d $3-7x=8$ **e** $19-4x=-1$ **f** $21-5x=8$

Example 10 **Self Tutor**

Solve the equation: $\dfrac{2x-3}{3}=-2$

$$\begin{aligned} \frac{2x-3}{3}&=-2 \\ \therefore\ \frac{2x-3}{3}\times 3&=-2\times 3 \quad \{\text{multiplying both sides by 3}\} \\ \therefore\ 2x-3&=-6 \\ \therefore\ 2x-3+3&=-6+3 \quad \{\text{adding 3 to both sides}\} \\ \therefore\ 2x&=-3 \\ \therefore\ \frac{2x}{2}&=\frac{-3}{2} \quad \{\text{dividing both sides by 2}\} \\ \therefore\ x&=-\tfrac{3}{2} \end{aligned}$$

Check: $\text{LHS}=\dfrac{2(-\frac{3}{2})-3}{3}=\dfrac{-6}{3}=-2=\text{RHS}$ ✓

4 Solve for x:

a $\dfrac{3x+2}{4}=5$ **b** $\dfrac{2x-7}{5}=6$ **c** $\dfrac{4x-3}{2}=-4$

d $\dfrac{5x+6}{4}=-6$ **e** $\dfrac{3x+1}{-5}=10$ **f** $\dfrac{6x-11}{-4}=-1$

Example 11 **Self Tutor**

Solve the equation: $3(2x-1)=-21$

$$\begin{aligned} 3(2x-1) &= -21 \\ \therefore\ \frac{3(2x-1)}{3} &= \frac{-21}{3} && \{\text{dividing both sides by 3}\} \\ \therefore\ 2x-1 &= -7 \\ \therefore\ 2x-1+1 &= -7+1 && \{\text{adding 1 to both sides}\} \\ \therefore\ 2x &= -6 \\ \therefore\ \frac{2x}{2} &= \frac{-6}{2} && \{\text{dividing both sides by 2}\} \\ \therefore\ x &= -3 && \textit{Check: } \text{LHS} = 3(2(-3)-1) = 3(-7) = -21 = \text{RHS}\ \checkmark \end{aligned}$$

5 Solve for x:

a $2(2x+1)=30$

b $3(4x-3)=-12$

c $4(2x-7)=20$

d $7(3x-4)=63$

e $6(3x+3)=-72$

f $-2(6x-3)=6$

6 Solve the following equations:

a $4a+3=19$

b $\frac{x}{4}+5=2$

c $\frac{y}{3}-1=6$

d $4(x+2)=20$

e $6(m-9)=18$

f $3x-5=28$

g $\frac{a}{4}+2=7$

h $\frac{x+14}{2}=8$

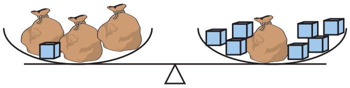
i $\frac{5m-4}{9}=4$

F EQUATIONS WITH A REPEATED UNKNOWN

If the unknown or variable appears more than once in the equation, we need to be systematic in our approach.

For example, consider the equation $3x+1=x+7$.

In this case the unknown appears twice, once on each side of the equation.

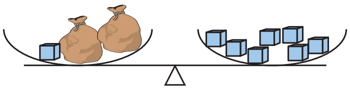


We can represent the equation using the set of scales shown.

The unknown x is the number of blocks in each bag.

DEMO

We can add or subtract bags or blocks on both sides of the scales to maintain the balance.



Removing a bag amounts to taking x from both sides. This gives us the equation $2x+1=7$.

We can then use inverse operations to solve for x.

In general, we follow these steps to solve equations:

Step 1: If necessary, expand out any **brackets**.

Step 2: Simplify each side of the equation by **collecting like terms**.

Step 3: If necessary, remove the unknown from one side of the equation using an **inverse operation**.

Step 4: **Solve** in the usual way.

Example 12 **Self Tutor**

Solve for d: $2d + 3(d - 1) = 7$

$$\begin{aligned} 2d + 3(d-1) &= 7 \\ \therefore \quad 2d + 3d - 3 &= 7 && \{\text{expanding brackets}\} \\ \therefore \quad 5d - 3 &= 7 && \{\text{collecting like terms}\} \\ \therefore \quad 5d - 3 + 3 &= 7 + 3 && \{\text{adding 3 to both sides}\} \\ \therefore \quad 5d &= 10 \\ \therefore \quad \frac{5d}{5} &= \frac{10}{5} && \{\text{dividing both sides by 5}\} \\ \therefore \quad d &= 2 \end{aligned}$$

Check: LHS $= 2(2) + 3(2-1) = 4 + 3 = 7 =$ RHS

EXERCISE 13F

1 Solve the following equations:

a $3x + 4x = 14$

b $3x + 4 - 6x = 46$

c $2x - 5 - x + 7 = 12$

d $2y - 15 + 3y + 3 = 24$

e $3x + 2(x - 3) = -8$

f $2(a + 3) + 3(a - 2) = 18$

g $x + \frac{x}{2} = 3$

h $3x + \frac{x}{2} = 15$

i $x - \frac{x}{3} = 4$

j $2x - \frac{x}{4} = 14$

Example 13 **Self Tutor**

Solve for x: $3x + 2 = x + 14$

$$\begin{aligned} 3x + 2 &= x + 14 \\ \therefore \quad 3x + 2 - x &= x + 14 - x && \{\text{subtracting } x \text{ from both sides}\} \\ \therefore \quad 2x + 2 &= 14 && \{\text{simplifying}\} \\ \therefore \quad 2x + 2 - 2 &= 14 - 2 && \{\text{subtracting 2 from both sides}\} \\ \therefore \quad 2x &= 12 && \{\text{simplifying}\} \\ \therefore \quad x &= 6 && \{\text{dividing both sides by 2}\} \end{aligned}$$

2 What operation must be done to both sides to remove x from the RHS?

a $2x+3=x+8$
b $3x-1=7+x$
c $4x+6=2x+6$
d $2x-8=7x-3$
e $2x+7=5x+25$
f $3x-9=10x-2$

3 Solve the following equations:

a $2x+3=x+8$
b $3x-1=7+x$
c $4x+6=2x+6$
d $2x-8=7x-3$
e $2x+7=5x+25$
f $3x-9=10x-2$

Example 14

Self Tutor

Solve for x: $4x+3=23-x$

$$\begin{aligned}
4x+3&=23-x\\
\therefore\quad 4x+3+x&=23-x+x &&\{\text{adding } x \text{ to both sides}\}\\
\therefore\quad 5x+3&=23 &&\{\text{simplifying}\}\\
\therefore\quad 5x+3-3&=23-3 &&\{\text{subtracting 3 from both sides}\}\\
\therefore\quad 5x&=20 &&\{\text{simplifying}\}\\
\therefore\quad x&=4 &&\{\text{dividing both sides by 5}\}
\end{aligned}$$

4 What operation must be done to both sides to remove x from the RHS?

a $x+4=8-x$
b $3x+4=7-3x$
c $6+5x=5-4x$
d $2x-11=7-x$
e $7-3x=2-8x$
f $9-3x=5-6x$

5 Solve the following equations:

a $x+4=8-x$
b $3x+4=7-3x$
c $6+5x=5-4x$
d $2x-11=7-x$
e $7-3x=2-8x$
f $9-3x=5-6x$

Example 15

Self Tutor

Solve for x: $3(x+2)=x-1$

$$\begin{aligned}
3(x+2)&=x-1\\
\therefore\quad 3x+6&=x-1 &&\{\text{expanding the brackets}\}\\
\therefore\quad 3x+6-x&=x-1-x &&\{\text{subtracting } x \text{ from both sides}\}\\
\therefore\quad 2x+6&=-1 &&\{\text{simplifying}\}\\
\therefore\quad 2x&=-7 &&\{\text{subtracting 6 from both sides}\}\\
\therefore\quad x&=-\tfrac{7}{2} &&\{\text{dividing both sides by 2}\}
\end{aligned}$$

6 Solve the following equations:

a $3(x-2)=x+4$
b $4(t-3)=3-t$
c $3(2y-1)=4y-5$
d $2(a+6)=7-2a$
e $4(2b+3)=b+12$
f $8(a+2)=1+3a$

7 **a** Try to solve $3(a+2) = 6 + 3a$. What do you notice?
b How many values of a satisfy the equation?

8 **a** Try to solve $2(2a+3) = 5 + 4a$. What do you notice?
b How many values of a satisfy the equation?

9 Solve the following equations:

a $2 - x = 3(x+3) + 1$
b $2x + 1 = 2(1-3x) + 7$
c $12 - x = 2(x-2) + x$
d $4 + 3p = 2 + 5(1-p)$
e $3x + 2(x+1) = 8$
f $4x - 1 + 3(2x-2) = 4$
g $x + 2 - 4(x-1) = 11$
h $2(x-1) - 5(x+2) = -8$
i $3(1-x) - 2(2-x) = 4$
j $5(x-2) + 3(1-2x) = 6$
k $4(2x-3) - 5(3x+1) = -2$
l $3 - 2x - (x+4) = -11$
m $2(1-3x) - (5-x) = 7$
n $4x + 2(x-1) + x = 9$

10 Solve the following equations:

a $12(x-1) = 3(2x+1) + 9$
b $3(2p-1) = 5(1-p) + 3$
c $2(5x+1) + 2 = 3(x-1)$
d $6(x-4) + 2(x+5) = 3(x-1) + 4$
e $x - 5 - (2x+1) = 4x + 3$
f $3 - 2x = 4(x+1) - 3(1-x)$
g $4x + 2(1-x) = 5 - (1-4x)$
h $5 - (2-3x) = 11 - (5-4x)$
i $x = 3(2x-1) - 4(2-x)$
j $2(x-2) - 3(5-x) = 4(2x+1)$

KEY WORDS USED IN THIS CHAPTER

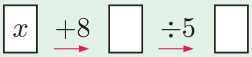
- algebraic equation
- inverse operation
- flowchart
- solution
- identity

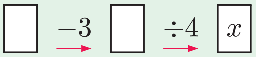
REVIEW SET 13A

1 **a** State the inverse of ×5.
b Find the result of adding 7 to both sides of $3x - 7 = 5$.
c Solve $2x = -8$ by inspection.

2 One of the numbers -3, 1, 2, or 5 is the solution to the equation $8 - 2x = 2 + x$. Find the solution by trial and error.

3 Copy and complete the following flowcharts:

a 

b 

4 Use a flowchart to show how the following expressions are built up from x:

a $\dfrac{x+4}{6}$
b $4x - 5$

5 Use a flowchart to show how to isolate x from the following expressions:

a $\dfrac{x}{5} + 8$
b $3(x-9)$

6 Solve for x:

a $4x + 5 = 12$ **b** $\dfrac{x}{3} = -4$ **c** $\dfrac{x}{3} - 5 = 7$

7 Solve for x:

a $3x - 5 = 2x - 10$ **b** $3(2x + 1) = -3$

c $4x + 3(2 - x) = 7$ **d** $3x + 2(x - 5) = 1 - x$

e $4(x + 2) - 3(1 - x) = x + 2$

REVIEW SET 13B

1 **a** Solve by inspection: $a \div 6 = 5$.

b Find the equation which results from adding 6 to both sides of $4x - 6 = -8$.

c State the inverse of dividing by 4.

d Solve $\dfrac{x}{7} = -3$ using a suitable inverse operation.

2 One of the numbers 2, -3, 10 or -13 is the solution to the equation $3x + 5 = 2(x - 4)$. Find the solution by trial and error.

3 Copy and complete the following flowcharts:

a $\boxed{x} \xrightarrow{\times 2} \boxed{} \xrightarrow{+3} \boxed{} \xrightarrow{\div 4} \boxed{}$ **b** $\boxed{} \xrightarrow{+6} \boxed{} \xrightarrow{\times 5} \boxed{x}$

4 Use a flowchart to show how the following expressions are built up from x:

a $2(3x - 7)$ **b** $\dfrac{2x + 3}{6}$

5 Use a flowchart to show how to isolate x from the following expressions:

a $\dfrac{5x - 3}{4}$ **b** $6(2x + 1)$

6 Solve for x:

a $4x - 11 = 25$ **b** $5 + 4x = 11$ **c** $3x - 2 = x + 6$

7 Solve for x:

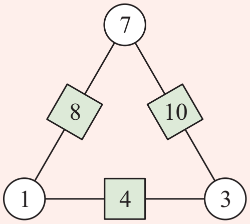
a $5 - 3x = -7$ **b** $2(3 - x) - 3x = 5$

c $\dfrac{3x - 5}{4} = 4$ **d** $3(2x - 1) - (4 - x) = 5$

e $3(2x + 1) = 5(2 - x) + 3(x + 4)$

ACTIVITY ARITHMAGONS

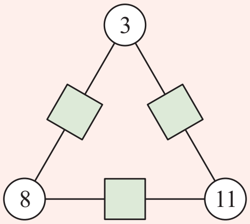
The diagram opposite is an **arithmagon**. The number in each square is equal to the sum of the numbers in the two circles connected to it.



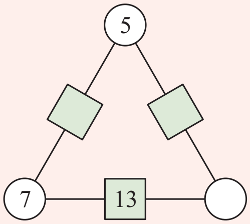
What to do:

1 Copy and complete these arithmagons:

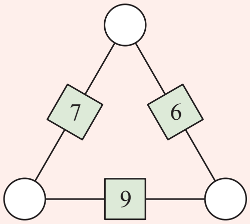
a



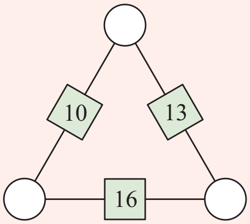
b



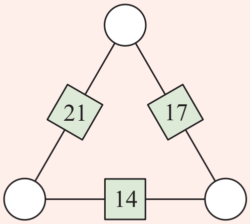
c



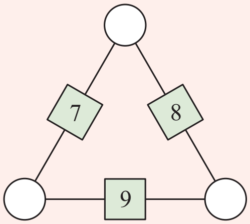
d



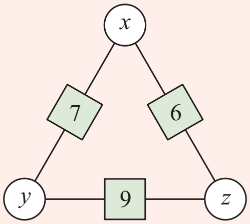
e



f



2 For some of the arithmagons in **1**, finding the numbers to go in the circles may not have been easy.
Consider the following approach for **c**:



$$x + y = 7$$
$$y + z = 9$$
$$x + z = 6$$

Adding these three equations gives

$$2x + 2y + 2z = 22$$
$$\therefore \quad x + y + z = 11$$
$$\therefore \quad x = 2, \quad y = 5 \quad \text{and} \quad z = 4.$$

Use this approach to check your answers to **1 d**, **e** and **f**.

Chapter 14

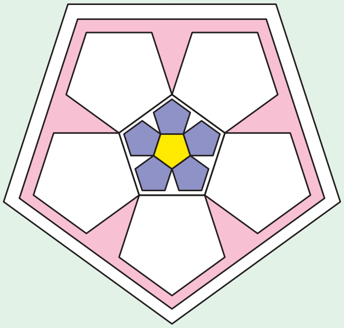
Polygons

Contents:

OPENING PROBLEM

Zac is asked to design a logo for the company he works for.

There are five departments and so the management decides to use a regular pentagon as the basis for the logo. This pentagon has sides of equal length and angles of equal size.



Things to think about:

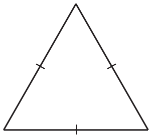
- What size are the angles of a regular pentagon?
- Is the angle size affected by the size of the pentagon?

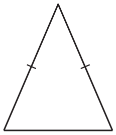
A CLASSIFYING TRIANGLES

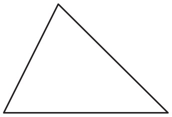
Triangles may be classified according to the measure of their sides or the measure of their angles.

CLASSIFICATION BY SIDES

A triangle is:
- **equilateral** if all its sides are equal in length
- **isosceles** if at least two of its sides are equal in length
- **scalene** if none of its sides are equal in length.

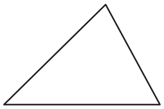

equilateral

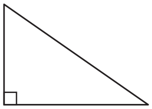

isosceles

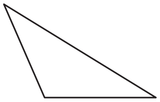

scalene

CLASSIFICATION BY ANGLES

A triangle is:
- **acute angled** if *all* its angles are acute
- **right angled** if one of its angles is a right angle (90^o)
- **obtuse angled** if one of its angles is obtuse.


acute angled

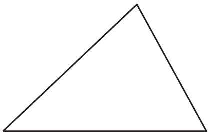

right angled


obtuse angled

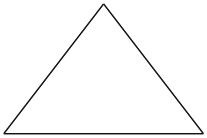
EXERCISE 14A

1 Measure the lengths of the sides of these triangles. Use your measurements to classify each as equilateral, isosceles or scalene.

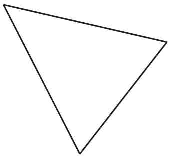
a



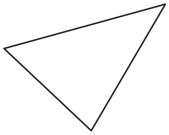
b



c

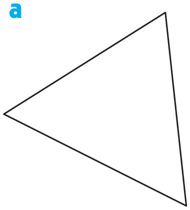


d

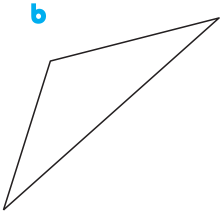


2 Measure the sizes of the angles of these triangles. Use your measurements to classify each as acute, obtuse or right angled.

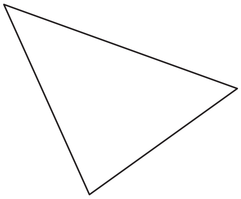
a



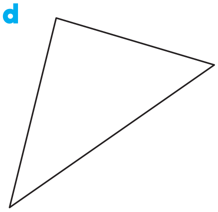
b



c

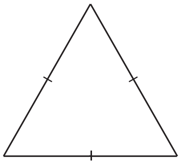


d

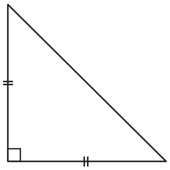


3 Use the indicated lengths of the sides *and* the sizes of the angles of each triangle to classify it. Each triangle should have *two* descriptions.

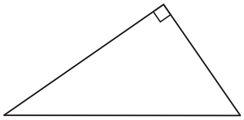
a



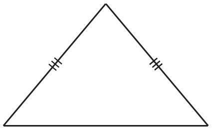
b



c

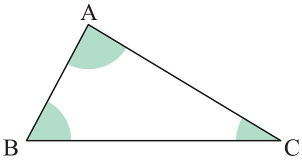


d



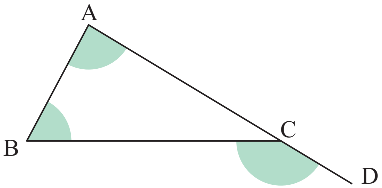
B ANGLES OF A TRIANGLE

When we talk about the angles of a triangle we actually mean the **interior angles** or the angles *inside* the triangle.



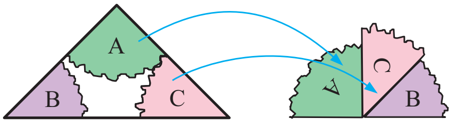
The shaded angles are the interior angles of triangle ABC.

If we extend a side of the triangle we create an **exterior angle**.



Angle BCD is an exterior angle of triangle ABC.
All triangles have six exterior angles.

In previous courses we discovered by tearing pieces of paper that the sum of the angles of a triangle is probably 180^o.

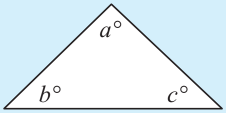


However, tearing paper is not accurate enough for us to say for certain that the sum is exactly 180^o. A more rigorous approach is necessary, and this is called a **geometric proof**.

THEOREMS

Angles of a triangle

The sum of the angles in a triangle is 180^o.

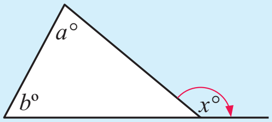


$a + b + c = 180$

GEOMETRY PACKAGE

Exterior angle of a triangle

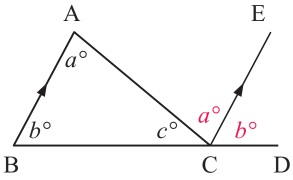
An exterior angle of a triangle is equal in size to the sum of its interior opposite angles.



$x = a + b$

GEOMETRY PACKAGE

Proof:



Let triangle ABC have angles of a^o, b^o and c^o at the vertices A, B and C.

Extend [BC] to D and draw [CE] parallel to [BA].

Now $A\widehat{C}E = B\widehat{A}C = a^o$ {equal alternate angles}

and $E\widehat{C}D = A\widehat{B}C = b^o$ {equal corresponding angles}

But $a + b + c = 180$ {angles on a line}

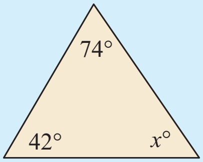
$\therefore$ $a + b + c = 180$

Also, the exterior angle $ACD = a^o + b^o = a^o + b^o$ which is the sum of its interior opposite angles.

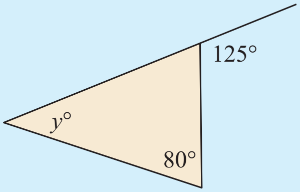
Example 1 **Self Tutor**

Find the unknowns in the following, giving brief reasons:

a



b



a $x + 42 + 74 = 180$ {angle sum of triangle}

$\therefore \quad x = 180 - 42 - 74$ {subtracting 42 and 74 from both sides}

$\therefore \quad x = 64$

b $y + 80 = 125$ {exterior angle of triangle}

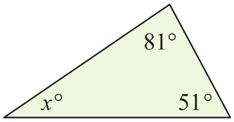
$\therefore \quad y = 125 - 80$ {subtracting 80 from both sides}

$\therefore \quad y = 45$

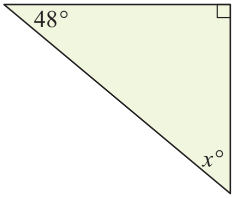
EXERCISE 14B

1 Find x in the following, giving brief reasons:

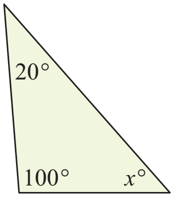
a



b

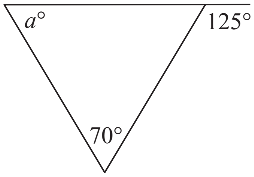


c



2 Find a in the following, giving brief reasons:

a



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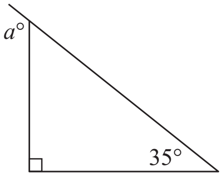
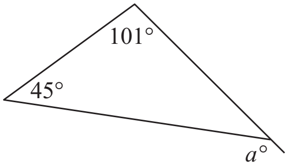


c



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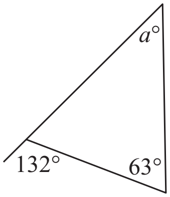
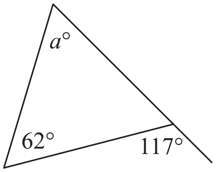


e



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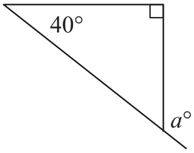
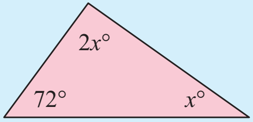


Example 2 **Self Tutor**

Find x in: **a**



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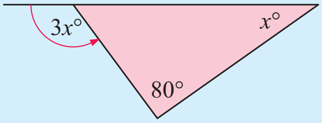


a $2x + x + 72 = 180$ {angles of a triangle}

$\therefore \quad 3x + 72 = 180$ {collecting like terms}

$\therefore \quad 3x + 72 - 72 = 180 - 72$ {subtracting 72 from both sides}

$\therefore \quad 3x = 108$

$\therefore \quad \dfrac{3x}{3} = \dfrac{108}{3}$ {dividing both sides by 3}

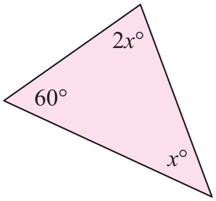
$\therefore \quad x = 36$

b

$$
\begin{aligned}
3x &= x + 80 && \{\text{exterior angle of a triangle}\}\\
\therefore\ 3x - x &= x + 80 - x && \{\text{subtracting } x \text{ from both sides}\}\\
\therefore\ 2x &= 80 &&\\
\therefore\ \frac{2x}{2} &= \frac{80}{2} && \{\text{dividing both sides by 2}\}\\
\therefore\ x &= 40 &&
\end{aligned}
$$

3 Find the unknowns in these triangles, giving brief reasons for your answers:

a



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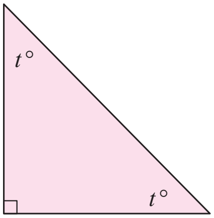

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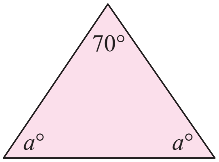

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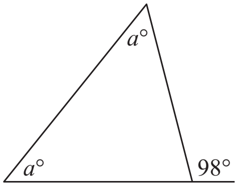
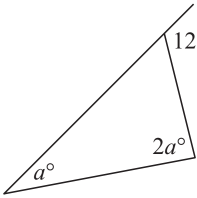


e



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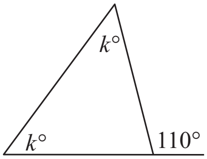
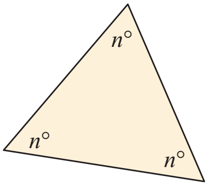
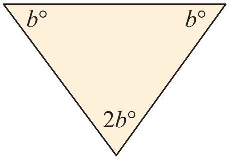


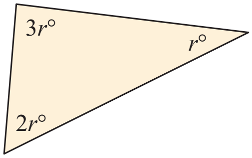
g



h



i



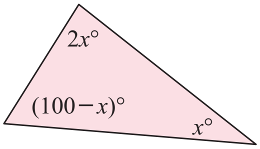
4 Explain why it is not possible to have a triangle which has:

a two obtuse angles

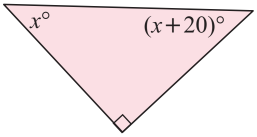
b one obtuse angle and one right angle.

5 Find x given:

a



b



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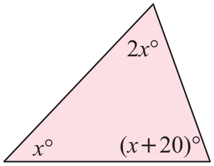

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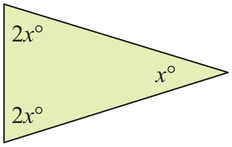
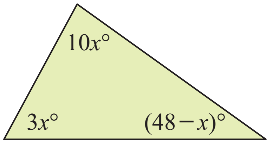


e



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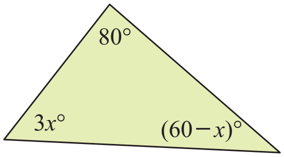


6 The figure shown can also be used to prove the sum of the angles of a triangle theorem.

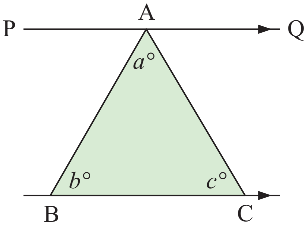
Copy and complete:

$Q\widehat{A}C = \ldots\ldots$ {equal alternate angles}

$P\widehat{A}B = \ldots\ldots$ {equal alternate angles}

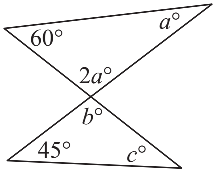
But $P\widehat{A}B + B\widehat{A}C + Q\widehat{A}C = \ldots\ldots$ {angles on a line}

So, $a + b + c = \ldots\ldots$

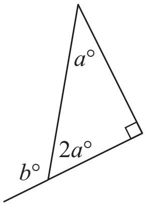


7 Find in alphabetical order, the values of the unknowns in:

a



b



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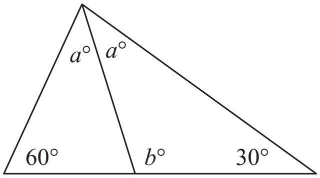
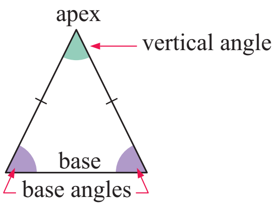


C ANGLES OF ISOSCELES TRIANGLES

An **isosceles triangle** is a triangle which has two sides equal in length.

We label parts of an isosceles triangle as follows:

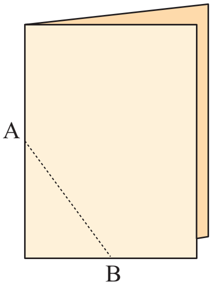
- the non-equal side is called the **base**
- the vertex between the equal sides is called the **apex**
- the angle at the apex is the **vertical angle**
- the angles opposite the equal sides are called the **base angles**.



MAKING AN ISOSCELES TRIANGLE

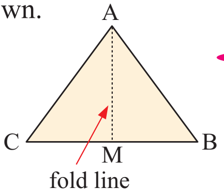
If we obtain a clean sheet of paper and fold it exactly down the middle we can make an isosceles triangle.

We draw a straight line [AB] as shown. Then, with the two sheets pressed tightly together, cut along [AB] through both sheets.



Keep the triangular piece of paper.

When you unfold it, you should obtain the isosceles triangle ABC shown.



DEMO

DISCUSSION

In the triangle ABC above, explain why:

- $A\widehat{C}B = A\widehat{B}C$
- M is the midpoint of [BC]
- [AM] is at right angles to [BC]

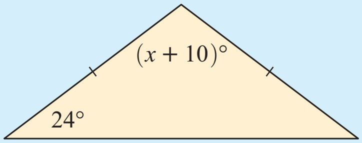
From the demonstrations above we conclude that:

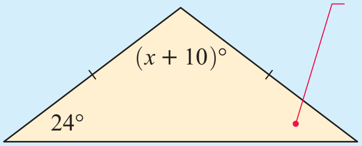
In any isosceles triangle:
- the base angles are equal
- the line joining the apex to the midpoint of the base is perpendicular to the base.

Example 3

Self Tutor

Find the value of the unknown in the figure:





As the triangle is isosceles the base angles are equal in size.

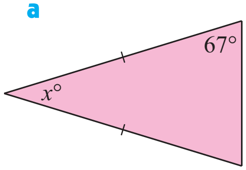
$\therefore$ this angle is also 24^o.

$$\begin{aligned} \text{But} \quad x+10+24+24 &= 180 \quad \{\text{angle sum of } \Delta\} \\ \therefore \quad x+58 &= 180 \\ \therefore \quad x+58-58 &= 180-58 \\ \therefore \quad x &= 122 \end{aligned}$$

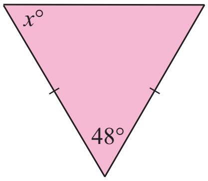
EXERCISE 14C

1 Find the unknowns in the following which have *not been drawn to scale*:

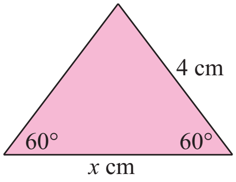
a



b



c



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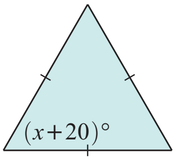

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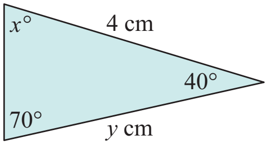

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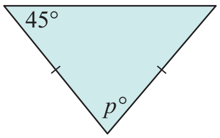
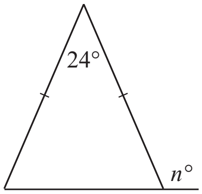


g



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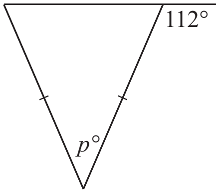
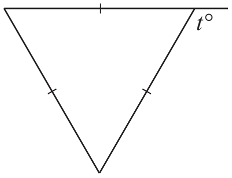
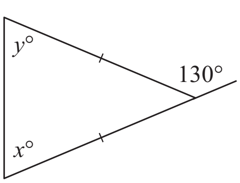


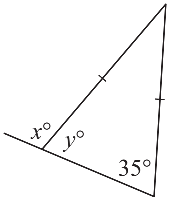
i



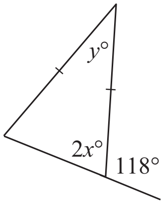
j



k

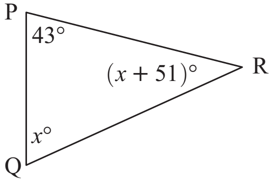


l

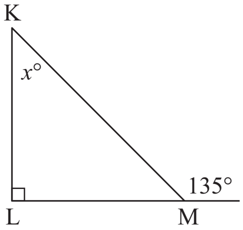


2 What can be deduced from these figures which have not been drawn to scale?

a



b



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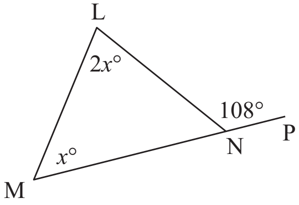


D POLYGONS

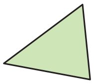
A shape that is drawn on a flat surface or plane is called a **plane figure**.

If the shape has no beginning or end it is said to be **closed**.

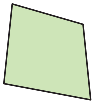
A **polygon** is a closed plane figure with straight line sides which do not cross.

Some simple examples of polygons are:

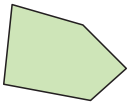
triangle
3 sides



quadrilateral
4 sides

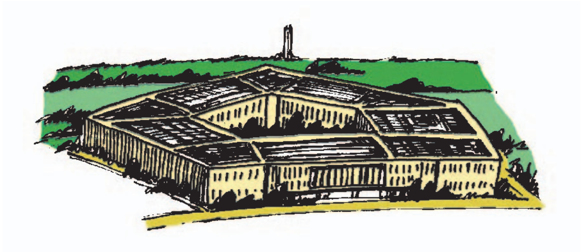


pentagon
5 sides



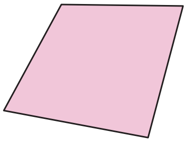
Polygons are named according to the number of sides they have. For example, a 9 sided polygon can be called a 9-gon. However, many polygons are known by other more familiar names. Here are the first few:

Number of Sides	*Polygon Name*
3	**Tri**angle
4	**Quadr**ilateral
5	**Penta**gon
6	**Hexa**gon
7	**Hepta**gon
8	**Octa**gon
9	**Nona**gon
10	**Deca**gon

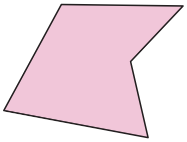


CONVEX POLYGONS

A **convex polygon** is a polygon which has no interior reflex angles.



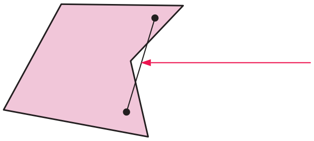
is a convex polygon.



is a non-convex polygon.

Every pair of points inside a convex polygon can be joined by a straight line segment which remains inside the polygon.

For example:

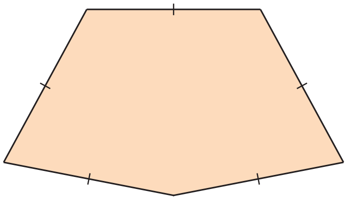


this line goes outside the polygon, so this polygon is not convex.

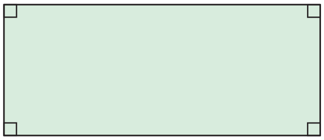
REGULAR POLYGONS

A **regular polygon** has sides of equal length **and** angles of equal measure.

This polygon is not regular even though its sides are equal in length. Its angles are not equal.



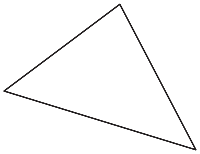
This polygon is not regular even though its angles are equal. Its sides are not equal in length.



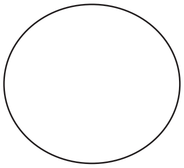
EXERCISE 14D

1 Which of these figures can be classified as a polygon?
Give a reason if the figure is not a polygon.

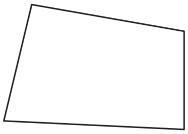
a



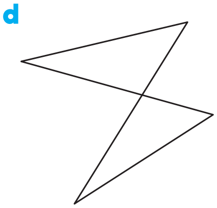
b



c



d



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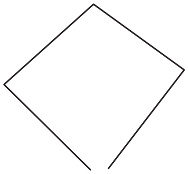

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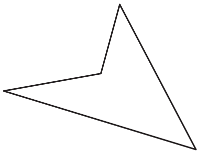

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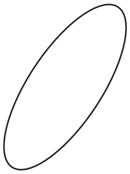

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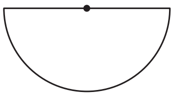
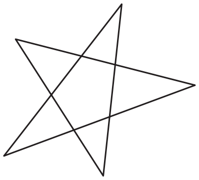


i



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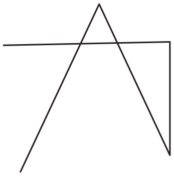
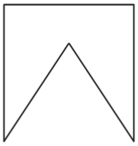
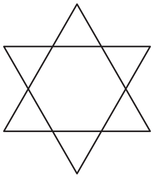


k



l



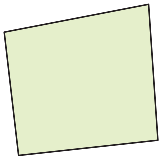
2 What special name is given to a polygon with:

a three sides **b** four sides **c** five sides

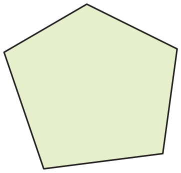
d six sides **e** eight sides **f** ten sides?

3 Name these polygons according to their number of sides and convexity:

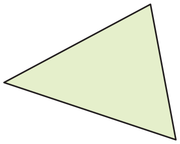
a



b



c



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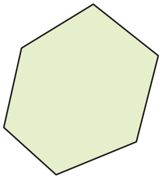

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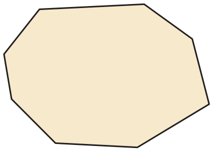

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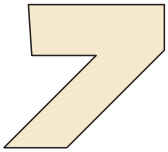

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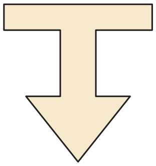

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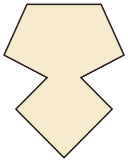


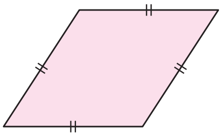
4 Draw a free-hand sketch of:

a a convex 4-sided polygon **b** a non-convex 4-sided polygon

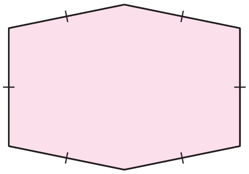
c a convex 5-sided polygon **d** a non-convex 6-sided polygon

5 Why are these figures not regular polygons?

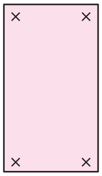
a



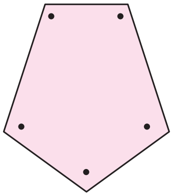
b



c



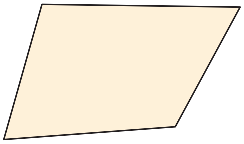
d



The same angle markings indicate equal degree measure.

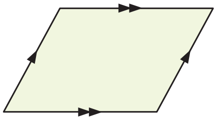
E QUADRILATERALS

A **quadrilateral** is a polygon with four sides.

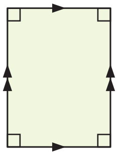


There are six special quadrilaterals:

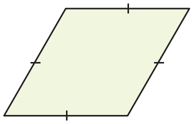
- A **parallelogram** is a quadrilateral which has opposite sides parallel.
- A **rectangle** is a parallelogram with four equal angles of 90^o.
- A **rhombus** is a quadrilateral in which all sides are equal.
- A **square** is a rhombus with four equal angles of 90^o.
- A **trapezium** is a quadrilateral which has a pair of parallel opposite sides.
- A **kite** is a quadrilateral which has two pairs of adjacent sides equal.



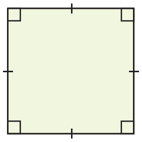
parallelogram



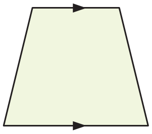
rectangle



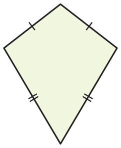
rhombus



square



trapezium

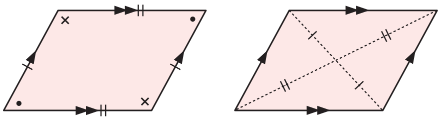


kite

PROPERTIES OF QUADRILATERALS

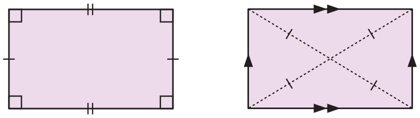
Parallelogram

- opposite sides are equal
- opposite angles are equal
- diagonals bisect each other (divide each other in half).



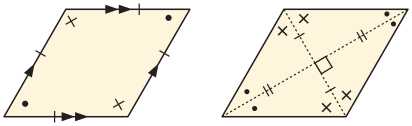
Rectangle

- opposite sides are equal in length
- diagonals are equal in length
- diagonals bisect each other.



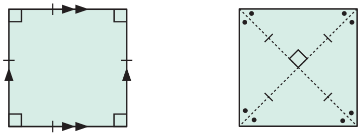
Rhombus

- opposite sides are parallel
- opposite angles are equal in size
- diagonals bisect each other at right angles
- diagonals bisect the angles at each vertex.



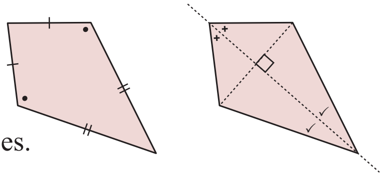
Square

- opposite sides are parallel
- diagonals bisect each other at right angles
- diagonals bisect the angles at each vertex.



Kite

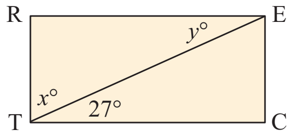
- *one* pair of opposite angles is equal in size
- diagonals cut each other at right angles
- one diagonal bisects *one* pair of angles at the vertices.



EXERCISE 14E

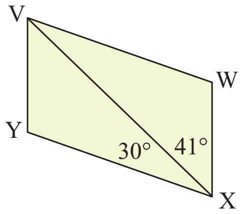
1 Solve the following problems:

- **a** RECT is a rectangle. Find the values of x and y.

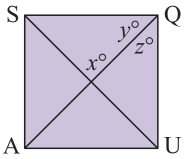


- **b** PARM is a parallelogram. Find the size of:
  - **i** $P\widehat{M}R$ **ii** $A\widehat{R}M$ **iii** $P\widehat{A}R$

- **c** VWXY is a parallelogram. Find the size of:
  - **i** $W\widehat{V}X$ **ii** $Y\widehat{V}X$
  - **iii** $V\widehat{Y}X$ **iv** $V\widehat{W}X$

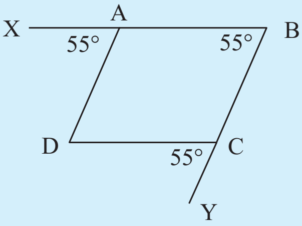


- **d** SQUA is a square. Find the values of:
  - **i** x **ii** y **iii** z



Example 4 **Self Tutor**

Is ABCD a parallelogram?



Since $X\widehat{A}D = A\widehat{B}C$

$\therefore$ [AD] $\parallel$ [BC] {equal corresponding angles}

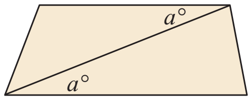
Likewise, $A\widehat{B}C = D\widehat{C}Y$

$\therefore$ [AB] $\parallel$ [DC]

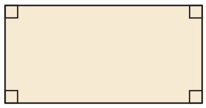
$\therefore$ ABCD is a parallelogram.
{opposite sides are parallel}

2 Using the information given in the diagrams, name the following quadrilaterals. Give brief reasons for your answers.

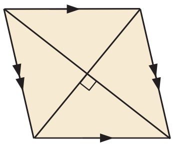
a



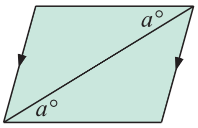
b



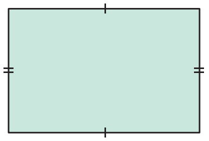
c



d



e



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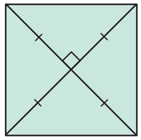
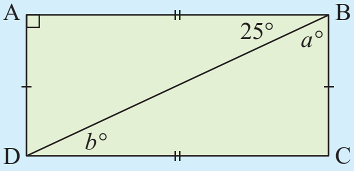


Example 5 **Self Tutor**

Classify the following quadrilateral, then find the values of the unknowns:



Since opposite sides are equal, we have a parallelogram.

In addition, $B\widehat{A}D$ is a right angle, so the quadrilateral is a rectangle.

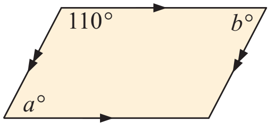
Thus $a + 25 = 90$ {as $A\widehat{B}C$ also measures 90^o}

$\therefore$ $a = 65$

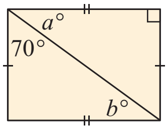
and $b = 25$ {$A\widehat{B}D$ and $C\widehat{D}B$ are equal alternate angles}

3 Use the information given to name the quadrilateral and find the values of the variables:

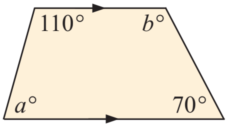
a



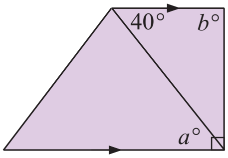
b



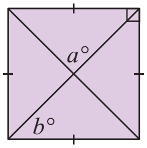
c



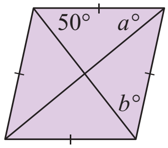
d



e



f

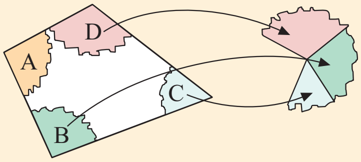


F ANGLES OF A QUADRILATERAL

INVESTIGATION 1 ANGLES OF A QUADRILATERAL

You will need: a large piece of paper, scissors, ruler and pencil.

What to do:



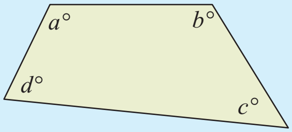
Step 1: Draw any quadrilateral on a piece of paper. Label the vertices A, B, C and D on the inside of the quadrilateral. Cut out the quadrilateral.

Step 2: Tear off each of the 4 angles. Place them adjacent to each other with vertices all meeting and no overlapping. What do you notice?

Step 3: Repeat this experiment with a few other quadrilaterals. What do you notice?

From **Investigation 1** you should have discovered that:

The sum of the angles of a quadrilateral is 360^o.

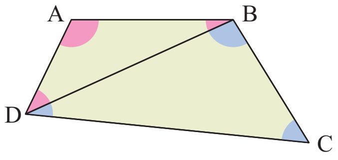


$$a + b + c + d = 360$$

GEOMETRY PACKAGE

Proof:

Suppose we divide quadrilateral ABCD into the two triangles ABD and BCD.



The sum of the interior angles of ABCD

$= \text{sum of angles of } \Delta\text{ABD} \;+\; \text{sum of angles of } \Delta\text{BCD}$

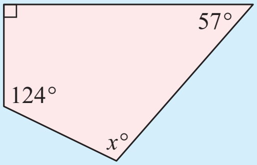
$= 180^o + 180^o$

$= 360^o$

Example 6

Self Tutor

Find the value of x, giving a brief reason:



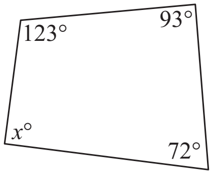
Using the angles of a quadrilateral result,

$$x + 57 + 90 + 124 = 360$$
$$\therefore \quad x + 271 = 360$$
$$\therefore \quad x = 89$$

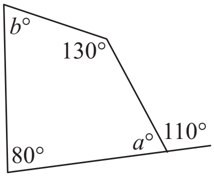
EXERCISE 14F

1 Find the values of the variables, giving brief reasons for your answers:

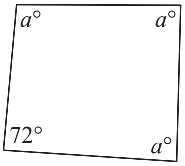
a



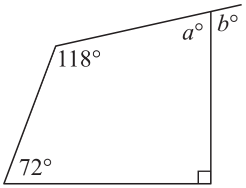
b



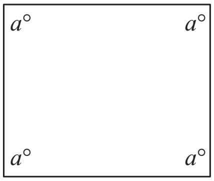
c



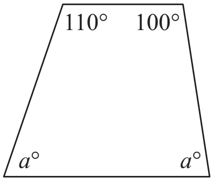
d



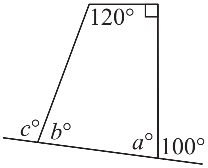
e



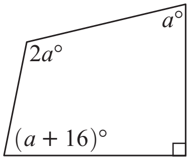
f



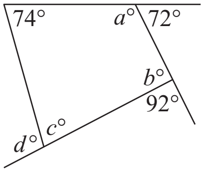
g



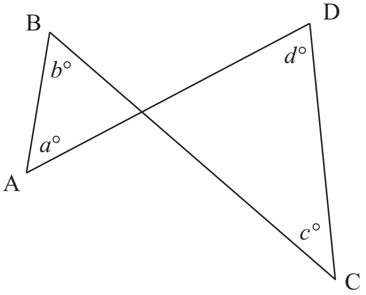
h



i



2



Consider the illustrated figure ABCD.

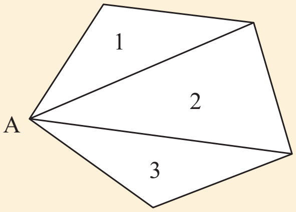
- **a** Why is it not a polygon?
- **b** Explain why $a + b = c + d$.
- **c** Show that $a + b + c + d$ must always be less than 360^o.

G INTERIOR ANGLES OF POLYGONS

INVESTIGATION 2 ANGLES OF AN n-SIDED POLYGON

What to do:

Step 1: Draw any pentagon (5-sided polygon) and label one of its vertices A. Draw in all the diagonals from A. Notice that 3 triangles are formed.



Step 2: Repeat with a hexagon, a heptagon (7-gon), and an octagon, drawing diagonals from one vertex only.

Step 3: Copy and complete the following table:

Polygon	*Number of sides*	*Number of diagonals from* A	*Number of triangles*	*Angle sum of polygon*
quadrilateral	4	1	2	$2 \times 180^o = 360^o$
pentagon				
hexagon				
heptagon				
octagon				
20-gon				

Copy and complete:

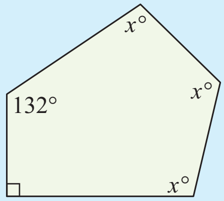
"*The sum of the sizes of the interior angles of any n-sided polygon is ......* $\times\ 180^o$."

From the **Investigation** you should have discovered that:

The sum of the sizes of the interior angles of any n-sided polygon is $(n-2) \times 180^o$.

Example 7 **Self Tutor**

Find x, giving a brief reason:



The pentagon has 5 sides

$\therefore$ the sum of its interior angles is $3 \times 180^o = 540^o$

$\therefore$ $x + x + x + 132 + 90 = 540$

$\therefore$ $3x + 222 = 540$

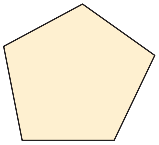
$\therefore$ $3x = 318$

$\therefore$ $x = 106$

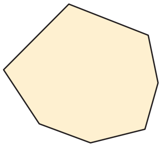
EXERCISE 14G

1 Find, without measuring, the sum of the angles in the following:

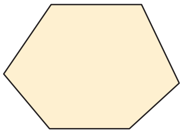
a



b



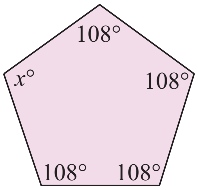
c



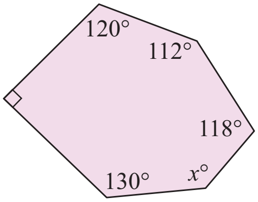
d a polygon with 12 sides **e** a 15-gon

2 Find the values of x in the following, giving brief reasons for your answers:

a



b



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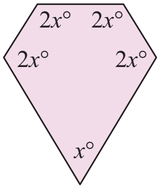
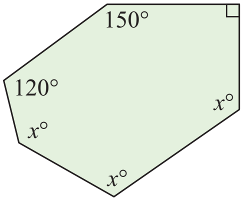
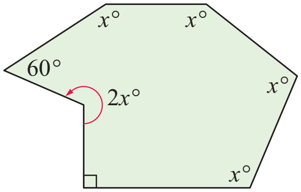


d



e



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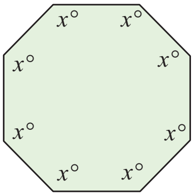


3 A pentagon has three right angles and two other equal angles. What is the size of each of the two equal angles?

4 Copy and complete the following table:

Regular polygon	*Number of sides*	*Sum of angles*	*Size of each angle*
square			
pentagon			
hexagon			
octagon			
decagon			

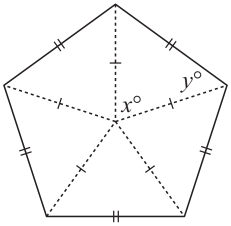
A regular polygon has all sides equal and all angles equal.

5 Write down a formula for calculating the size θ of the interior angle of an n-sided regular polygon.

6 A regular polygon has interior angles of 156^o. How many sides does the polygon have?

7 Jason claims to have found a regular polygon with interior angles of 161^o exactly. Comment on Jason's claim.

8



Five identical isosceles triangles are put together to form a regular pentagon.

a Explain why $5x = 360$.

b Find x and y.

c Explain why the regular pentagon has angles of 108^o.

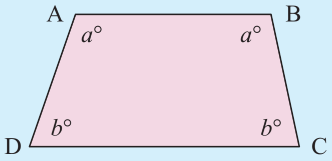
H DEDUCTIVE GEOMETRY (EXTENSION)

We can use the theorems studied in this chapter to prove other geometric facts. This process is called **deductive geometry** because we use the information given to *deduce* facts.

Example 8 **Self Tutor**

In the given figure $D\widehat{A}B = C\widehat{B}A$ and $A\widehat{D}C = B\widehat{C}D$.

Show that [AB] is parallel to [DC].



Let $D\widehat{A}B = C\widehat{B}A = a^o$ and
$A\widehat{D}C = B\widehat{C}D = b^o$

Now $a + a + b + b = 360$
{angles of a quadrilateral}

$\therefore \quad 2a + 2b = 360$

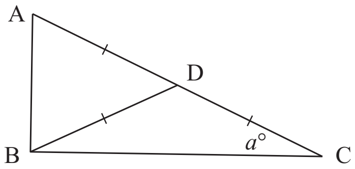
$\therefore \quad a + b = 180$ {dividing by 2 throughout}

$\therefore \quad [AB] \parallel [DC]$

{as co-interior angles add to 180^o}

EXERCISE 14H

1



The given figure is not drawn to scale.

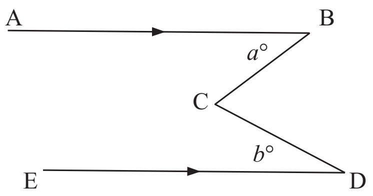
Let $D\widehat{C}B = a^o$.

a Find, in terms of a and giving reasons:

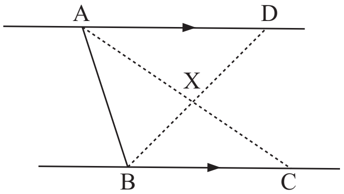
i $D\widehat{B}C$ **ii** $A\widehat{D}B$ **iii** $D\widehat{B}A$

b Show that $A\widehat{B}C$ is a right angle.

2 In the given figure [AB] and [ED] are parallel. By adding in another line segment *or* by extending an existing line segment, show that $\widehat{BCD} = (a+b)^o$.



3



(AD) and (BC) are parallel lines.

[AC] is the angle bisector of $\widehat{DAB}$.

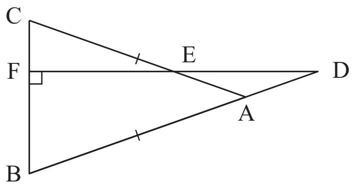
[BD] is the angle bisector of $\widehat{ABC}$.

Prove that [BD] is perpendicular to [AC].

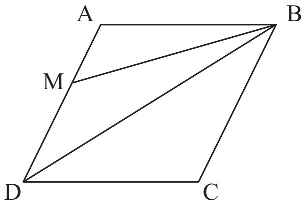
4 ABC is an isosceles triangle with $AB = AC$. [FD] is drawn perpendicular to [BC] so it intersects [CA] at E, and the extension of [BA] at D.

Show that triangle ADE is isosceles.

Hint: Start by letting $\widehat{CBA} = \alpha$.



5



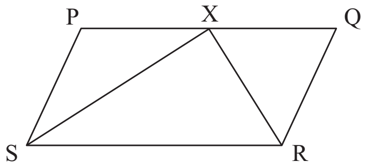
ABCD is a rhombus. [BM] bisects $\widehat{ABD}$.

Show that $\widehat{AMB}$ is three times larger than $\widehat{ABM}$.

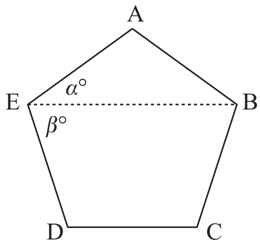
Hint: Start by letting $\widehat{ABM} = \alpha$.

6 PQRS is a parallelogram. The angle bisectors of $\widehat{PSR}$ and $\widehat{QRS}$ meet at X on [PQ].

Show that $\widehat{SXR}$ is a right angle.



7



ABCDE is a regular pentagon.

a Find α and β.

b Prove that [EB] is parallel to [DC].

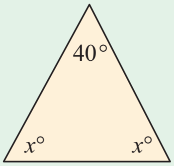
KEY WORDS USED IN THIS CHAPTER

- acute
- equilateral
- kite
- polygon
- rhombus
- trapezium
- apex
- exterior angle
- obtuse
- quadrilateral
- right angled
- base angles
- interior angle
- parallelogram
- rectangle
- scalene
- convex polygon
- isosceles
- plane figure
- regular polygon
- square

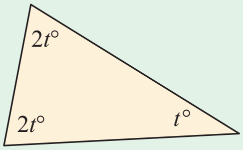
REVIEW SET 14A

1 Find the values of the variables in the following, giving brief reasons:

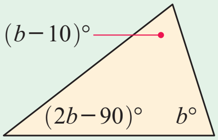
a



b

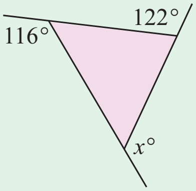


c



2 Find, giving reasons, the value of x:

a



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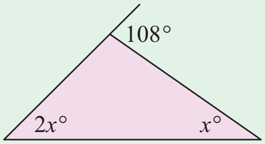

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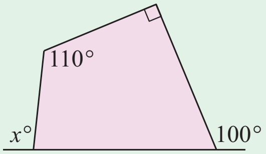
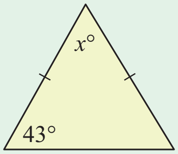
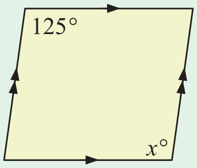


d



e



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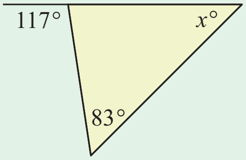
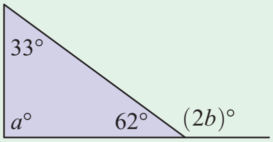
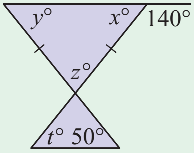


3 Find the values of the variables in the following diagrams:

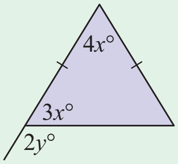
a



b

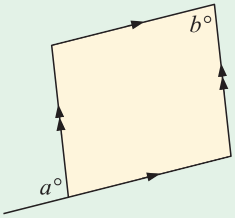


c

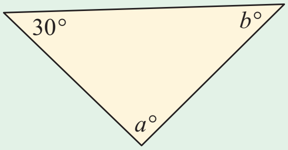


4 Find equations connecting the variables in each diagram. Give reasons for your answers.

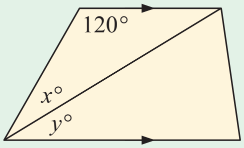
a



b

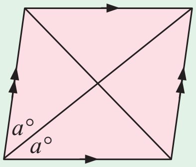


c

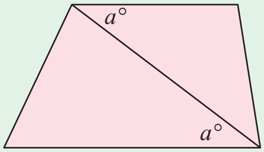


5 Using the information given, name each of the following quadrilaterals. Give reasons for your answers.

a



b



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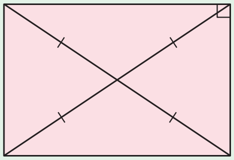
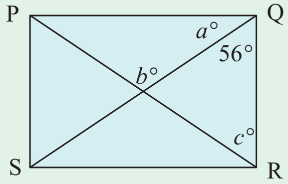


6 **a** Is a square a kite? **b** Is a parallelogram a trapezium?

7 PQRS is a rectangle. Find the values of:

a a **b** b **c** c



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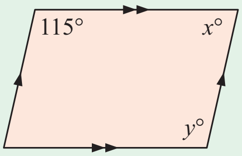
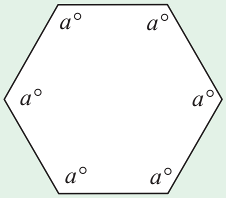


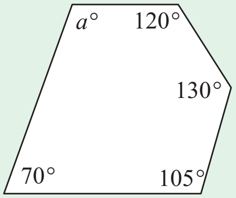
Using the information given on the diagram, name the figure and find the values of x and y.

9 Find the value of a in each of the following polygons:

a



b



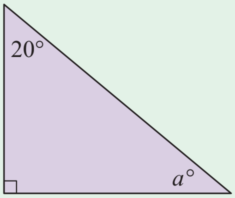
10 Find the size of the angles of a regular 12-sided polygon.

11 A regular polygon has angles of 162^o. How many sides does it have?

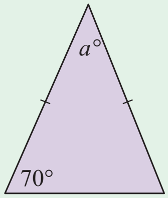
REVIEW SET 14B

1 Find the value of a in each diagram, giving brief reasons for your answers:

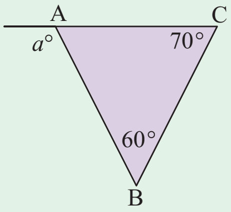
a



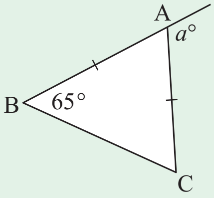
b



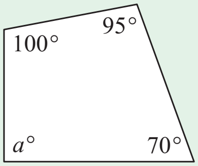
c



d



e



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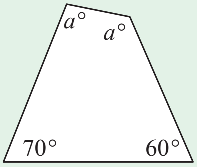
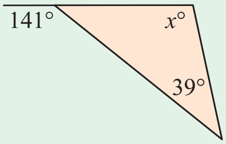


2 Find the value of x, giving reasons:

a



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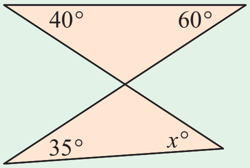

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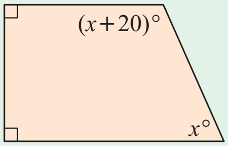
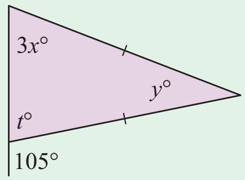
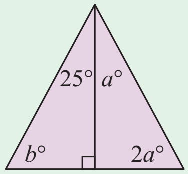


3 Find the values of the variables in the following diagrams:

a



b



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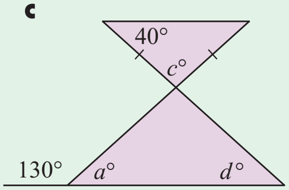
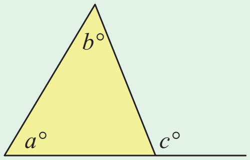
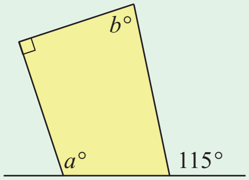


4 Find the equation connecting the variables in each figure. Give reasons for your answers.

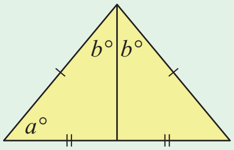
a



b

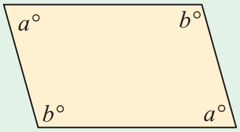


c

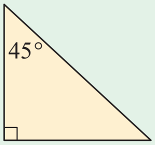


5 Using the information given, name each of the following figures.
Give reasons for your answers.

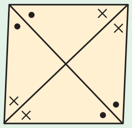
a



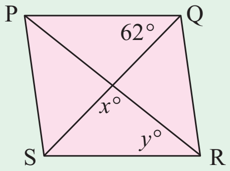
b



c

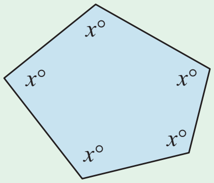


6 PQRS is a rhombus. Find the values of x and y.

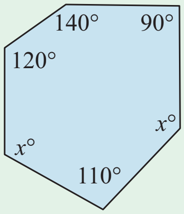


7 Find the value of x in these diagrams:

a



b



8 **a** Is a square a rectangle? **b** Is a rectangle a parallelogram?

9 Find the size of the angles of a regular 9-gon.

10 Can a regular polygon have angles of size 155^o?

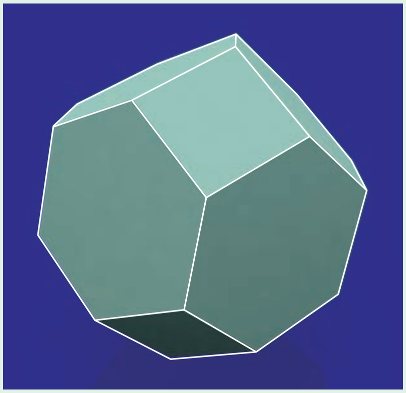
Chapter 15

The geometry of solids

Contents:

OPENING PROBLEM

The illustrated solid has two types of faces: hexagons and squares. It is called a **truncated octahedron**.



a How many hexagonal faces does it have?

b How many square faces does it have?

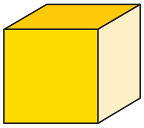
c A net used to make a model of this solid can be drawn on a single sheet of A4 paper. Can you draw the net accurately?

You should visit http://www.software3d.com

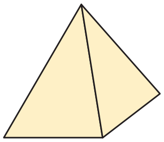
A SOLIDS

A **solid** is a three-dimensional body which occupies space.

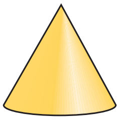
The diagrams below show a collection of solids. Each solid has the three dimensions: *length*, *width* and *height*.



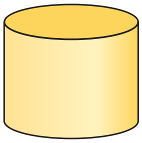
cube



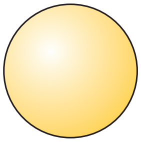
square-based pyramid



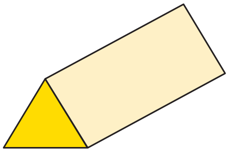
cone



cylinder

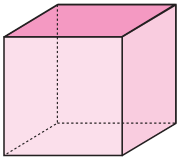


sphere

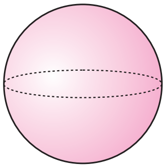


triangular prism

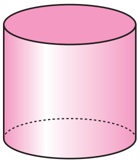
The boundaries of a solid are called **surfaces**. These may be flat surfaces, curved surfaces, or a mixture of both.



A cube is bounded by six flat surfaces.



A sphere is bounded by one curved surface.

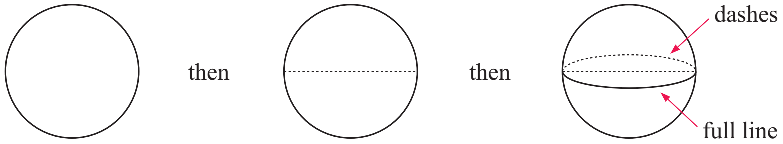


A cylinder is bounded by two flat surfaces and one curved surface.

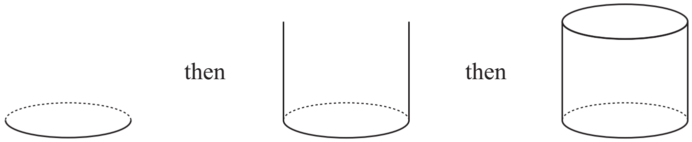
When we draw solids, we often used dashed lines to show edges which are hidden at the back of the solid. The dashed lines remind us these edges are there, even if we cannot normally see them. The dashed lines also help us to appreciate the 3-dimensional nature of the solids.

Here are step-by-step drawings to help you to draw your own diagrams of some special solids.

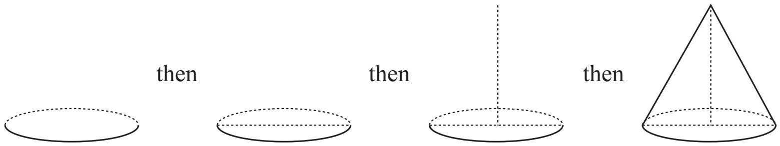
- **sphere:**



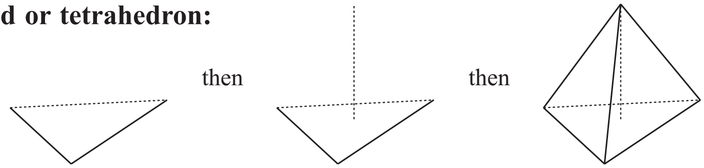
- **cylinder:**



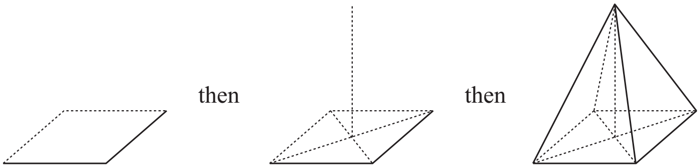
- **cone:**



- **triangular-based pyramid or tetrahedron:**



- **square-based pyramid:**



POLYHEDRA

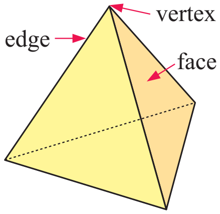
A **polyhedron** is a solid which is bounded by flat surfaces only.
The plural of polyhedron is **polyhedra**.

Cubes and pyramids are examples of polyhedra. Spheres and cylinders are not.

Each flat surface of a polyhedron is called a **face** and has the shape of a polygon.

Each corner point of a polyhedron is called a **vertex**. The plural of vertex is **vertices**.

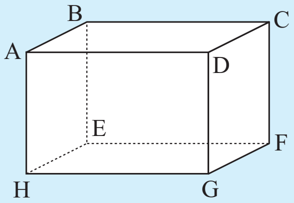
Two flat surfaces intersect in a line called an **edge**.



The solid shown is a triangular-based pyramid or **tetrahedron**. It has 4 vertices, 4 faces and 6 edges. The tetrahedron is the simplest example of a solid for which all faces are polygons.

Example 1 **Self Tutor**

This figure is a rectangular box since all of its faces are rectangles.



a Name the vertices of the figure.
b [AB] is an *edge*. Name the other 11 edges.
c ABCD is a *face*. Name the other 5 faces.
d Name all edges that are parallel to [AH].
e Name all edges which meet at D.

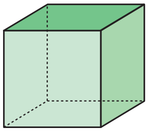
a A, B, C, D, E, F, G, H.
b [BC], [CD], [AD], [AH], [BE], [CF], [DG], [EH], [EF], [FG], [GH]
c BCFE, HEFG, ADGH, ABEH, DCFG
d [BE], [CF], [DG]
e [AD], [CD], [DG]

When naming faces we move around the face in order, usually in a clockwise direction.

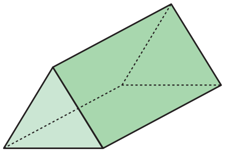
PRISMS

A **prism** is a solid with a uniform cross-section that is a polygon.

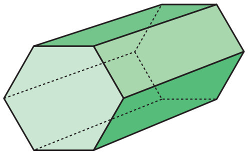
Examples of prisms:



cube



triangular prism



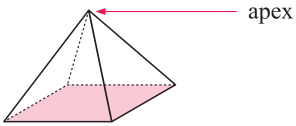
hexagonal prism

A **cube** is a rectangular prism with 6 square faces. This is why rectangular prisms are sometimes called **cuboids**.

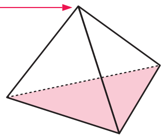
PYRAMID

A **pyramid** is a solid with a polygon for a base, and triangular faces which come from the edges of the base to meet at a point called the **apex**.

Examples of pyramids:



square-based pyramid



triangular-based pyramid

EXERCISE 15A

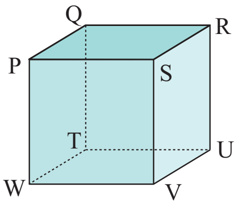
1 Draw a diagram to represent:

- **a** a cone
- **b** a cube
- **c** a rectangular-based pyramid
- **d** a cylinder
- **e** a tetrahedron
- **f** a pentagonal prism
- **g** a sphere
- **h** an octagonal prism
- **i** a hexagonal-based pyramid

2 Name the solid which best resembles:

- **a** a witch's hat
- **b** a shuttlecock tube
- **c** a computer
- **d** a planet
- **e** a carrot
- **f** a four-sided die
- **g** a coin

3
- **a** Name all the vertices of this cube.
- **b** Name all the faces of this cube.
- **c** Name all the edges of this cube.
- **d** Name all edges parallel to [SV].
- **e** Name all edges that are concurrent at P.
- **f** Name a face which is parallel to face PQRS.

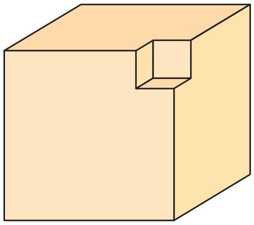


4 What shape are the side faces of any: **a** prism **b** pyramid?

5 Draw a solid which has:

- **a** only a curved surface
- **b** a curved and a flat surface
- **c** two flat and one curved surface

6

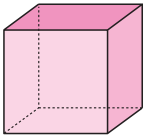
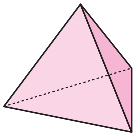
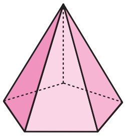
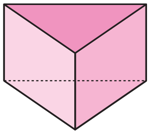
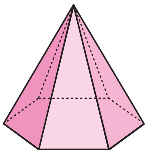
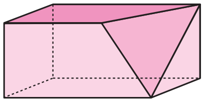
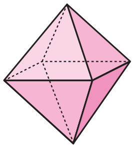
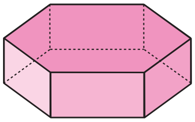


A large cube of cheese has a smaller cube removed from one corner.

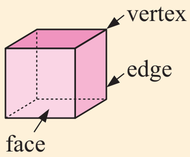
- **a** How many faces, vertices and edges does the resulting piece of cheese have?
- **b** If identical smaller cubes are removed from all corners of the original cube, how many faces, vertices and edges does the resulting piece of cheese have?

INVESTIGATION 1 EULER'S FORMULA

For all polyhedra, there is a relationship between the number of edges, the number of faces, and the number of vertices. Consider the diagrams of eight polyhedra below:

a 	**b** 	**c** 	**d** 
e 	**f** 	**g** 	**h** 

For a given figure we let:
E represent the number of **edges**,
F represent the number of **faces**, and
V represent the number of **vertices**.



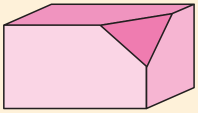
What to do:

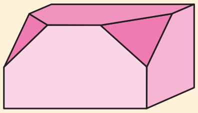
1 Copy and complete the table for the solids above:

Figure	F	V	$F+V$	E
a				
b				
c				
d				
e				
f				
g				
h				

2 Look carefully at the last two columns of your table. Can you see a relationship? Describe it in words or symbols. This is **Euler's formula**.

3 Investigate the truth of Euler's formula with solids of your own construction. For example, try cutting corners off a cuboid.

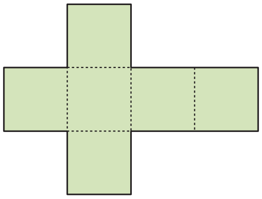




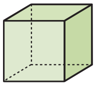
B NETS OF SOLIDS

A **net** is a two-dimensional shape which may be folded to form a solid.

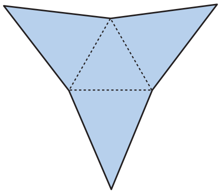
For example, the following nets may be cut out and folded along the dotted lines to form common solids:



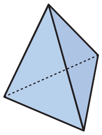
becomes



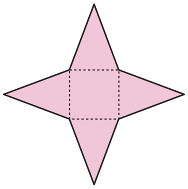
cube



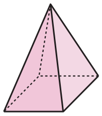
becomes



triangular-based pyramid
or **tetrahedron**



becomes



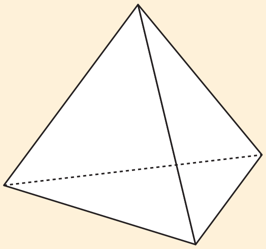
square-based pyramid

Click on the icon to view demonstrations of how the nets form the solids.

INVESTIGATION 2 TRIANGULAR-BASED PYRAMIDS

If we start with an acute angled triangle drawn on paper, how can we construct a model of a triangular-based pyramid?

What to do:

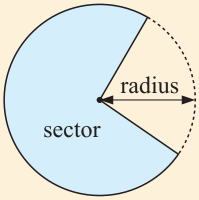


1. On separate sheets of paper, draw acute angled triangles of different shapes. Cut them out with scissors.
2. Using three folds of the paper, construct models of a triangular based pyramid.
3. Will your method work if the original triangle is:
   - **a** right angled
   - **b** obtuse angled?

INVESTIGATION 3 MAKING CONES

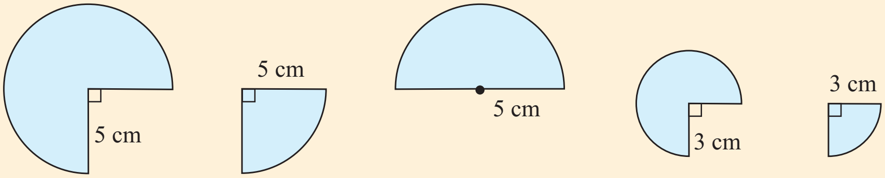
The net for the *curved surface* of a cone is actually a **sector** of a **circle**.

The shape of the cone depends on the **radius** or size of the circle, and also on the size of the sector.



What to do:

1. Using your compass, draw *two* circles of radius 5 cm and *one* circle of radius 3 cm.
2. Cut the circles to produce the illustrated sectors.

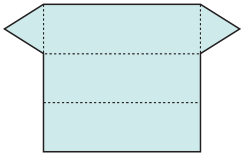


3. Use your five sectors to form the curved surfaces of cones.
4. Complete the following statements:
   - **a** The radius of the sector affects the ...... of the cone.
   - **b** The larger the sector cut from the circle, the ..... the cone will be.

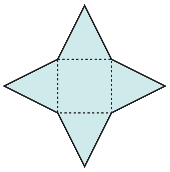
EXERCISE 15B

1 For each of the following nets, draw and name the corresponding solid:

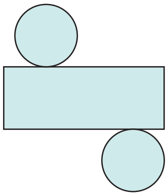
a



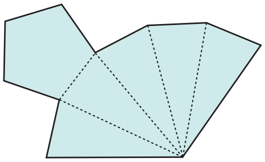
b



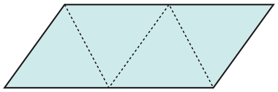
c



d

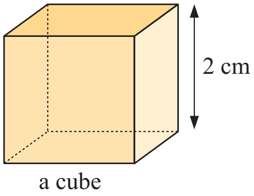


e

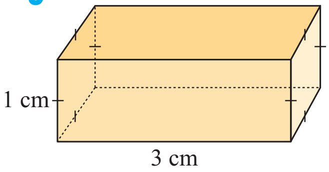


2 For each of the following 3-dimensional solids, draw nets with lengths clearly marked:

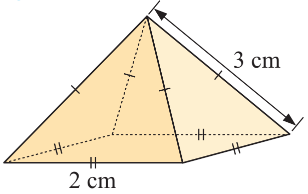
a



b



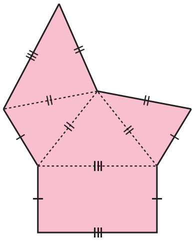
c



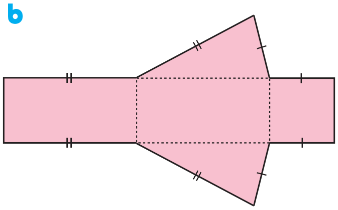
3 Draw as many *different* nets as you can which can all be made into the same sized cube.

4 Draw and name the solids which would be formed from the following nets:

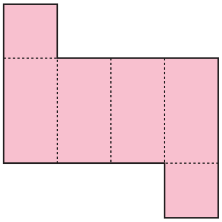
a



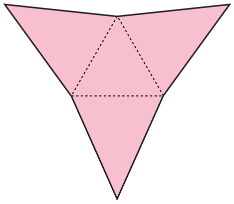
b



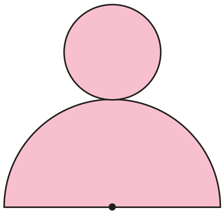
c



d



e

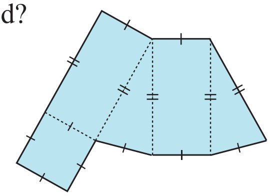


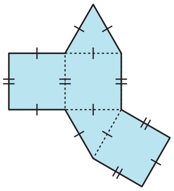
5 Consider the following nets:

a Do these nets form *exactly* the same solid?

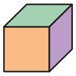
b Do both of these nets form a triangular prism?

c Explain the difference between the two nets. Which net forms a *regular* triangular prism?

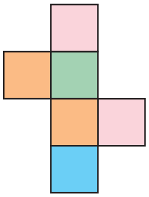




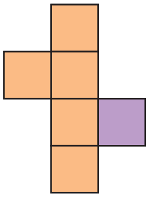
6 Which of the following nets can be used to make this cube?



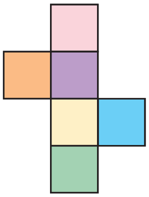
A



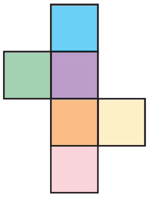
B



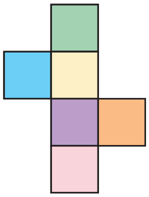
C



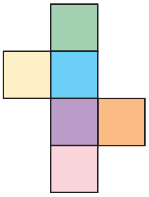
D



E

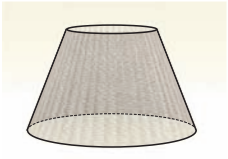


F



7 A lampshade is a **truncated cone**, which is a cone with a smaller cone cut from it. Sketch a net for a truncated cone.

8 Answer the questions in the **Opening Problem** on page **296**.



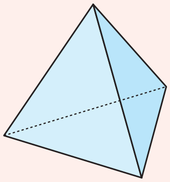
ACTIVITY 1 PLATONIC SOLIDS

We have already seen that a **polyhedron** is a solid which is bounded by flat surfaces only.

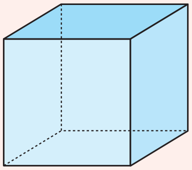
One particular set of polyhedra consists of solids whose surfaces are identical *regular* shapes with all sides of equal length.

They are called the **Platonic solids** after the Greek philosopher **Plato** (427-347 BC).

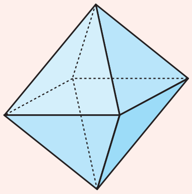
There are only five Platonic solids:



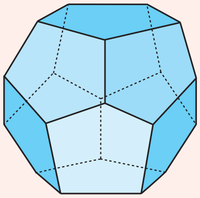
Tetrahedron



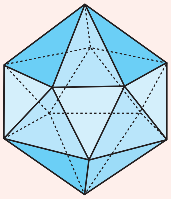
Hexahedron



Octahedron



Dodecahedron



Icosahedron

The names of the solids refer to their number of faces.
For example: *tetra* means four, *icosa* means twenty.

NETS

What to do:

1. Identify the regular polygon which forms the faces of each Platonic solid.
2. Print the nets for making the Platonic solids.
3. Photocopy them onto light card and make the solids.

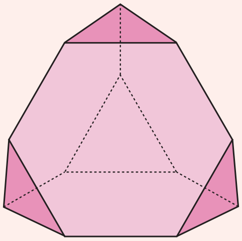
ACTIVITY 2 FURTHER SOLID CONSTRUCTIONS

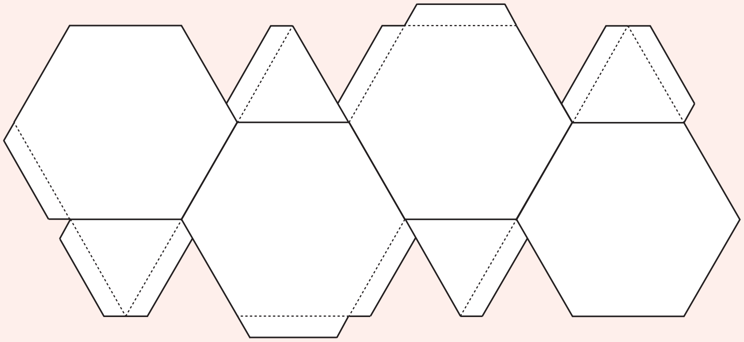
In this investigation we will construct a truncated tetrahedron and a cuboctahedron.

What to do:

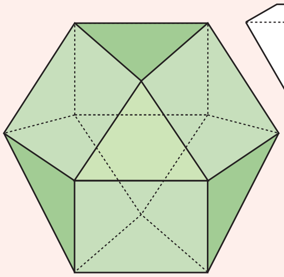
1. Print each net from the CD then photocopy it onto light cardboard or stiff paper.
2. Cut around the solid lines on the outside of the net.
3. Crease along all the lines, always folding away from you.
4. Lightly form the net into a ball with your cupped hands, so that you can see which flaps to join.
5. Glue the flaps under the matching faces, leaving the glue on each section to dry before going on to the next section.

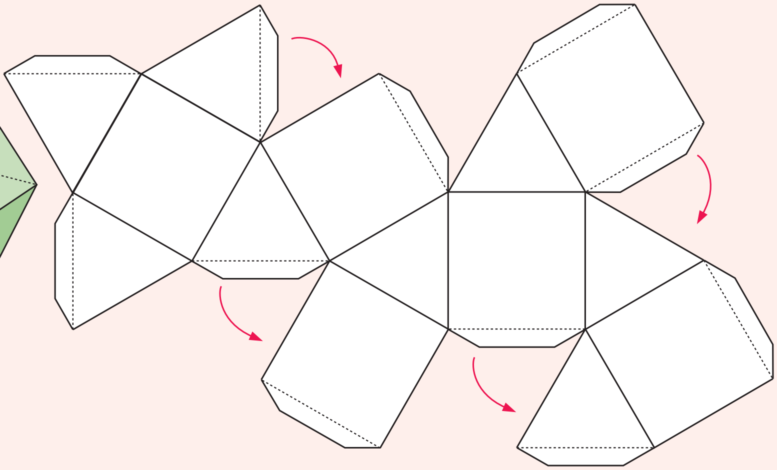
A truncated tetrahedron





A cuboctahedron





RESEARCH OTHER THREE-DIMENSIONAL OBJECTS DRAWN ON THE TWO-DIMENSIONAL PLANE

Alongside is a sphere which has been computer drawn using a 3-D modelling program.

1 Search for more of these drawings in the library or on the internet.

2 Can a net be drawn for a sphere?

C DRAWING RECTANGULAR SOLIDS

We sometimes need to draw three-dimensional solids with rectangular faces on two-dimensional sheets of paper. To do this we can start with either a face or an edge of the figure, and this choice leads to two different types of **projection**. We call them projections because we **project** the image of the three-dimensional solid onto the two-dimensional paper.

OBLIQUE

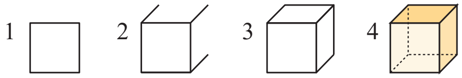
When using an **oblique projection**, the lines going back from the front face are inclined at 45^o and their lengths are shown shorter than those of the front face.
For example, to draw a cube we use the following steps:

Step 1: Draw a square for the front face.

Step 2: Draw edges back from the front face at 45^o and shorter than those of the front face.

Step 3: Complete the cube.

Step 4: Draw in dashed lines which show the hidden edges.



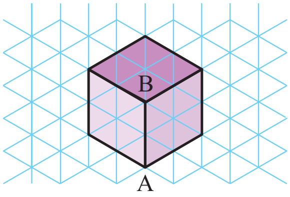
ISOMETRIC

When drawing a rectangular object using an **isometric projection**, we use special graph paper made up of equilateral triangles. Isometric graph paper is designed specifically for drawing objects of this type.

ISOMETRIC GRAPH PAPER

We start with a vertical **edge** of the solid. The horizontal edges are drawn inclined at 30^o, and their lengths are maintained.

The diagram alongside shows the isometric projection of a cube. The edge [AB] appears closest to us, and this is often the **starting edge** of the figure, or first edge drawn.

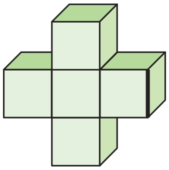


EXERCISE 15C

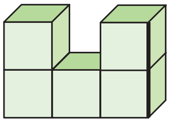
1 Draw an oblique projection of a box which has sides 3 units by 2 units by 1 unit. Start with a 3 unit by 2 unit rectangle as the front face.

2 Redraw the following on isometric paper. Use the darker lines as the starting edges.

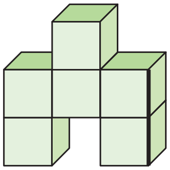
a



b

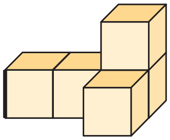


c

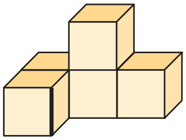


3 Copy these objects and then draw each of them as isometric projections. Use the darker lines as the starting edges.

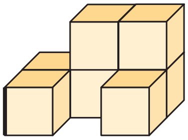
a



b

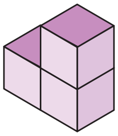


c

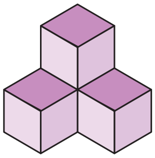


4 Redraw the following as oblique projections:

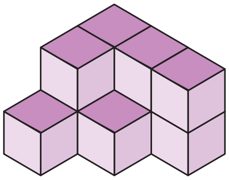
a



b

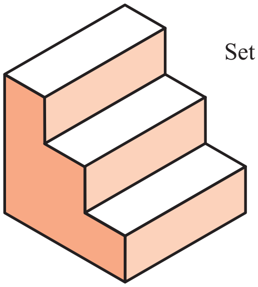


c



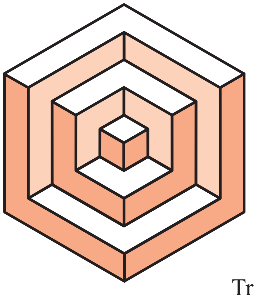
5 Use isometric paper to draw:

a



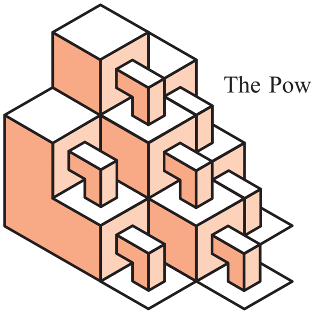
Set of steps

b



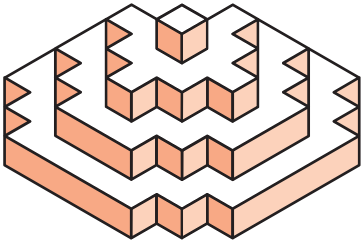
Tri-cube

c



The Power Station

d

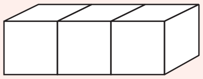


Fancy Pyramid

ACTIVITY 3 CUBIC REPRESENTATIONS

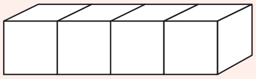
In this activity we construct rectangular prisms using cubes.

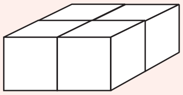
If we are given 3 cubes, there is only one rectangular prism that we can form:



Notice that $3 = 1 \times 1 \times 3$. The length, width, and depth measurements are therefore 1, 1 and 3, though not necessarily in that order.

If we are given 4 cubes then there are *two* possible prisms:





This is because $4 = 1 \times 1 \times 4$ and $1 \times 2 \times 2$ are the only possible products.

What to do:

1 Find how many different prisms can be made if we are given the following numbers of cubes:

a 5 **b** 6 **c** 8 **d** 12 **e** 24

2 How many different prisms can be made if we are given:

a a prime number of cubes

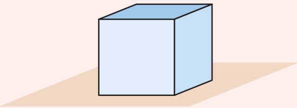
b a number of cubes that is the product of two different primes?

3 Without sketching them, list the possible prisms that can be made using:

a 36 **b** 120 **c** 360 cubes.

ACTIVITY 4 HUMBLE HOUSES

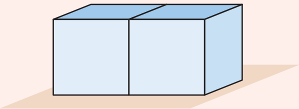
The Humble House factory manufactures cubic living quarters for use in countries where the conditions most of the year are dry and hot. Heat enters every roof and exposed wall by exactly the same amount.



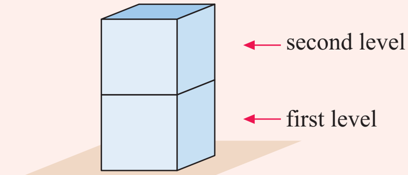
For a house made of one cube, heat enters in equal amounts from 5 sides, but not the floor.

There are two possible house designs made from **two cubes** placed together, face to face:

A



B



What to do:

1 How many exposed faces are there in each of the designs **A** and **B**? Which one would be the more suitable for hot conditions?

2 Draw the 4 possible housing arrangements using 3 adjacent cubes. The blocks must touch face to face and the building must be free-standing. For example, columns for support are not acceptable, and in any case we do not want heat to come in through the floor.

3 Determine from your models the 'best' 3-cube structure which minimises heat intake.

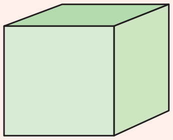
4 Investigate the possible 4-cube buildings and determine the model which would take in the least amount of heat.

5 How many different possible 5-cube buildings are there? Which one is 'best'?

ACTIVITY 5

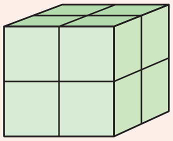
PAINTED CUBES

A cube is painted and then cut into 8 smaller cubes.



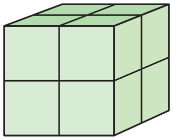
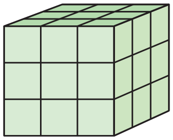
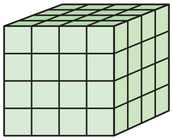
On dismantling the 2 × 2 × 2 cube we see that all 8 cubes have paint on exactly 3 faces.

How many cubes are painted the same when the cube is cut into 3 × 3 × 3, 4 × 4 × 4, and so on?



What to do:

1 Copy and complete:

Cube cut	*3 faces painted*	*2 faces painted*	*1 face painted*	*No faces painted*
	8	0	0	0
				
				

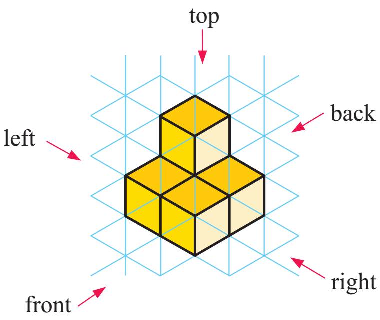
2 From the results in your table, what patterns do you notice?

D CONSTRUCTING BLOCK SOLIDS

When an architect draws plans of a building, separate drawings are made from several viewing directions.

Given a drawing on isometric graph paper, there are 5 directions we consider:

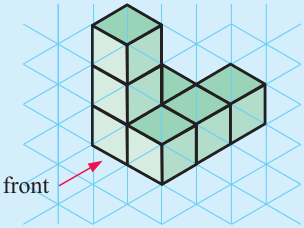
The top view is also called the **plan**. We use numbers on the plan to indicate the height of each pile.



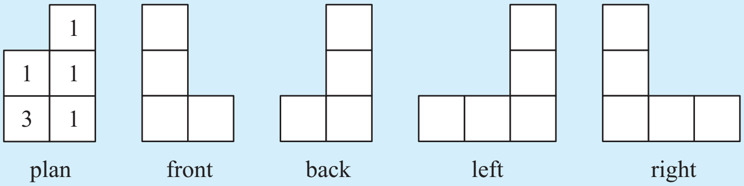
Example 2

Self Tutor

Draw the plan, front, back, left and right views of:



The views are:

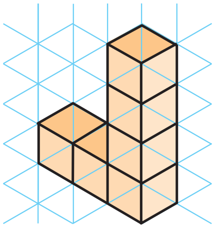


EXERCISE 15D

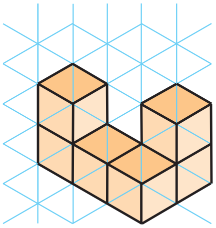
BLOCK BUSTER

1 Draw the plan, front, back, left and right views of:

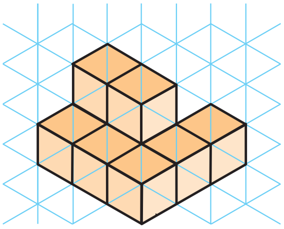
a



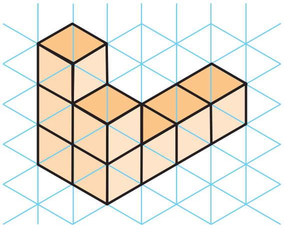
b



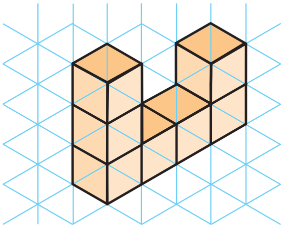
c



d



e



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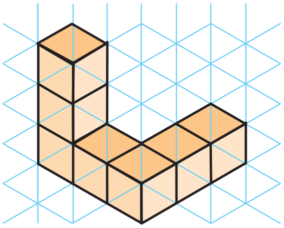
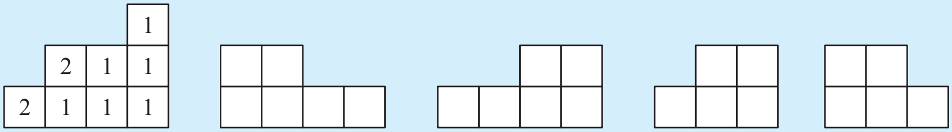
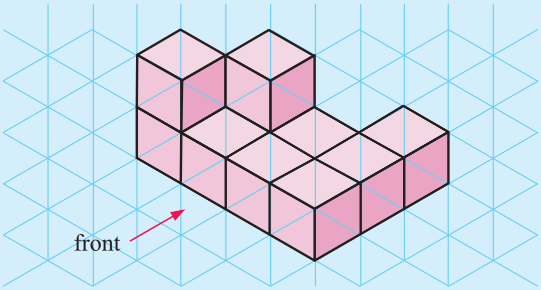


Example 3 **Self Tutor**

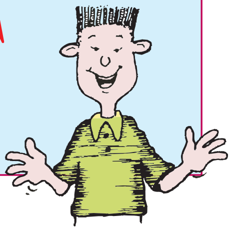
These diagrams show the different views of a block solid. Draw the object on isometric paper.



plan front back left right

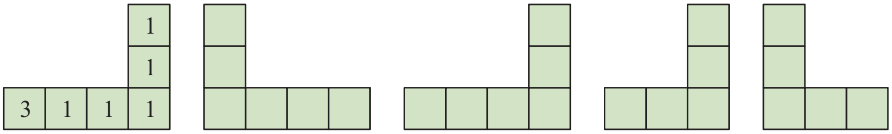


The **plan** is the view from above, or the *bird's-eye-view*.



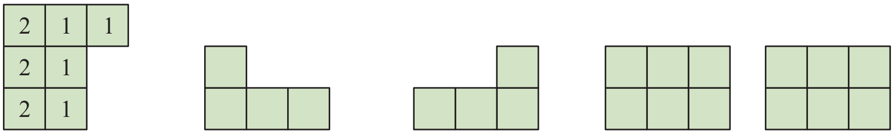
2 Draw a 3-dimensional object with these views:

a



plan front back left right

b

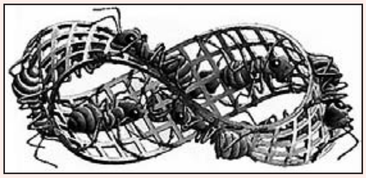


plan front back left right

ACTIVITY 6 INTERESTING MODELS

THE MOEBIUS STRIP

A mathematician confided
That the Moebius strip is
one-sided
And you get quite a laugh
When you cut it in half
Because it stays in one
piece when divided.

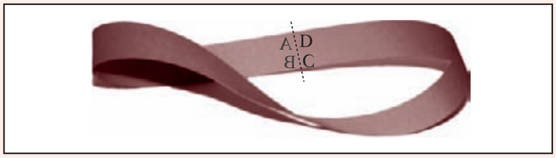


The Moebius strip was discovered in 1858 by **A F Moebius**.

What to do:

1. Take a long strip of paper. Twist one end of it through 180^o and join the ends A to D, and B to C.

   A band is formed with one twist in it. This band is called a **Moebius strip**.



2. How many sides does the Moebius strip have?
   Start colouring on one side and see what happens.
3. Cut the Moebius strip lengthwise down the middle and see what happens.
4. Make another Moebius strip and make two cuts along its length.
   What happens?

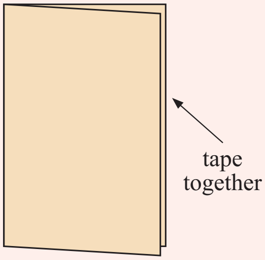
THE KLEIN BOTTLE

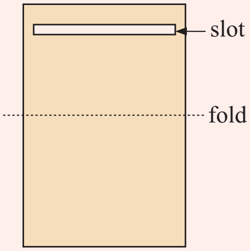
In 1882 **Felix Klein** invented a bottle which has one side and no edges. It does not have an inside or an outside.

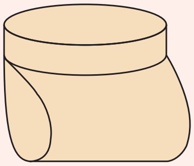
Three jolly sailors from
Blaydon-on-Tyne,
They went to sea in a
bottle by Klein,
Since the sea was entirely
inside the hull,
The scenery seen was
exceedingly dull.
Frederick Winsor

What to do:

1. Make a **Klein bottle** out of paper using the following steps:
   - **a** Take a rectangular piece of paper, fold it in half, then tape the edges opposite the fold together.
   - **b** About a quarter of the distance down from the top of the tube, cut a slot through the thickness of paper nearest to you.
   - **c** Fold the bottom half of the tube up and push the lower end through the slot.
   - **d** Tape the edges together all the way around the top of the model.







2. The Klein bottle has only one surface. Investigate this for yourself using a pencil to mark the paper before you construct a second Klein bottle.

THE TWO LOOPS PROBLEM

What would result when a paper loop is cut lengthwise using scissors? It is obvious that two loops are created.

However, the following **two loops problem** is much more interesting.

What to do:

1 Cut out two identical strips of paper and make two loops with them. Stick the loops together at right angles to each other. Use plenty of sticky tape so that overlapping paper is stuck down.

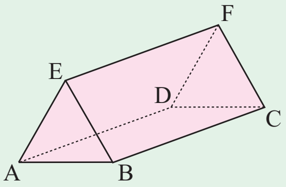
2 With scissors, cut lengthwise along the middle of both strips. Can you predict the final result?

KEY WORDS USED IN THIS CHAPTER

- apex
- cylinder
- face
- oblique projection
- prism
- sphere
- tetrahedron
- cone
- edge
- isometric projection
- Platonic solid
- pyramid
- square-based pyramid
- triangular-based pyramid
- cube
- Euler's formula
- net
- polyhedron
- solid
- surface
- vertex

REVIEW SET 15A

1



The figure alongside is a triangular prism.

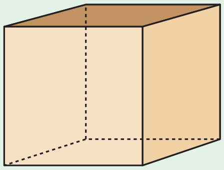
a Name all line segments parallel to [BC].

b Name 3 line segments which are concurrent at E.

c Name a face parallel to face ABE.

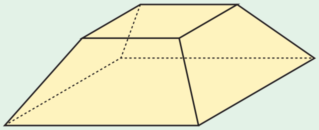
2 Draw a diagram to represent a triangular-based pyramid.

3 How many edges has this solid?

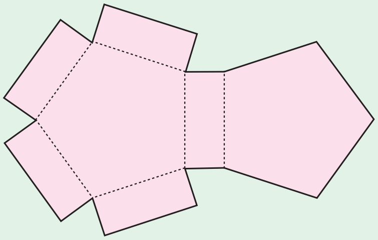


4 Draw a net for making a rectangular prism 5 cm by 3 cm by 1 cm.

5 How many vertices, faces and edges has the given figure?

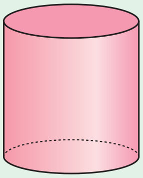


6 Draw and name the polyhedron that can be made by folding this net.

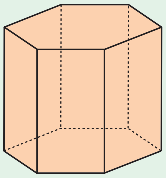


7 Name the following solids:

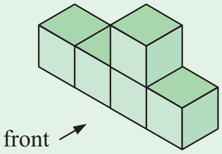
a



b

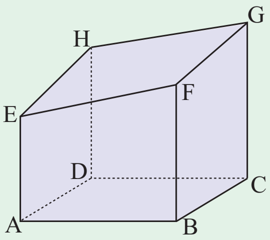


8 Draw the plan, front, back, left and right views of:



REVIEW SET 15B

1



a Name all line segments parallel to line segment [BF].

b Name 3 line segments which are concurrent at H.

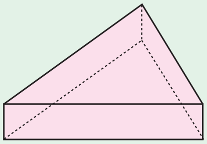
c Name a face parallel to face AEHD.

2 Draw a diagram to represent a pentagonal prism.

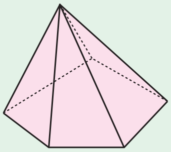
3 Draw a net for a triangular-based pyramid.

4 Name the following polyhedra:

a



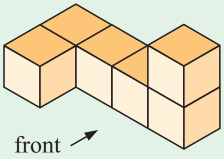
b



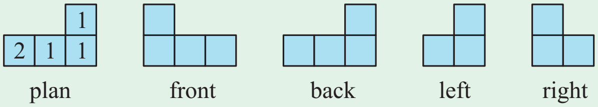
5 How many faces, edges and vertices has the figure in question **4b**?

6 Draw diagrams of two different types of solids with surfaces that are only rectangles and triangles.

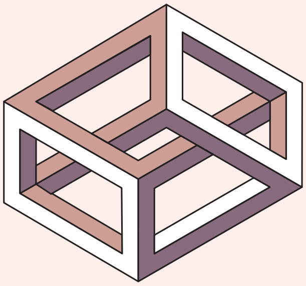
7 Draw the plan, front, back, left and right views of:

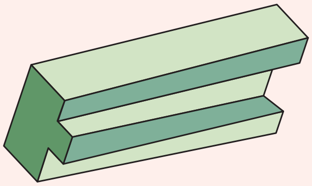


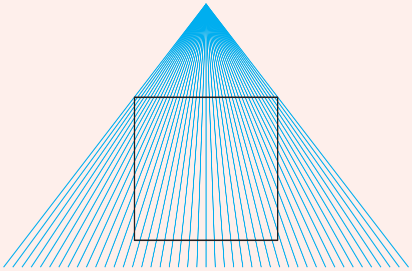
8 Draw the three-dimensional object whose views are:



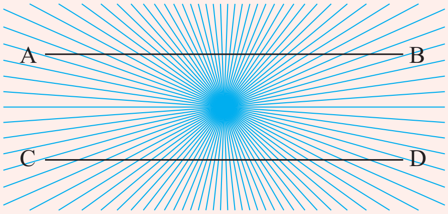
ACTIVITY — VISUAL ILLUSION







Is this a square or not?



Are [AB] and [CD] parallel?

What to do:

Organise a competition amongst your class to see who can draw the best visual illusions.

Chapter 16

Problem solving

Contents:

OPENING PROBLEM

Problems solved by the Ancient Egyptians were recorded by the scribe **Ahmes** around 1650 BC.

The Ahmes papyrus was bought by **Henry Rhind** in 1858. It is now housed in the British Museum in London and is referred to as the **Rhind papyrus**.

Below are four problems from this papyrus. The Egyptians solved these problems using an early form of algebra which did not use any special symbols as we do today. The unknown was referred to as the '*heap*' or the '*aha*'.

Problem 1: A 'heap' and its seventh added together give 19. What is the 'heap'?

Problem 2: An 'aha' and its $\frac{1}{2}$ added together become 16. What is the 'aha'?

Problem 3: There are seven houses. In each are seven cats. Each cat kills seven mice. Each mouse would have eaten seven ears of wheat. Each ear of wheat will produce seven hekats of grain. What is the total of all these?

Problem 4: Divide 100 loaves among 10 men, including a boatman, a foreman, and a doorkeeper who receive double portions. What is the share of each?

When you have completed this chapter you should try to solve these problems using modern algebraic symbols and methods.

A WRITING EQUATIONS USING SYMBOLS

One of the hardest things to do when solving problems is to write the information as an equation using the language of algebra.

We write each step of the problem using symbols, taking care to use brackets when necessary.

Example 1 **Self Tutor**

When a number multiplied by 3 is added to 5, the result is 23.
Write this as an algebraic equation.

Suppose we represent the number with the letter n.

Start with a number	n
multiply it by 3	$3n$
add 5	$3n+5$
the result is 23	$3n+5=23$

So, the algebraic equation which represents the problem is $3n+5=23$.

Example 2 **Self Tutor**

Express the following problem algebraically:

"Start with a number, add 4, multiply by 3, and the result is 33."

Start with a number	n
add 4	$n+4$
multiply by 3	$3(n+4)$
the result is 33	$3(n+4)=33$

So, the equation $3(n+4)=33$ represents the problem.

WRITING EQUATIONS AND SENTENCES

We can also take algebraic equations and write them in words in sentences.

Example 3 **Self Tutor**

Rewrite the equation $3(n+1)+4=16$ as a sentence.

We start with the unknown number, n.

n	I think of a number
$n+1$	add 1 to it
$3(n+1)$	multiply the result by 3
$3(n+1)+4$	add 4 to the new result
$3(n+1)+4=16$	and the answer is 16

I think of a number, add 1 to it, multiply the result by 3, add 4 to the new result, and the answer is 16.

EXERCISE 16A

1 Starting with the number n, rewrite the following in algebraic form:

- **a** I think of a number and subtract 5 from it, getting an answer of 16.
- **b** I think of a number, multiply it by 4 and then add 6. The result is 18.
- **c** I think of a number and divide it by 5, getting an answer of 7.
- **d** I think of a number, add 5, multiply the result by 3 then add 4. The result is 40.
- **e** I think of a number, add 6 to it, divide the result by 2, and get an answer of 9.
- **f** I think of a number, add 10, multiply the result by 4 then add 2. The answer is 50.
- **g** I think of a number, multiply it by 3, add 5, then multiply the result by 3. The answer is 51.
- **h** I think of a number, add 12, then divide the result by 2. The answer is 7.
- **i** I think of a number, divide it by 2 and add 7 to the result, getting an answer of 23.
- **j** I think of a number, multiply it by 7 and then subtract 6. The result is 64.

2 Write the following equations about the number n in English sentences:

a $n+5=2$ **b** $n-9=3$ **c** $2n-5=7$

d $3n+6=21$ **e** $2(n+1)=16$ **f** $3(n-4)=9$

g $3(2n+3)=15$ **h** $9(4n-1)=27$ **i** $2(n+3)+4=24$

j $3(2n-4)-5=25$ **k** $4(3n-9)-8=40$ **l** $5(2n+1)-4=41$

m $\dfrac{3(n+1)}{7}=6$ **n** $\dfrac{4(2n-5)}{3}=20$ **o** $\dfrac{3(2-n)}{7}=6$

B PROBLEM SOLVING WITH ALGEBRA

In the first section we saw how worded problems can be written as algebraic equations. We can solve these equations using the techniques in **Chapter 13**, thus finding solutions to the problems.

For example: What number when divided by 7 gives a result which is 2 more than the original number?

If x is the number, then the problem in algebraic form is $\dfrac{x}{7}=x+2$.

This equation is very difficult to solve by *inspection* or *trial and error*. Try to do this and you will see!

However, algebra can be used to solve more difficult problems like this more easily.

The following steps should be followed:

Step 1: Examine the problem carefully.
Step 2: Draw a diagram if necessary.
Step 3: Select a letter or symbol to represent the unknown quantity to be found.
Step 4: Set up an equation using the information in the question.
Step 5: Solve the equation.
Step 6: Write the answer to the question in a sentence.
Step 7: *Check* that your answer satisfies the given conditions.

Example 4 **Self Tutor**

Find the unknown numbers using equations:

a A number plus 6 is equal to 11.

b When a number is divided by 5, the answer is 11.

a Let x be the number.
The number plus 6 is $x+6$.

$$\begin{aligned} \therefore \quad x+6 &= 11 \\ \therefore \quad x+6-6 &= 11-6 \\ \therefore \quad x &= 5 \end{aligned}$$

The number is 5.
Check: $5+6=11$ ✓

b Let y be the number.
The number divided by 5 is $\frac{y}{5}$.

$$\begin{aligned} \therefore \quad \frac{y}{5} &= 11 \\ \therefore \quad \frac{y}{5}\times 5 &= 11\times 5 \\ \therefore \quad y &= 55 \end{aligned}$$

The number is 55.
Check: $55 \div 5 = 11$ ✓

Example 5 **Self Tutor**

I think of a number, treble it, then subtract 7. Find the number given the result is 10.

Let x be the number, so $3x$ is the number trebled.

$\therefore \quad 3x-7$ is the number trebled, minus 7.

$$\begin{aligned} \therefore \quad 3x-7 &= 10 \\ \therefore \quad 3x-7+7 &= 10+7 \\ \therefore \quad 3x &= 17 \\ \therefore \quad \frac{3x}{3} &= \frac{17}{3} \\ \therefore \quad x &= 5\tfrac{2}{3} \end{aligned}$$

Thus, the number is $5\frac{2}{3}$.

Check: LHS

$$\begin{aligned} &= 3\times 5\tfrac{2}{3} - 7 \\ &= (3\times \tfrac{17}{3}) - 7 \\ &= 17-7 \\ &= 10 \\ &= \text{RHS} \end{aligned}$$

EXERCISE 16B

1 Find the unknown numbers using equations:

- **a** If a certain number is doubled, the result is -6. Find the number.
- **b** A quarter of a number is 13. Find the number.
- **c** 7 more than a number is 22. Find the number.
- **d** When a number is decreased by 16, the result is 31.
- **e** When a number is doubled, the result is 9.
- **f** When a number is subtracted from 13, the result is 7.

Always check that your answer makes the original equation true **and** answers the original question.

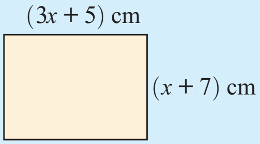
2 Use equations to solve the following problems:

- **a** I think of a number, double it, then subtract 5. The result is -6. Find the number.
- **b** From twice a certain number, 11 is subtracted, giving a result of 6. Find the number.
- **c** When twice a certain number is increased by 3, the result is 37. Find the number.
- **d** When one is added to a certain number and the result is halved, 7 results. What is the number?
- **e** When a number is halved and one is added, the result is 7. What is the number?

C MEASUREMENT PROBLEMS

Example 6 **Self Tutor**

The perimeter of the rectangle is 40 cm. Find x.



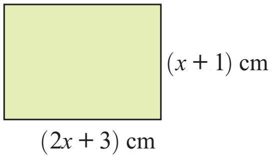
The two sides will add to half the perimeter.

$$\begin{aligned} \therefore \quad 3x+5+x+7 &= \tfrac{1}{2}\times 40 \\ \therefore \quad 4x+12 &= 20 && \{\text{simplifying}\} \\ \therefore \quad 4x &= 8 && \{\text{subtracting 12 from both sides}\} \\ \therefore \quad x &= 2 && \{\text{dividing both sides by 4}\} \end{aligned}$$

EXERCISE 16C

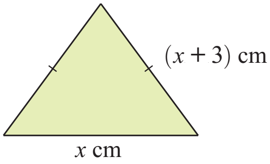
1 Set up an equation and hence find the value of x:

a



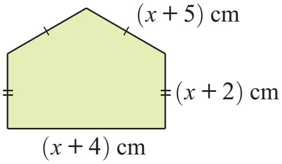
Perimeter = 26 cm

b



Perimeter = 27 cm

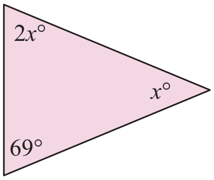
c



Perimeter = 23 cm

2 Write an equation and hence find the value of x in these figures:

a



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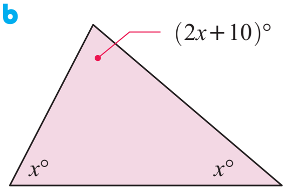

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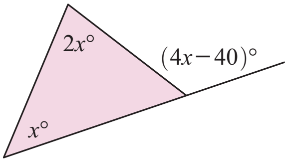

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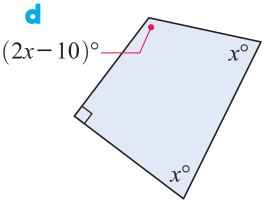

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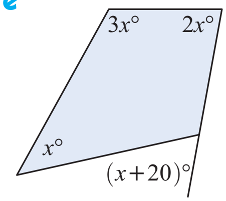

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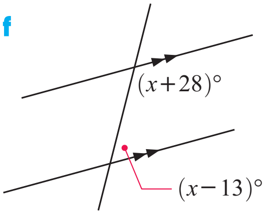


Example 7 **Self Tutor**

A rectangle has length 2 cm longer than its width. If the rectangle has perimeter 52 cm, find its width.

Let the width be x cm, so the length is $(x+2)$ cm.

$$\begin{aligned}\therefore\quad 2x+2(x+2)&=52 &&\{\text{the perimeter is 52 cm}\}\\ \therefore\quad 2x+2x+4&=52 &&\{\text{expanding}\}\\ \therefore\quad 4x+4&=52 &&\{\text{simplifying}\}\\ \therefore\quad 4x+4-4&=52-4 &&\{\text{subtracting 4 from both sides}\}\\ \therefore\quad 4x&=48 &&\{\text{simplifying}\}\\ \therefore\quad x&=12 &&\{\text{dividing both sides by 4}\}\end{aligned}$$

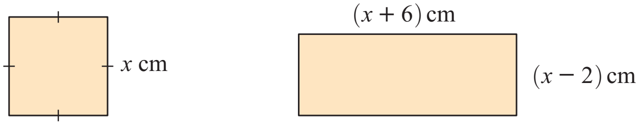
So, the width of the rectangle is 12 cm.

3 A triangle has sides of length x cm, $2x$ cm and $(x+3)$ cm. Find x given that the perimeter is 23 cm.

4 A rectangle has length 5 cm longer than its width. Its perimeter is 82 cm. Find the width of the rectangle.

5 A rectangle has length 3 times its width, and its perimeter is 36 cm. Find the width of the rectangle.

6 Find x given that the square and the rectangle have equal areas.



D MONEY PROBLEMS

When we solve problems involving money, we need to make sure we use the same units on both sides of the equation.

Example 8 Self Tutor

I have two identical shovels and one rake for sale. Each shovel is worth \$7 more than the rake. If the total value is \$59, find the value of the rake.

Let the rake cost $\$r$. $\therefore$ each shovel costs $\$(r+7)$.

$$\begin{aligned}\therefore\quad r+2(r+7)&=59 &&\\ \therefore\quad r+2r+14&=59 &&\{\text{expanding}\}\\ \therefore\quad 3r+14&=59 &&\{\text{simplifying}\}\\ \therefore\quad 3r+14-14&=59-14 &&\{\text{subtracting 14 from both sides}\}\\ \therefore\quad 3r&=45 &&\\ \therefore\quad r&=15 &&\{\text{dividing both sides by 3}\}\end{aligned}$$

$\therefore$ the rake is for sale at \$15. *Check:* LHS $=15+2(15+7)=59=$ RHS.

Example 9 **Self Tutor**

I have twice as many 20-cent coins as 50-cent coins, and the total value of all these coins is \$27.00. How many 50-cent coins do I have?

Let the number of 50-cent coins be x.

Type of coin	*Number of coins*	*Total value*
50-cent	x	$50x$ cents
20-cent	$2x$	$40x$ cents
	Total	2700 cents

$$\begin{aligned} \text{Now} \quad 50x + 40x &= 2700 \\ \therefore \quad 90x &= 2700 \\ \therefore \quad \frac{90x}{90} &= \frac{2700}{90} \\ \therefore \quad x &= 30 \end{aligned}$$

Check: $\text{LHS} = 50 \times 30 + 40 \times 30$
$= 2700 = \text{RHS}$

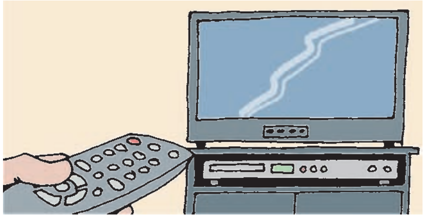
So, there are thirty 50-cent coins.

EXERCISE 16D

1 An orange costs twice as much as an apple. The two of them cost a total of 72 cents. Find the cost of an apple.

2 A TV set costs \$100 more than a DVD player. If the two of them cost a total of \$854, find the cost of the TV and DVD player.

3 A handbag costs twice as much as a purse. If Jade buys 2 handbags and a purse, the total bill is €87.50. Find the cost of the purse.



4 A bowl of noodles costs half as much as a chicken and mushroom hot pot. If I buy 3 bowls of noodles and 5 hot pots, the total cost is RM13.65. Find the cost of one chicken and mushroom hot pot.

5 I have three times as many 10-cent coins as 20-cent coins, and their total value is €30.50. How many 20-cent coins do I have?

6 John has only 20-pence and 50-pence coins. He has 23 more 20-pence coins than 50-pence coins, and their total value is £26.30. How many 50-pence coins does John have?

E MISCELLANEOUS PROBLEM SOLVING

To solve these problems you should first set up an algebraic equation.

EXERCISE 16E

1 I am thinking of two numbers. One of them is 11 more than the other, and their sum is 131. What are the numbers?

2 One number is twice as large as another, and a third number is 11 more than the smaller one. If the sum of the three numbers is 79, find the smallest number.

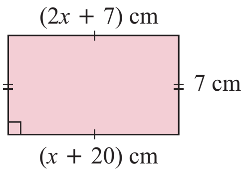
3 The figure shown is a rectangle.
Find the value of x.

4 A triangle has sides of length a cm, $(25 - a)$ cm and $(2a + 3)$ cm. If its perimeter is 40 cm, find the length of the longest side.

5 The equal sides of an isosceles triangle are 3 cm longer than the third side. If the perimeter is 18.9 cm, find the length of the third side.

6 A rectangular paddock has a length 300 m greater than its width. If the perimeter of the paddock is 1400 m, find: **a** the width **b** the length.

7

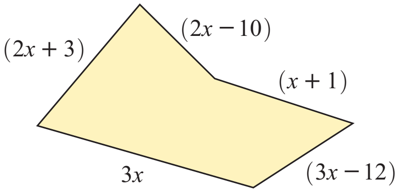


Find the perimeter of the rectangle shown.
Your answer must *not* contain x.

8 The perimeter of this pentagon is 70 units.

a What is the value of x?

b What is the length of the shortest side?



9 Adjacent sides of a rhombus have lengths $(2x + 11)$ cm and $(4x + 5)$ cm. Find x and hence find the perimeter of the rhombus.

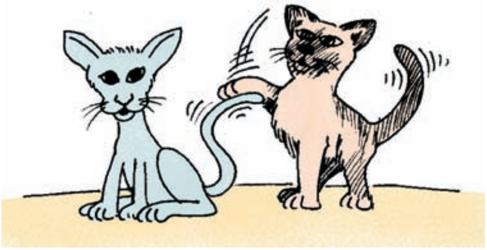
10 Jane has saved £12 less than twice the amount Jason has saved. Together they have £144. How much has been saved by:

a Jason **b** Jane?

11 A Siamese kitten costs \$150 less than a Burmese kitten. If three Siamese and two Burmese kittens cost \$1800 in total, find the cost of:

a a Siamese kitten

b a Burmese kitten.



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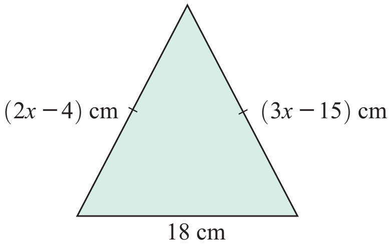


The isosceles triangle illustrated has a base of length 18 cm.

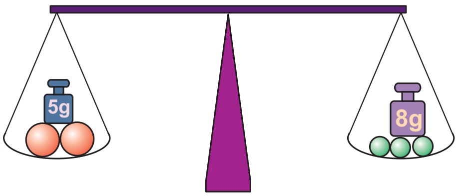
a What is the value of x?

b Is the triangle equilateral? Explain your answer.

13 Four less than a certain number is half the number. What is the number?

14 When nine is added to a certain number and the result is divided by three, the answer is eight. What is the number?

15 Juan has two sets of metallic balls. The balls in one set each weigh twice as much as each ball in the other set. He knows that two of the heavier balls plus a five gram weight balance three of the lighter balls plus an eight gram weight. Find the weight of one of the heavier balls.



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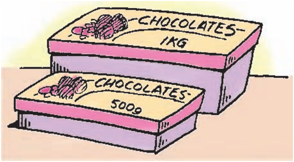


A one kilogram box of chocolates costs €0.90 less than twice the cost of a 500 g box. Two one kilogram boxes and three 500 g boxes cost €34.60. Find the cost of a:

a one kilogram box **b** 500 g box.

17 5 times an unknown number is the same as twice the number subtracted from 35. What is the number?

18 Six less than a certain number is 11 more than treble the number. Find the number.

19 Three less than a number is 8 less than a third of the number. Find the number.

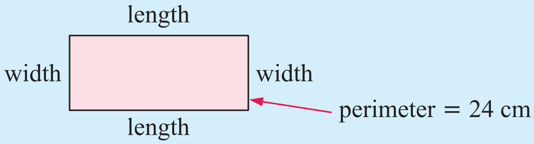
F PROBLEM SOLVING BY SEARCH

For some problems it is either difficult to write down an equation, or the equation may be difficult to solve. In some cases the equation may have more than one solution.

If the number of possibilities for the solution is small, it may be quickest to search through them all to find the solutions which work. In other cases a search through particular values may reveal what the correct solution must be.

Example 10 **Self Tutor**

What rectangle with perimeter 24 cm has the greatest area?



{Draw a diagram to show **all** given information.}

We notice that length + width = 12. {Write down any observations.}

Trials:

Length	*Width*	*Area*
1	11	$1 \times 11 = 11$
2	10	$2 \times 10 = 20$
3	9	$3 \times 9 = 27$
4	8	$4 \times 8 = 32$
5	7	$5 \times 7 = 35$
6	6	$6 \times 6 = 36$
7	5	$7 \times 5 = 35$
8	4	$8 \times 4 = 32$

It appears that the area is largest when the rectangle is a 6 cm by 6 cm square.

EXERCISE 16F

1 Consider the equation $5x + 2y = 50$ where x is a positive integer and y is positive.

a Explain why x cannot be greater than 10.

b Copy and complete the following table for possible values of x and y.

For example, when $x = 1$, $5 \times 1 + 2 \times y = 50$

$\therefore \quad 2 \times y = 45$

$\therefore \quad y = 22\frac{1}{2}$

x	1	2	3	4	5	6	7	8	9	10
y	$22\frac{1}{2}$									

c If x and y are *both* positive integers, what are their values?

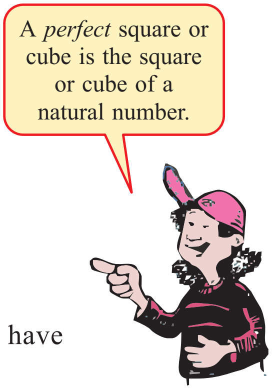
2 Find positive integers x and y that satisfy the rule $12x + 5y = 100$.

3 Find the smallest positive whole number greater than 2 which, when divided by 3 and 5, leaves a remainder of 2. Try 3, 4, 5, 6, ... in that order until you find the answer.

4 Find all positive integers less than 40 which cannot be written as a sum of a perfect square and a perfect cube. Do not forget that $0^2 = 0$ and $0^3 = 0$.

Hint: Be systematic.

Try: $0^2 + 1^3$ $\quad 1^2 + 1^3$ and so on.
$0^2 + 2^3$ $\quad 1^2 + 2^3$
$0^2 + 3^3$ $\quad 1^2 + 3^3$
$0^2 + 4^3$



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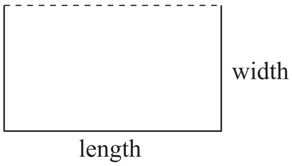


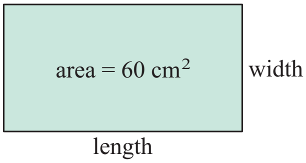
Three sides of a rectangle have lengths which add to 24 cm.

What are the dimensions of the rectangle if its area is as large as possible?

Hint: Copy and complete:

Width	*Length*	*Area*
1	22	
2	20	
⋮		

6



A rectangle has area 60 cm^2. Copy and complete the table below, stopping when you have discovered something.

Write down exactly what you have discovered.

Area	*Length*	*Width*	*Perimeter*
60	1	60	$2 \times (1 + 60) = 122$ cm
60	2	30	
60	3		
⋮	⋮		

7 If a and b are positive integers and $2a+b=40$, find the greatest possible value of ab^2.

G PROBLEM SOLVING BY WORKING BACKWARDS

Often in mathematics we begin with an initial condition and work systematically towards an end point or goal.

For example: A farmer had 40 cattle. A quarter died from a mysterious disease and he bought 5 more at a recent sale. How many did he then have?

40	The farmer started with 40 cattle.
$\frac{1}{4}$ of $40=10$	10 cattle died.
$40-10=30$	After the disease he had 30 left.
$30+5=35$	After the purchase he had 35.

However, sometimes we are given details about the final situation and need to determine what the situation was initially. In these cases we need to **work backwards** using **inverse operations**.

Example 11 **Self Tutor**

My friend lost \$5 from her wallet, but she then doubled the amount in her wallet by selling a book to me. She then had \$50 in her wallet. How much did she have in her wallet initially?

My friend doubled her money to reach \$50.

Double means multiply by 2, so working backwards we need to divide by 2.

So, she had \$25 in her wallet before selling the book to me.

Lose \$5 means subtract \$5, so working backwards we need to add \$5.

Before losing the \$5 she must have had $\$25+\$5=\$30$.

She had \$30 in her wallet initially.

Example 12 **Self Tutor**

A teacher offers members of the class a peppermint for every problem they get right, but he takes back *two* for each incorrect answer. Barry stored up his peppermints over a week. On Thursday he got 8 questions right and 2 wrong. On Friday he got 9 right and 1 wrong. He finished the week with 16 peppermints.

How many did he have at the start of Thursday?

Friday: 9 right and 1 wrong is a gain of $9 - 2 = 7$ peppermints.
$\therefore$ he had $16 - 7 = 9$ peppermints at the start of Friday.

Thursday: 8 right and 2 wrong is a gain of $8 - 2 \times 2 = 4$ peppermints.
$\therefore$ he had $9 - 4 = 5$ peppermints at the start of Thursday.

Barry had 5 peppermints at the start of Thursday.

Example 13

Self Tutor

Eduardo wishes to fly to Barcelona to visit his grandparents. He saves $\frac{1}{4}$ of the money, and his grandfather gives him the remaining €120.

How much does Eduardo's flight cost?

Eduardo saves $\frac{1}{4}$ of the cost so his grandfather gives him $\frac{3}{4}$.

$\therefore$ $\frac{3}{4}$ of the cost is €120.

$\therefore$ $\frac{1}{4}$ of the cost is €40.

$\therefore$ the cost of Eduardo's flight is $4 \times €40 = €160$.

EXERCISE 16G

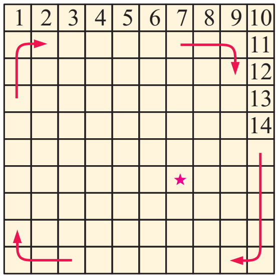
1 Mrs Smith bought some chocolates at the supermarket.
Her children ate $\frac{1}{3}$ of them and her husband ate 5 chocolates.
When she came to eat some herself she found only 7 left. How many chocolates did she buy?

2 I think of a number. I multiply it by 8, subtract 6, divide by 2, then add 3.
The result is 12. What was my number?

3 A breeder sold half of his horses, then another two. He then sold half of his remaining horses, then another two. He then only had his one favourite horse left. How many horses did he have initially?

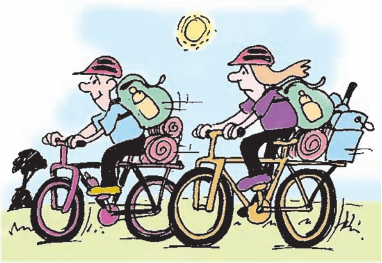
4 Xuen and Xia are sisters. Xia grew 4 cm in 2004 and Xuen grew 3 cm more than Xia in that year. In 2005 Xuen grew 3 cm and Xia grew 2 cm. When measured at the end of 2005 both were 140 cm tall. What was the height of each girl in 2003?

5 If you were to continue this number pattern until you got to the star, what number would you put in the star's square?
Can you determine this without filling in the numbers or counting all the numbers between?



6 Two bottles X and Y contain some water. Bottle X contains more than bottle Y. From bottle X we pour into bottle Y twice as much water as bottle Y already contains. We then pour from bottle Y into bottle X as much water as bottle X now contains. Both bottles now contain 200 mL. How much water was in each bottle to start with?

7 Joe and Jerry went for a bike trek during the holidays. On the first day they rode $\frac{1}{3}$ of the total distance. On the second day they were tired and only rode $\frac{1}{4}$ of the remaining distance. On the third day they rode half of the distance left. The last day they rode the remaining 18 km. How far did they ride altogether?



KEY WORDS USED IN THIS CHAPTER

- algebraic equation
- inverse operation
- positive integer

REVIEW SET 16A

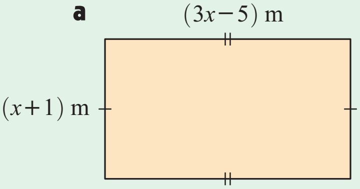
1 Use equations to solve the following problems:

a I think of a number, treble it, then add 4. If the result is 28, find the number.

b When twice a certain number is subtracted from 21, the result is 9. Find the number.

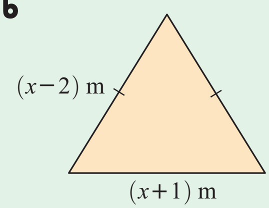
2 Set up an equation and hence find the value of x:

a



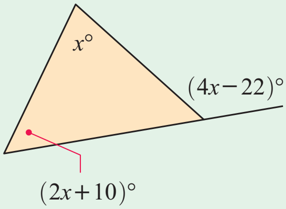
perimeter = 24 m

b



perimeter = 15 m

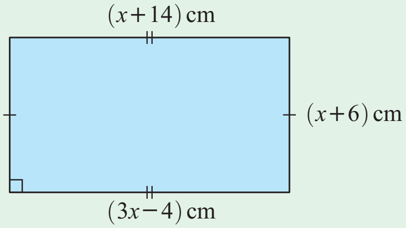
c



3 A rectangle has length 6 cm longer than its width. If its perimeter is 56 cm, find its length.

4 A CD costs twice as much as a book. If I buy 3 CDs and 2 books for a total of €120, find the cost of each item.

5 Find the perimeter of the rectangle. Your answer must *not* contain x.



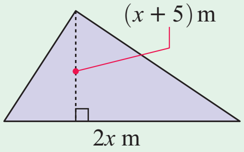
6 The sum of three consecutive numbers is 72. Find the largest of the three numbers.

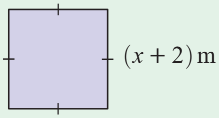
7 Scott has 12 more 50-pence coins than 20-pence coins, and their total value is £10.20. How many 20-pence coins does Scott have?

8 Find positive integers x and y that satisfy the rule $7x + 9y = 60$.

9 Jake has a brother named Phil. Their mother's name is Sue. A year ago, Sue was three times as old as Jake. Four years ago, Sue was four times as old as Phil. If Phil is 13 years old, how old is Jake?

10 Find x if the square and the triangle have the same area:





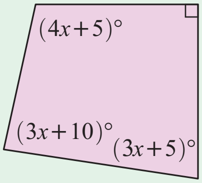
REVIEW SET 16B

1 **a** When a certain number is trebled, the result is 21. Find the number.
  b I think of a number, halve it, then add 3. The result is 11. What is the number?

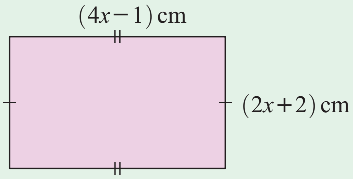
2 At the supermarket a carton of juice costs £1.50 more than a bottle of milk. If 3 bottles of milk and 2 cartons of juice cost £8, how much does a carton of juice cost?

3 Set up an equation and hence find the value of x:

a

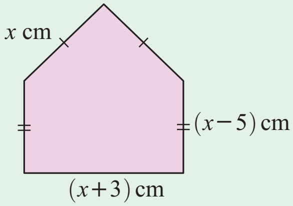


b



perimeter = 62 cm

c



perimeter = 53 cm

4 The equal sides of an isosceles triangle are 2 cm shorter than the length of the third side. If the perimeter of the triangle is 35 cm, find the length of the third side.

5 When a certain number is trebled, the result is the same as when the number is subtracted from 56. What is the number?

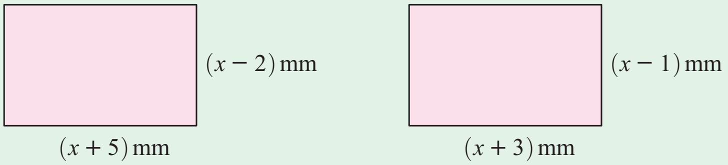
6 George has three times as many marbles as Sam. If George gives 5 of his marbles to Sam, George will have twice as many marbles as Sam. How many marbles does Sam have?

7 I have three times as many 20-cent coins as 10-cent coins, and their total value is \$4.20. How many 10-cent coins do I have?

8 If x and y are positive integers and $3x + y = 50$, find the greatest possible value of xy^2.

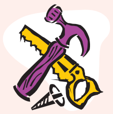
9 Vincent spent half of the money in his wallet on a new jumper.
He then spent \$15 on lunch.
After lunch he spent two thirds of the money remaining in his wallet on a hat.
If he is left with \$10 in his wallet, how much did he have in his wallet initially?

10 Find x if the two rectangles have the same area:



ACTIVITY

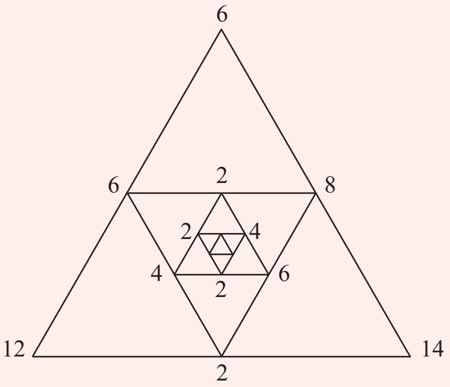
VANISHING TRIANGLES



Draw an equilateral triangle and on each vertex write any positive integer. At the midpoint of each side write down the positive difference between the integers at the ends of each side. Now join these midpoints and repeat the procedure on the newly formed triangle.

Continue this procedure until either the numbers created are identical in consecutive triangles or a pattern emerges.

For example, if the numbers 14, 12 and 6 are chosen, after 3 steps the figure will be as shown alongside.



What to do:

1 Copy and complete the given example making sure your original triangle is large enough. You may need several triangles.

2 Try triangles with the following combinations of integers:

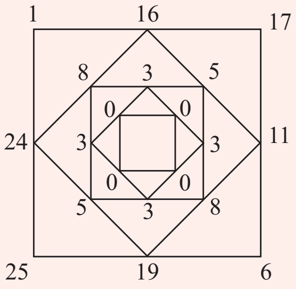
a three even integers **b** two even integers

c one even integer **d** no even integers.

Can you draw any general conclusions?

3 Now try the same procedure starting with a square instead of an equilateral triangle. For example, starting with 1, 17, 6 and 25, a square produces the figure alongside.

Can you suggest a general result for the case of a square?

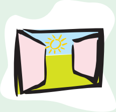


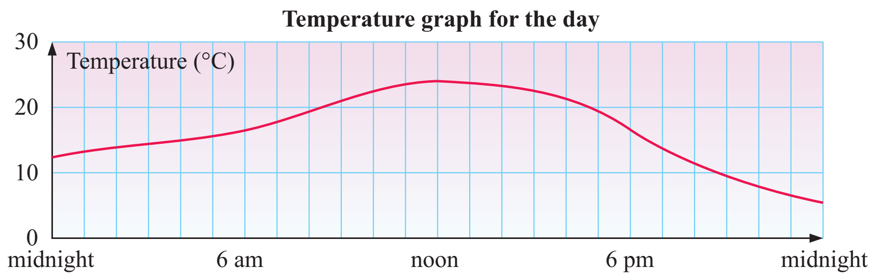
Chapter 17

Line graphs

Contents:

OPENING PROBLEM





The graph above shows the temperature inside a room over a 24 hour period. The temperature was measured in degrees Celsius at one minute intervals, and the results plotted to form the graph.

Can you determine:

- the temperature at noon and at 7 pm
- the **minimum** or smallest temperature during the period and the time when it occurred
- the times of the day when the temperature was 20^oC
- the time of the day when the **maximum** or highest temperature occurred
- the period when the temperature was increasing?

A PROPERTIES OF LINE GRAPHS

Line graphs consist of straight line segments or curves.
They are used to show how one quantity varies in relation to another.

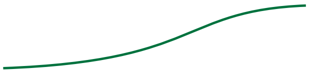
Since the quantities may take different values, we say they are **variables**.

In general when quantities vary, one is *dependent* on the other. We say that the **dependent variable** depends on the **independent variable**.

The independent variable is placed on the horizontal axis and the dependent variable is placed on the vertical axis.

For example, in the **Opening Problem** the temperature *depends* on the time of day. The time of day is the independent variable and is placed on the horizontal axis. The temperature is the dependent variable and is placed on the vertical axis.

INCREASING AND DECREASING GRAPHS



If the line or curve slopes upwards from left to right, the dependent variable increases as the independent variable increases. We say the graph is **increasing**.

If the line or curve slopes downwards from left to right, the dependent variable decreases as the independent variable increases. We say the graph is **decreasing**.

Some graphs may have sections which are increasing and others which are decreasing. For example, the graph in the **Opening Problem** is increasing from midnight to noon and then decreasing from noon to midnight the next night.

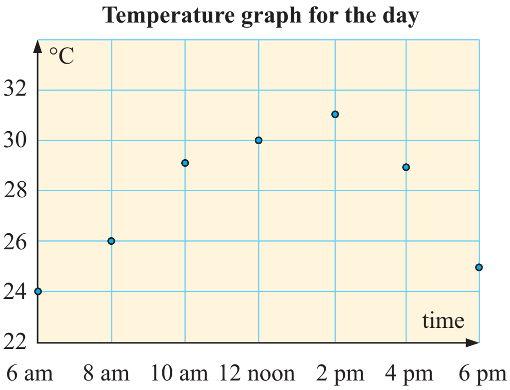
CONSTRUCTING LINE GRAPHS

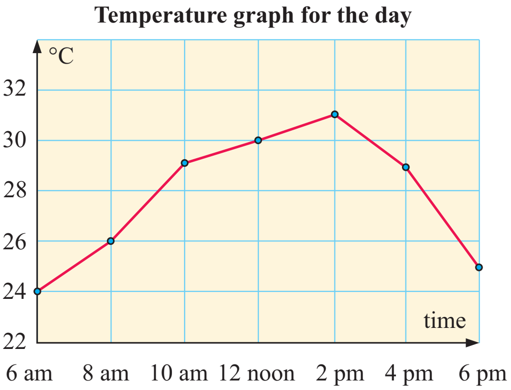
When we are given data, we can plot it on a set of axes to produce a **scatterplot** or **point graph**. The information is often given at **regular intervals** of the independent variable.

For example, consider the following temperature information collected at 2-hour intervals:

Time	6 am	8 am	10 am	noon	2 pm	4 pm	6 pm
Temperature (°C)	24	26	29	30	31	29	25

The graph on the left is a **point graph** of the temperature data. However, we know that the temperature has a value at all times. We therefore join the dots together either with a smooth curve or with straight line segments. The graph on the right is a **line graph** which connects the dots.



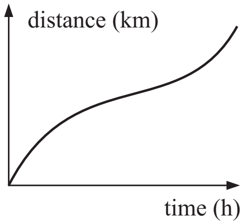


In reality, the lines between the dots would almost certainly not be straight, but because we do not have more accurate data we use straight lines to give good estimates.

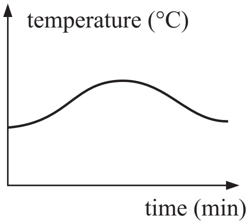
EXERCISE 17A

1 For each of the following graphs, state the dependent and independent variables:

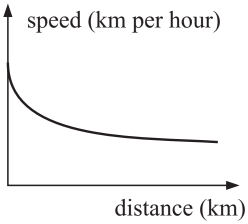
a



b



c



2 For each graph in **1**, state whether it is:

A increasing

B decreasing

C increasing in some sections and decreasing in others.

3 This data set shows the speed of a runner at various times during a cross country race.

time (min)	0	5	10	15	20	25	30
speed (km per h)	17	15	11	14	13	13	12

- **a** State which is the dependent and which is the independent variable.
- **b** Draw a point graph to display the data. Make sure the independent variable is on the horizontal axis.
- **c** Connect the points on your graph with straight line segments to form a line graph.

4 The data set below records the progress of a speed skater in a 500 m race.

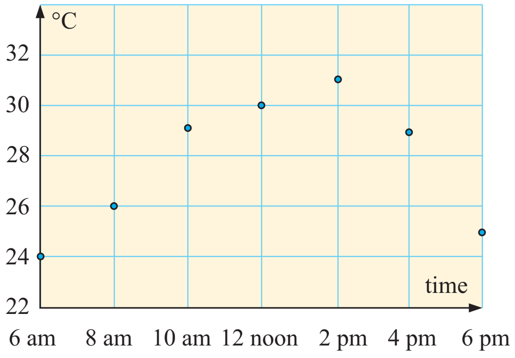
time (s)	0	5	10	15	20	25	30	35	39.6
distance (m)	0	34	79	142	226	291	350	429	500

- **a** State which is the dependent and which is the independent variable.
- **b** Draw a point graph to display the data. Make sure the independent variable is on the horizontal axis.
- **c** Connect the points on your graph with a smooth curve.
- **d** Comment on whether the graph is decreasing or increasing.

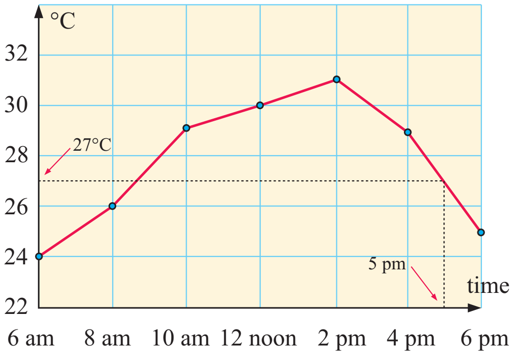
B ESTIMATING FROM LINE GRAPHS

Consider again the point and line graphs of temperature in **Section A**.

Temperature graph for the day



Temperature graph for the day



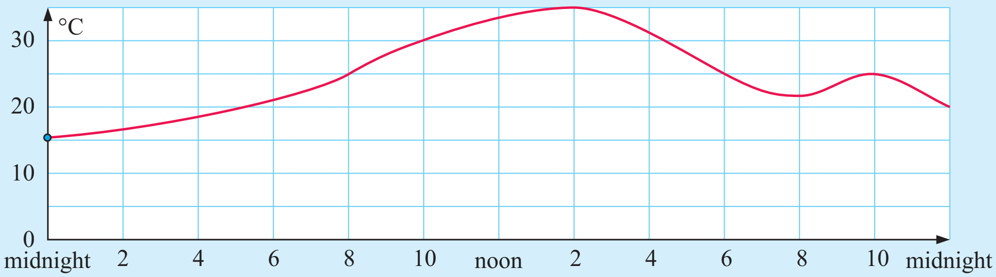
From both graphs we can estimate information such as:

- the highest temperature recorded for the day was at 2 pm
- the lowest temperature was at 6 am
- the temperature increased from 6 am to 2 pm and then decreased until 6 pm
- the temperature at 8 am was 26°C.

For this information, the line graph is more useful than the point graph because values *in between* data points can be **estimated** from the graph.

For example, the temperature at 5 pm is estimated to be 27°C, as shown by the dotted line on the line graph.

Example 1 **Self Tutor**



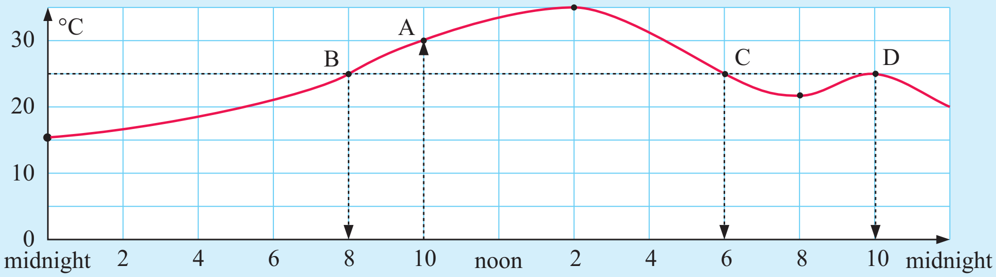
Use the temperature-time graph to find:

a the temperature at 10 am

b the times when the temperature was 25^{o}C

c the periods when the temperature was: **i** increasing **ii** decreasing

d the maximum temperature and when it occurred.



a The temperature at 10 am was 30^{o}C. (point A)

b The temperature was 25^{o}C at 8 am, 6 pm and 10 pm. (points B, C, D)

c The temperature was:

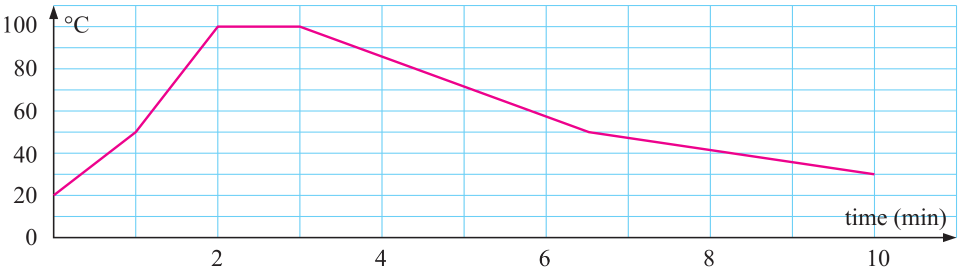
  i increasing from midnight to 2 pm and from 8 pm to 10 pm

  ii decreasing from 2 pm to 8 pm and from 10 pm to midnight.

d The maximum temperature was 35^{o}C at 2 pm.

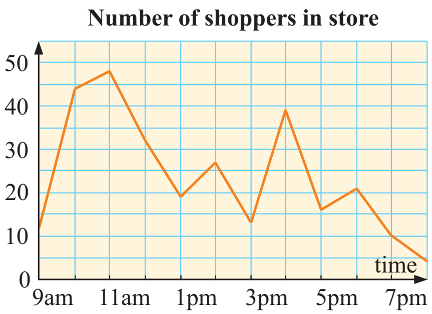
EXERCISE 17B

1 The temperature of water in a kettle is graphed over a 10 minute period.



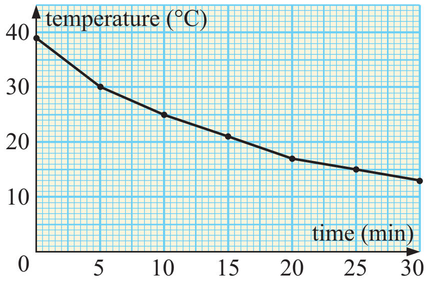
- **a** What was the room temperature when the kettle was switched on?
- **b** How long did it take for the water to boil?
- **c** For how long did the water boil?
- **d** At what times was the water temperature 50^{o}C?
- **e** During what period was the temperature decreasing?

2 Managers of a retail store conduct a customer count to help them decide how to roster their sales staff. The results are shown in the line graph.



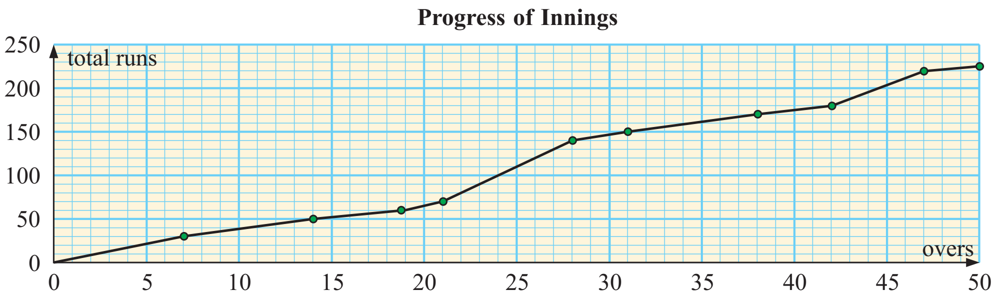
- **a** At what time was the number of people in the store greatest?
- **b** At what time was the number of people in the store least?
- **c** Describe what happened in the store between 3 pm and 4 pm.
- **d** Use the graph to estimate the number of people in the store at 9:30 am.

3 When a bottle of soft drink was placed in a refrigerator, its temperature was measured at 5-minute intervals. The results are graphed alongside.



- **a** Determine the temperature of the liquid when it was first placed in the refrigerator.
- **b** Find the time taken for the temperature to drop to 20^{o}C.
- **c** Find the temperature after 10 minutes in the refrigerator.
- **d** Find the fall in temperature during:
  - **i** the first 15 minutes
  - **ii** the next 15 minutes.

4 The graph below shows the progress of the batting team in a game of cricket. It shows the runs accumulated at the times when the batsmen were given 'out'.



Use the graph to estimate:

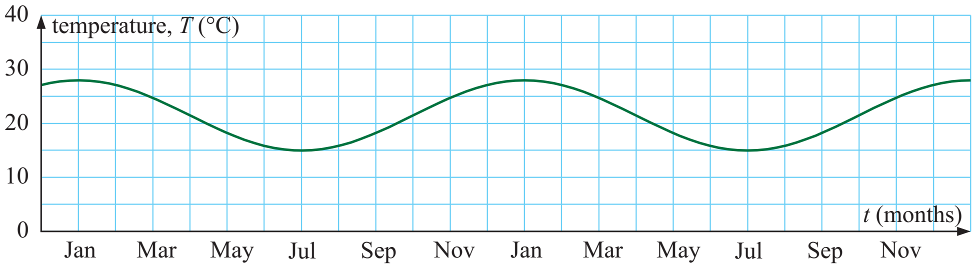
- **a** the total runs at the end of the innings
- **b** how many runs were achieved before the first batsman was out

c during which over the third batsman got out

d which batsman got out in the 38th over

e during which over the score reached 200.

5 The graph below shows the average temperature for Sydney over a 2 year period.



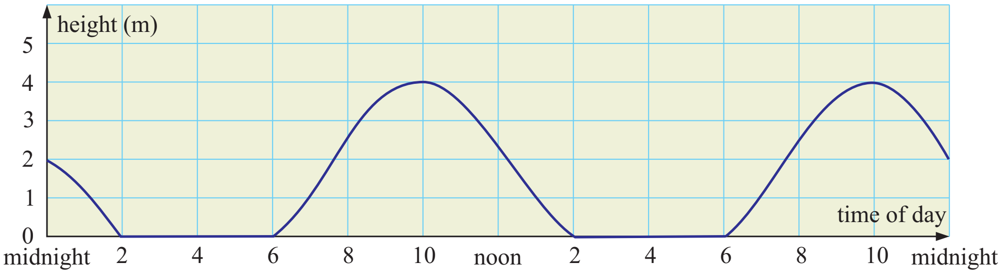
a In which month was the average temperature:

  i a maximum **ii** a minimum?

b What happens to the temperature in Sydney from July to January?

c In which month(s) was the average temperature about 25°C?

6 The graph below shows the fluctuations in the tide at the local estuary. The vertical axis indicates the height of the water above the sand in metres.



a During what period(s) is the tide out so far that the sand is exposed?

b What is the highest tide depth and at what times does it occur?

c At what times is the water depth 1 m?

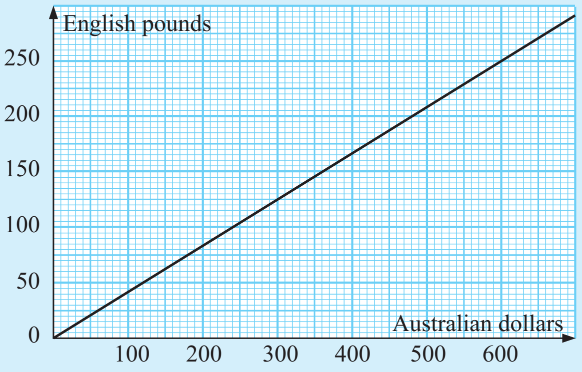
d If a water depth of $2\frac{1}{2}$ m is needed to get the fishing fleet out to sea, what is the earliest time in the day when this can happen?

C CONVERSION GRAPHS

Conversion graphs are special line graphs which enable us to convert from one quantity to another. We can use conversion graphs to convert between currencies and units of measurement. Conversion graphs usually are straight lines, since units are usually related in a **linear** way.

Example 2 **Self Tutor**

The graph shows the relationship between Australian dollars and English pounds on a particular day.



Determine:

a the number of dollars in 250 pounds

b the number of pounds in 480 dollars

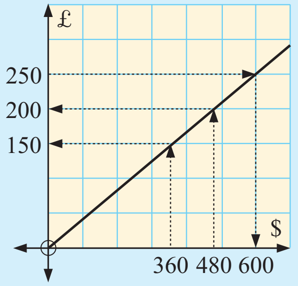
c whether a person with $360 could afford to buy an item valued at 200 pounds.

a 250 pounds is equivalent to $600.

b $480 is equivalent to 200 pounds.

c $360 is equivalent to 150 pounds.

$\therefore$ the person cannot afford to buy the item.



EXERCISE 17C

1 Use the currency conversion graph of **Example 2** to estimate:

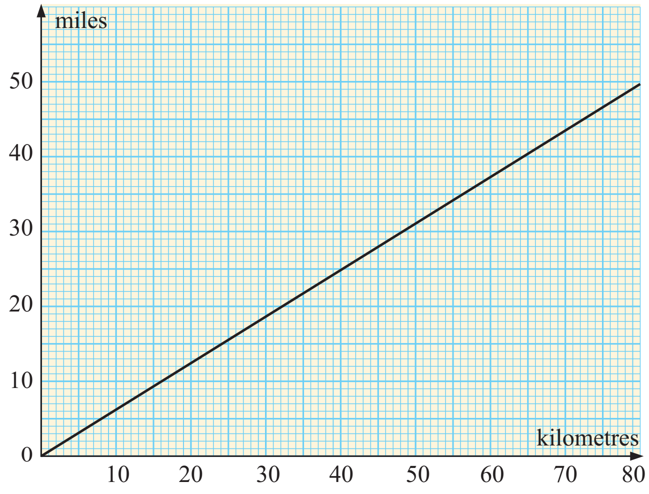
a the number of dollars in: **i** 130 pounds **ii** 240 pounds

b the number of pounds in: **i** $400 **ii** $560

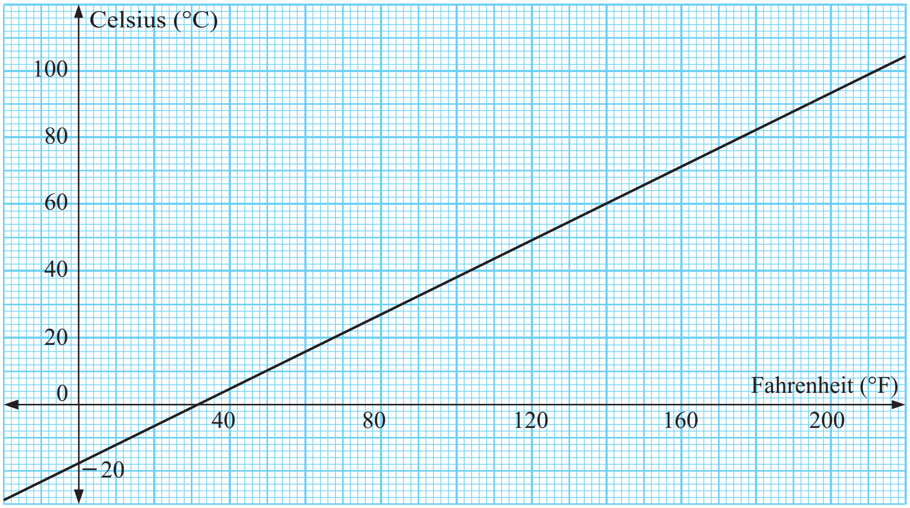
2 This graph shows the relationship between distances measured in miles and kilometres. Convert:

a **i** 45 km to miles
ii 28 km to miles

b **i** 48 miles to km
ii 30 miles to km.



3 Fahrenheit and Celsius are two scales for measuring temperature. The graph below shows how to convert from one unit of measure to the other.



a If pure water boils at 212^{o}F, find the equivalent temperature in degrees Celsius.

b Determine the degrees Fahrenheit equivalent to:

i 40^{o}C **ii** 75^{o}C **iii** 0^{o}C **iv** -20^oC.

c If it is 85^{o}F today, what is the temperature in degrees Celsius?

RESEARCH **CURRENCY TRENDS**

Over a period of a month, collect the currency rates which compare your local currency with either the US dollar or the euro. These are easily available on the internet.

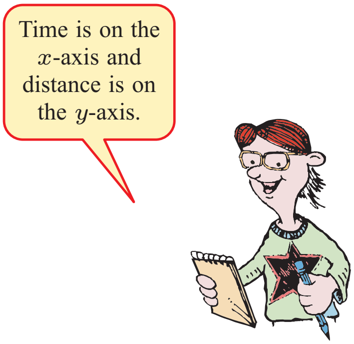
1 Graph your results, updating the graph each day.

2 How does the currency exchange rate affect the conversion graph between currencies?

D TRAVEL GRAPHS

Travel graphs show the relationship between the distance travelled and the time taken to travel that distance.

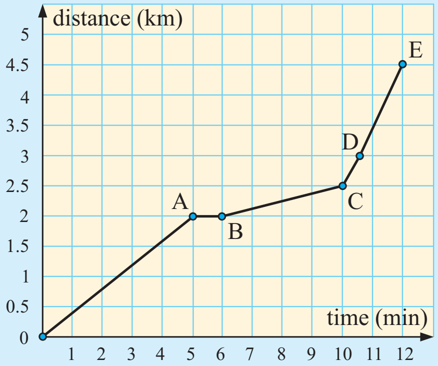
For these graphs, time is the independent variable on the x-axis and distance is the dependent variable on the y-axis.



Example 3 **Self Tutor**

The graph shows the progress of Juen as he cycles to school.

On the x-axis is the time in minutes since leaving home. On the y-axis is the distance in kilometres he has travelled from home.



a How long did it take Juen to cycle to school?

b How far is it from his home to the school?

c On his way to school, Juen crosses an intersection where there are traffic lights. How far are these lights from his home? For what length of time was he delayed at these lights?

d Juen's friend lives on the way to school and 3 km from Juen's house. How long did it take Juen to reach his friend's house?

e How far did Juen cycle:
- **i** in the first 5 minutes
- **ii** between the 6th and 10th minutes
- **iii** between the 10th and 12th minutes?

f There is a steep hill on Juen's way to school. When did he reach the top?

---

a 12 minutes {point E} **b** $4\frac{1}{2}$ km {point E}

c The lights are 2 km from Juen's home. He was delayed for 1 minute. {between points A and B}

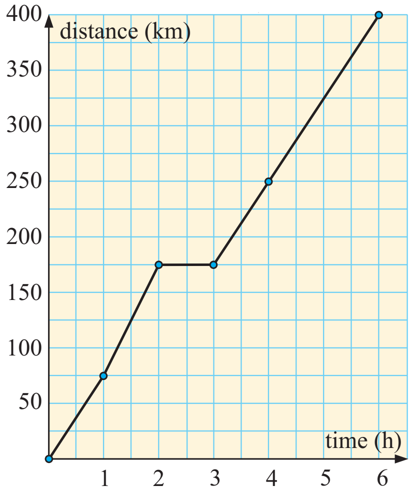
d $10\frac{1}{2}$ minutes {point D}

e **i** 2 km {O to A} **ii** $\frac{1}{2}$ km {B to C} **iii** 2 km {C to E}

f Juen rode more slowly from B to C. He reached the top after 10 minutes.

EXERCISE 17D

1 The graph alongside shows the distance travelled by a family car on a holiday trip. Use the graph to answer the following questions:



a How many hours did the trip take?

b What was the total distance of the trip?

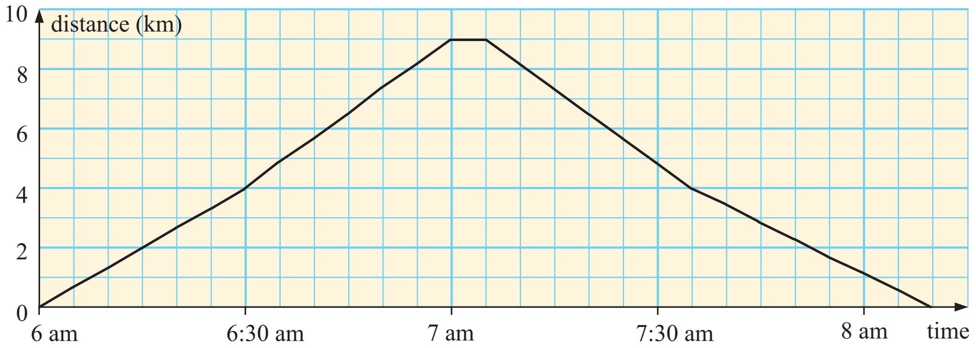
c How far had they travelled after 5 hours?

d How long did it take them to travel the first 250 km?

e What distance was travelled in the:
- **i** first hour
- **ii** second hour
- **iii** third hour
- **iv** last 3 hours?

f What happened during the third hour?

2 The graph below indicates the distance of a jogger from his home at various times:



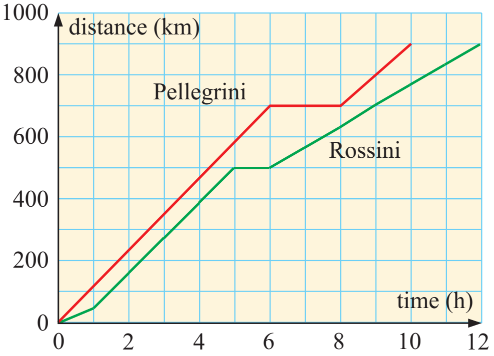
Use the graph to determine:

- **a** the time the jogger left home
- **b** the total time between the jogger's departure and arrival times
- **c** how far the jogger was from home at: **i** 6:45 am **ii** 7:30 am
- **d** how far the jogger travelled:
  - **i** between 6:15 am and 6:55 am **ii** between 7:20 am and 8:10 am
- **e** the total distance travelled by the jogger
- **f** for how long the jogger rested at the half-way point.

3 Two families travel 900 km by car from Rome to Lugano. The journeys are shown on the graph alongside:

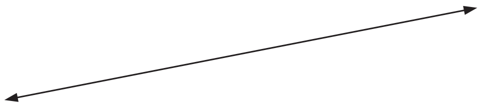
Use the graph to determine:

- **a** which family arrived in Lugano first
- **b** how long the Pellegrini family took for lunch in Como
- **c** the distance travelled by the Rossini family between the 3rd and 8th hours.



E CONTINUOUS AND DISCRETE GRAPHS

A **straight line graph** consists of an infinite number of points in the same direction.



represents a straight line

Consider the following example:

Daniel cycles to the local fruit and vegetable shop to buy potatoes and onions. He carries home exactly 5 kg of these vegetables.

If we let x kg be the weight of potatoes and y kg be the weight of onions that Daniel purchases, then we can construct a table and list some possible combinations.

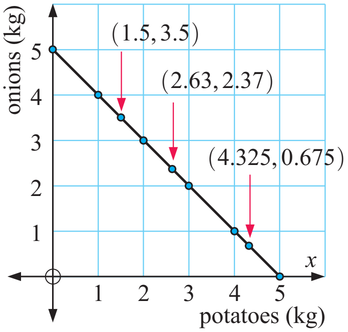
Amount of potatoes (x kg)	0	1	2	3	4	5	1.5	2.63	4.325
Amount of onions (y kg)	5	4	3	2	1	0	3.5	2.37	0.675

In algebraic form the rule connecting x and y is $x + y = 5$.

x and y do not have to be whole numbers, so we could list a very large number of combinations which add up to 5.

It makes sense to **join** all the points. The data is said to be **continuous**.

The graph is a **straight line segment** since neither x nor y may be negative.



Now consider this similar example:

Caitlin takes a piece of fruit to school each day to eat at recess. During the family's weekly shopping trip she has to choose five pieces of fruit for the coming school week. This week she can choose oranges or pears.

If we let x be the number of oranges and y be the number of pears she buys, then we can draw up a table of *all* possible combinations.

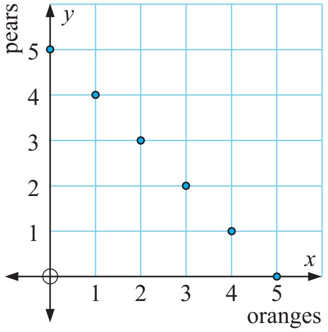
Number of oranges (x)	0	1	2	3	4	5
Number of pears (y)	5	4	3	2	1	0

In each case the total number of pieces of fruit she buys is 5, and the rule connecting x and y is again $x + y = 5$.

However, in this case x and y can only be **whole numbers**, so we can only plot six points on the graph.

The points are collinear but it does not make sense to draw a straight line segment through them.

The data is said to be **discrete**.



Collinear points lie in a straight line.

Example 4 **Self Tutor**

Ellyn has a part-time job in a pizza restaurant. She receives $12 per hour for her work and is required to work between 1 and 4 hours in a shift.

a Complete the following table of values:

Hours worked (x)	1	2	3	4
Total pay (y)				

b Graph the points on a number grid.

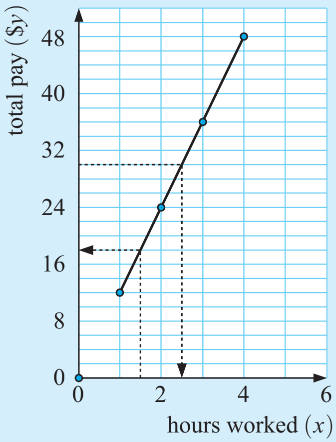
c Does it make sense to join the points? Why or why not?

d Use your graph to find:
  i how much Ellyn earns if she works 1.5 hours
  ii how long Ellyn must work to earn $30.

a

Hours worked (x)	1	2	3	4
Total pay (y)	12	24	36	48

b



c Yes, it makes sense to join the points because it is possible for Ellyn to work part of an hour.

d **i** If Ellyn works 1.5 hours she earns $18.
  ii To earn $30 Ellyn must work for 2.5 hours.

EXERCISE 17E

1 Daniel has £8 to spend on flowers for his garden bed. He buys x punnets of marigolds and y punnets of petunias. Each of the punnets costs £1, and he spends all of his money.

a Copy and complete this table which shows the different combinations he could buy.

Punnets of marigolds (x)	0	1	2	3	4	5	6	7	8
Punnets of petunias (y)									

b Graph the information in the table on a number grid.

c Are the points collinear?

d Write a relationship between x and y.

e Is it meaningful to join the points with straight line segments?

2 Sally sells coloured glass sea-shells by the seashore. She receives €2 commission for each shell she sells. Suppose she sells x sea-shells and the total commission is €y.

a Copy and complete the following table:

Number of sea-shells Sally sells (x)	0	1	2	3	4	5
Total commission (€y)	0					

- **b** Graph the information in the table on a number grid.
- **c** Are the points collinear?
- **d** Write a relationship between x and y.
- **e** Is it meaningful to join the points with straight line segments?
- **f** Is it meaningful to continue the line segment in either direction?

3 Joel and Jennifer are friends who were born exactly two years apart. Joel was born on Jennifer's second birthday.

- **a** Complete the following table which shows the relationship between their ages:

Joel's age (x)	0	1	2	3	4	5	6	7	8
Jennifer's age (y)	2	3							

- **b** Graph the information in the table on a number grid.
- **c** Are the points collinear?
- **d** Does it make sense to join the points? Why or why not?
- **e** Use the graph to find:
  - **i** Jennifer's age when Joel is $5\frac{1}{2}$ years old
  - **ii** Joel's age when Jennifer is $2\frac{1}{4}$ years old.
- **f** Explain how you could use your graph to find Jennifer's age when Joel is 28.
- **g** Write a rule connecting Joel's age x and Jennifer's age y.

F GRAPHING LINEAR RELATIONSHIPS

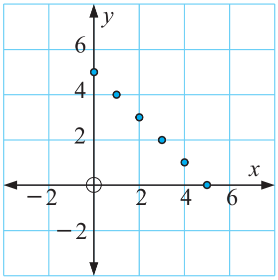
In the previous section we saw two graphs which obeyed the rule $x + y = 5$.

If x and y are **whole numbers** which cannot be negative then we have **discrete points** as in *Graph 1*.

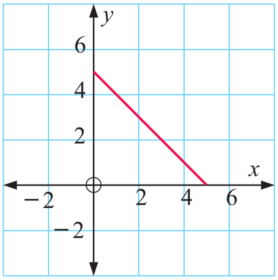
If x and y can be **any positive numbers** then the graph is **continuous** as in *Graph 2*.

If x and y can take **any value** we obtain the **continuous** line in *Graph 3*. We use arrowheads at each end of the line to show it continues infinitely in both directions.

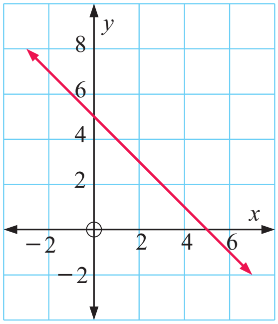
Graph 1:



Graph 2:



Graph 3:



If we are given an equation we can plot a graph using the following procedure:

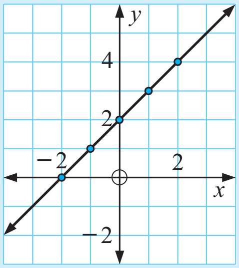
- Draw up a table of values which fit the rule. Start with 5 different values of x and find the corresponding values of y.
- Plot the points on a number grid.
- Join the points with a straight line.
- Place arrows at the ends to indicate that the line extends in both directions.

Example 5 **Self Tutor**

Draw graphs of the lines with equations: **a** $y = x + 2$ **b** $y = \frac{1}{2}x - 1$

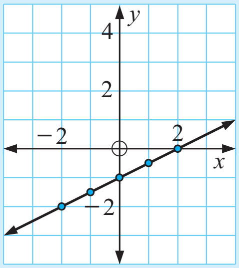
a $y = x + 2$

x	-2	-1	0	1	2
y	0	1	2	3	4



b $y = \frac{1}{2}x - 1$

x	-2	-1	0	1	2
y	-2	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0



EXERCISE 17F

1 Copy and complete the following tables for the equations provided. Plot each set of ordered pairs on separate axes and draw straight lines through the points.

a $y = x$

x	-2	-1	0	1	2
y					

b $y = x + 3$

x	-4	-2	0	2	4
y					

c $y = -x$

x	-4	-2	0	2	4
y					

d $y = 4 - x$

x	-2	-1	0	1	2
y					

e $y = 2x - 2$

x	-2	-1	0	1	2
y					

f $y = 3 - 2x$

x	-2	-1	0	1	2
y					

g $y = 4x - 5$

x	-2	-1	0	1	2
y					

h $y = 7 - 3x$

x	-2	-1	0	1	2
y					

2 Construct tables of values and then draw graphs of the lines with equations:

a $y = x - 4$ **b** $y = x + 4$ **c** $y = 2x$

d $y = 1 - x$ **e** $y = \frac{1}{2}x$ **f** $y = 2x + 3$

g $y = 2x - 3$ **h** $y = 2 - 3x$ **i** $y = 3x - 2$

INVESTIGATION 1 LOOKING AT LINES

This investigation is best done using technology. Click on the icon to run a graphing package.

GRAPHING PACKAGE

What to do:

1 On the same set of axes, graph the lines with equations:
$y = 3x, \quad y = 2x, \quad y = x, \quad y = \frac{1}{2}x, \quad y = \frac{1}{3}x.$

- **a** What is common to each of the equations?
- **b** What makes the equations differ?
- **c** What effect do the differences in **b** have on the resulting graphs?

2 On the same set of axes, graph the lines with equations:
$y = 2x, \quad y = 2x + 2, \quad y = 2x + 5, \quad y = 2x - 3.$

- **a** What is common to each of the equations?
- **b** What makes the equations differ?
- **c** Copy and complete:
  "Since the coefficients of x are the same, the lines are ...."

3 On the same set of axes, graph the lines with equations:
$y = 2x + 3, \quad y = 3x + 3, \quad y = x + 3, \quad y = -2x + 3.$

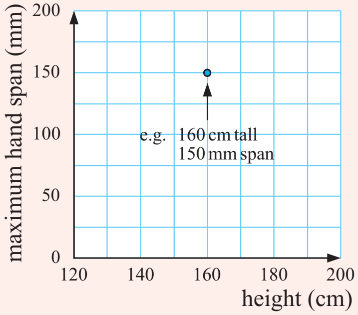
- **a** What is common to each of the equations?
- **b** What makes the equations differ?
- **c** Copy and complete:
  "The number added to the x term determines where the graph cuts ...."

ACTIVITY HEIGHT VERSUS HAND SPAN

You will need: A tape measure and a book.

What to do:

1 For each member of your class, measure their *height* to the nearest cm, and *maximum hand span* to the nearest mm. Record your results in a table.
You should use a book to help get accurate height measurements. The right angle formed helps to mark the height on the wall.



2 On graph paper plot points corresponding to each person's height and maximum hand span. Place the height on the horizontal axis.

3 Can a person's maximum hand span be accurately predicted if their height is known? Is there a feature of the graph that supports your argument?

4 What is it about your **scatterplot** which may enable you to deduce that: "*in general as a person's height increases, so does their maximum hand span*"?

INVESTIGATION 2 NON-LINEAR GRAPHS (EXTENSION)

There are a vast number of interesting graphs which are not linear. In this activity you are encouraged to use the graphing package on the CD.

GRAPHING PACKAGE

What to do:

1 Copy and complete tables of values for the following equations:

a $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

b $y = \sqrt{x}$

x	-4	0	1	4	9
y					

c $y = \dfrac{1}{x}$

x	-8	-2	-1	0	1	2	8
y							

d $y = \sqrt{1 - x^2}$

x	-3	-1	0	1	3
y					

2 On separate axes, try to graph each equation.

3 Now click on the icon to check your graphs.

4 Can you explain the strange feature of your graph in **1 c**?

KEY WORDS USED IN THIS CHAPTER

- collinear
- decreasing
- increasing
- maximum
- scatterplot
- continuous
- dependent variable
- independent variable
- minimum
- straight line graph
- conversion graph
- discrete
- line graph
- point graph
- travel graph

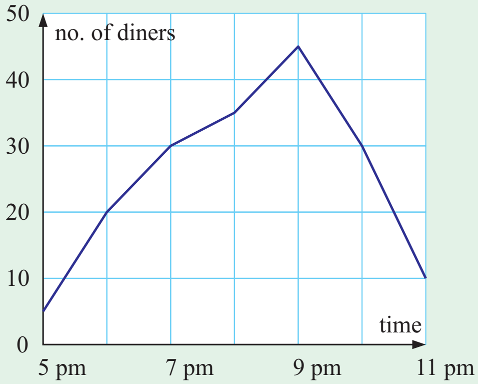
LINKS
click here

HOW ARE TAXI FARES CALCULATED?

Areas of interaction:
Human ingenuity, Approaches to learning

REVIEW SET 17A

1 The number of diners in a restaurant was recorded throughout an evening. The results are shown in the line graph.



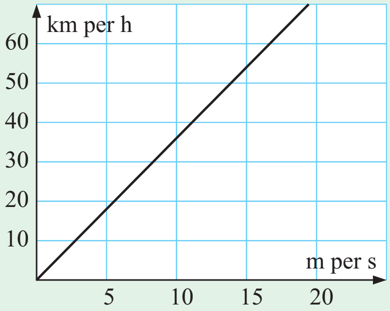
a At what time was the number of diners in the restaurant:

i greatest **ii** least?

b Use the graph to estimate the number of diners in the restaurant at 7:30 pm.

c Use the graph to estimate when there were 20 diners in the restaurant.

2



This graph shows the relationship between speeds in kilometres per hour and in metres per second.

Convert:

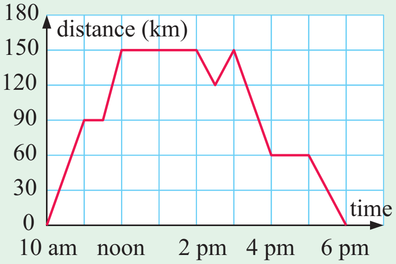
a **i** 60 km per h to m per s

ii 25 km per h to m per s

b **i** 15 m per s to km per h

ii 4 m per s to km per h

3 The graph alongside measures the distance of a family's car from their home as they went for a drive to their friend's house.



a How long did they spend at their friend's house?

b How far did they travel between:

i 11:30 am and noon

ii 3 pm and 4 pm?

c For how long had they been driving home before realising they had left something at their friend's house?

d Determine the times when the car was 30 km from home.

4 Owen and Hannah are sharing a 375 mL can of soft drink. Between them they drink the whole can. Suppose Owen drinks x mL and Hannah drinks y mL.

a Copy and complete the following table:

Amount Owen drinks (x *mL*)	0	75	150	225	300	375
Amount Hannah drinks (y *mL*)						

b Write an equation linking x and y.

c Plot the data from the table on a number grid.

d Does it make sense to join the points? Why or why not?

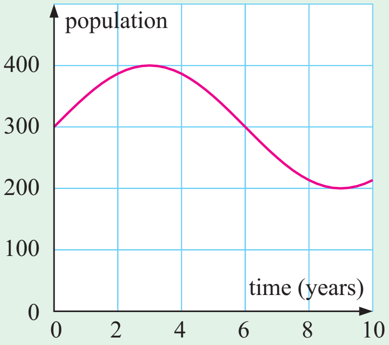
5 **a** Copy and complete the table of values for the equation $y = 2 - x$:

x	-2	-1	0	1	2
y					

b Plot these points on a number grid and draw a straight line through them.

REVIEW SET 17B

1 The population of salmon in a lake over a 10 year period is displayed in the graph alongside:



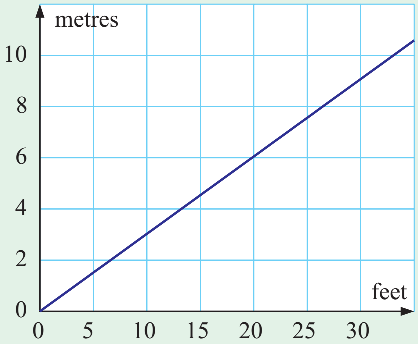
a At what time was the population greatest?

b What was the minimum population, and when did it occur?

c At what times was the population:
  i increasing **ii** decreasing?

d At what times was the population 350?

2



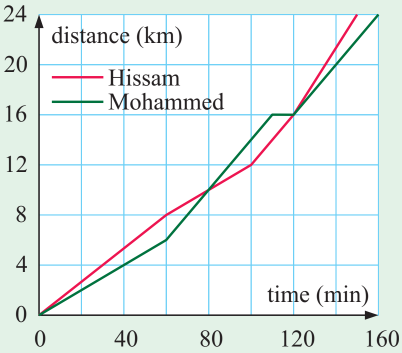
The graph alongside shows the relationship between two units of distance: metres and feet.

a Convert: **i** 8 metres to feet
  ii 2 metres to feet.

b Convert: **i** 6 feet to metres
  ii 20 feet to metres.

c There are 12 inches in 1 foot. Who is taller: a woman 5 feet 6 inches tall, or a woman 1.5 metres tall?

3 Two long distance runners Hissam and Mohammed decided to have a race one afternoon. The distance of each runner from the starting point is shown on the graph. Use the graph to determine:



a the distance of the race

b who was leading the race after 1 hour

c the time at which Mohammed overtook Hissam

d how far Mohammed had run before he developed cramp and had to stop and rest

e who won the race.

4 Rebecca has €4 to spend in the stationery shop. She spends it all buying x erasers for €1 each and y pencils for 50 cents each.

a Copy and complete the following table showing the different possible combinations:

Number of erasers (x)	0	1	2	3	4
Number of pencils (y)					

b Graph the possibilities in the table on a number grid.

c Are the points collinear?

d Does it make sense to join the points? Why or why not?

5 Bill and Ben are filling their 5 L fish tank with water from the kitchen tap.

a Copy and complete the following table:

Amount poured in by Bill (x *L*)	0	1	2	3	4	5
Amount poured in by Ben (y *L*)						

b Does the table show all possible ways that the fish tank can be filled?

c Write an equation linking x and y.

d Plot the data from the table on a number grid.

e Does it make sense to join the points? Why or why not?

6 **a** Copy and complete the table of values for the equation $y = 3x + 1$:

x	-2	-1	0	1	2
y					

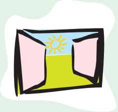
b Plot these points on a number grid and draw the straight line through them.

Chapter 18

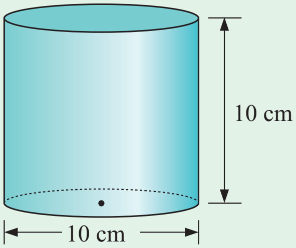
Circles

Contents:

OPENING PROBLEM



A cylindrical can has height 10 cm and width 10 cm.



Things to think about:

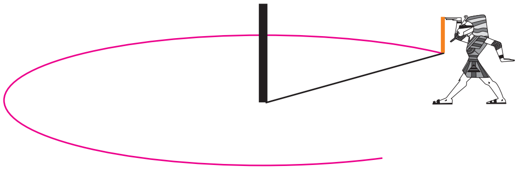
- What is the *distance* around the circular top?
- What is the *area* of the circular top?
- What is the *capacity* of the cylinder?

Suppose we make a loop at the end of a length of light rope. We place it over a fixed spike in the ground. The rope is made taut and a stick is placed at the opposite end to the fixed spike.

By keeping the rope taut and moving the stick around the spike, a **circle** is produced.

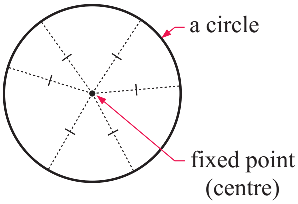
The fixed spike is the circle's **centre**.

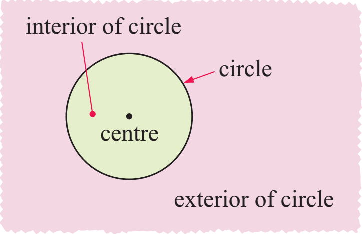
This method of drawing a circle was known and used by builders in ancient Egypt.



A PARTS OF A CIRCLE

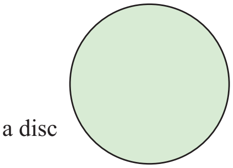
A **circle** is a two-dimensional shape. It is the set of all points which are a constant distance from a fixed point.





The fixed point is called the circle's **centre**. It lies inside the circle, or in the circle's **interior**. The region outside the circle is called its **exterior**.

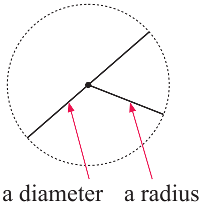
A **circular disc** consists of all points on and inside a circle.



A **diameter** of a circle is a straight line segment which passes through the circle's centre and has end points on the circle.

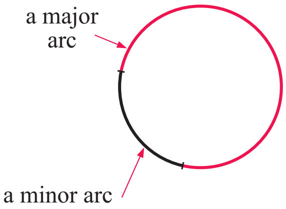
A **radius** of a circle is a straight line segment which joins a circle's centre to any point on the circle. **Radii** is the plural of radius.

However, it is common to refer to the radius of a circle as the length of any of its radii and the diameter of a circle as the length of any of its diameters.

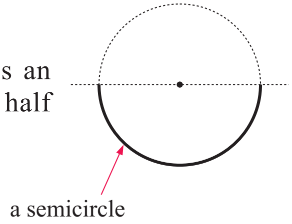


An **arc** is a part of a circle. It joins any two different points on the circle.

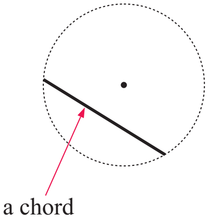
For any two points we can define a **minor arc** and a **major arc** which are the shorter and longer arcs around the circle respectively.



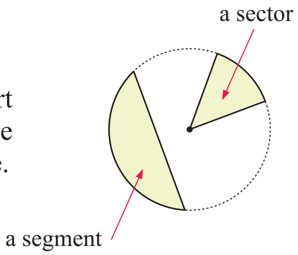
A **semi-circle** is an arc which is a half of a circle.



A **chord** of a circle is a line segment which joins any two points of the circle.



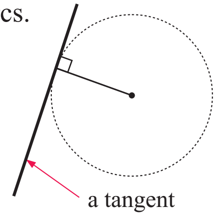
A **segment** of a circle is a part of the interior of a circle between a chord and the circle.



A **sector** of a circle is a part of the interior of a circle between two radii and the circle.

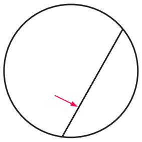
We can define major and minor segments and sectors just as we did for arcs.

A **tangent** to a circle is a line which *touches* the circle but does not enter its interior. A tangent is always at right angles to the radius at that point.



EXERCISE 18A

1 Match the part of the figure indicated to the phrase which best describes it:

a


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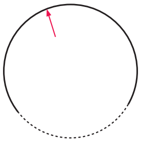

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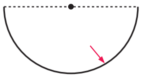

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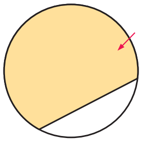

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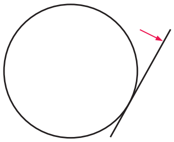

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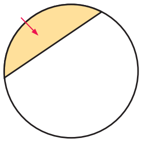

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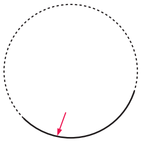

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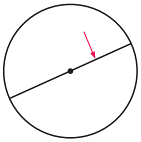

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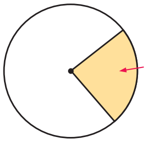

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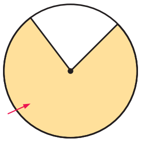

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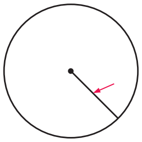
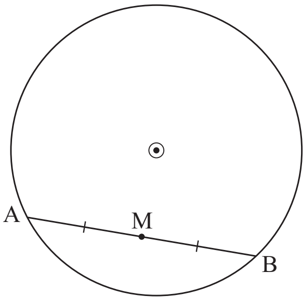


A a semi-circle
B a radius
C a minor arc
D a major arc
E a diameter
F a chord
G a minor segment
H a major segment
I a major sector
J a minor sector
K a tangent

2 What name can be given to the longest chord that you can draw in a circle?

3 Use a compass to draw a circle with radius of length 23 mm.
  a Find the length of a diameter of the circle.
  b Draw on the circle a chord [AB] with length 4 cm.
  c Label the major arc of the circle with end points A and B.
  d Shade the minor segment of the circle which can be formed using points A and B.

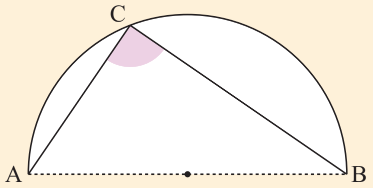
4 You are given a circular disc of paper which does not have the circle's centre marked. Explain how you could find the centre of the circle by folding the paper.

5 

A circle has centre O and chord [AB] as shown. M is the midpoint of [AB], or the point midway between A and B.
  a Explain why triangle AOB is isosceles.
  b Using **a**, what can be deduced about [OM] and [AB]?
  c Check your answer to **b** by making a disc from paper, drawing a chord, and making one fold.

INVESTIGATION 1 THE ANGLE IN A SEMI-CIRCLE

Suppose we have a semi-circle with diameter [AB]. We choose any point C in its perimeter. In this investigation we consider the measure of the angle ACB.



What to do:

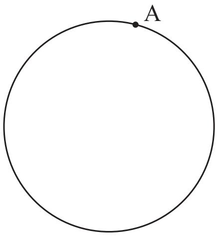
1 Draw a circle of radius greater than 5 cm.
2 Draw any diameter [AB] of the circle so you divide it into two semi-circles.
3 Choose any point C on one of the semi-circles.
4 Measure the angle ACB.
5 Now choose any point D on the second semi-circle.
6 Measure the angle ADB.
7 What do you suspect about the angle in a semi-circle?
8 Click on the icon to run software for measuring the angle in a semi-circle.

GRAPHING PACKAGE

9 Comment on the statement: *The angle in a semi-circle is always a right angle.*

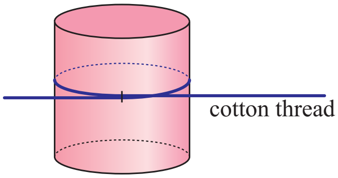
B CIRCUMFERENCE

The **circumference** of a circle is its perimeter.



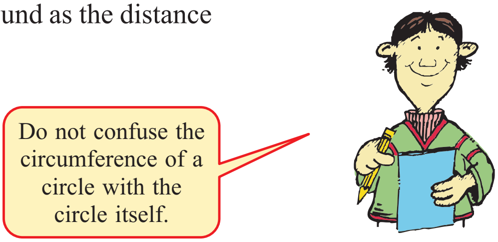
For example, if an ant starts at point A on the circle and walks around it until it gets back to A, then the total distance walked by the ant is the circumference of the circle.

To find the circumference of a circle you could use a cotton thread. For example, you could wrap a cotton thread around a tin can.



Hold the cotton tightly and get someone to mark across the cotton with a fine pen.

When two marks are made and the cotton is placed on a ruler, the circumference can be found as the distance between the marks.



INVESTIGATION 2 CIRCUMFERENCE

In this investigation we look for a connection between the circumference of a circle and the length of a diameter.

What to do:

1. Gather some cylinders such as a drink can, a tin can, a toilet roll, and a length of water pipe.
2. Use the cotton thread method to find the circumference of each circle.
3. Use a ruler to find the diameter of each object.
4. Construct a table to record the circumference and diameter of each object.

Object	*Circumference*	*Diameter*	$\dfrac{\textit{Circumference}}{\textit{Diameter}}$
⋮			

5. Fill in the last column by calculating the fraction $\dfrac{\text{circumference}}{\text{diameter}}$ for each object.
6. What do you notice from the results in **5**?

From the **Investigation** you should have found that the fraction $\dfrac{\text{circumference}}{\text{diameter}}$ has the same value for any circle. This value lies between 3.1 and 3.2.

We know, in fact, that the fraction $\dfrac{\text{circumference}}{\text{diameter}}$ is an exact number which we write as π, the **Greek** letter pi.

For a circle we denote the radius by r, the diameter by d, and the circumference by C.

So, $\dfrac{C}{d} = \pi$ and hence $C = \pi d$ {multiplying both sides by d}

The diameter is always twice as long as the radius, so $\boldsymbol{d = 2r}$

So, for a circle with circumference C, diameter d and radius r,

$$\boldsymbol{C = \pi d} \quad \text{or} \quad \boldsymbol{C = 2\pi r}$$

The exact value of π cannot be written down because its decimal places go on forever without repeating. Its value is 3.141 592 653 589 79......

In practice we use $\pi \approx 3.14$ or the $\boxed{\pi}$ key on a calculator.

Example 1 **Self Tutor**

Find the circumference of a circle of:

a diameter 10 cm **b** radius 2 m

a $C = \pi d$
$\approx 3.14 \times 10$
≈ 31.4 cm

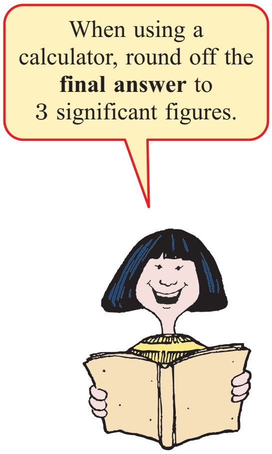
b $C = 2\pi r$
$\approx 2 \times 3.14 \times 2$
≈ 12.6 m

Example 2 **Self Tutor**

Find the circumference of a circle of radius 4.35 m.

$C = 2\pi r$
$= 2 \times \pi \times 4.35$
≈ 27.3 m

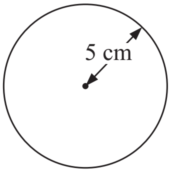
Calculator: 2 $\boxed{\times}$ $\boxed{\pi}$ $\boxed{\times}$ 4.35 $\boxed{=}$



EXERCISE 18B.1

1 Using $\pi \approx 3.14$, find the circumference of:

a



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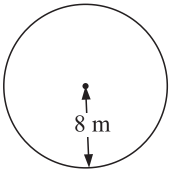

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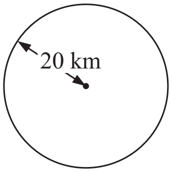

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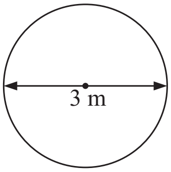

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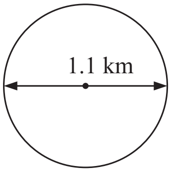

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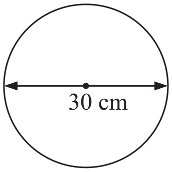

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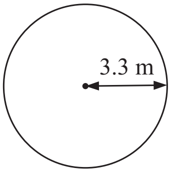

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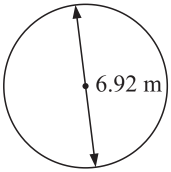
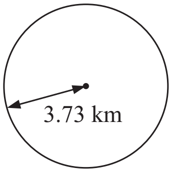


i



2 Find the circumference of a circle of:

a diameter 8.5 cm **b** diameter 11.3 m **c** radius 2.45 km

d radius 841 m **e** diameter 12.75 m **f** radius 11.72 km

3 A cylindrical tank has radius 1.5 m. What is the circumference of its base?

4 A circular pond has diameter 8 m and needs to be fenced for the protection of children.

a What length of fencing is required?

b Fencing comes in 1 m lengths. How many lengths are needed?

c What is the total cost of the fencing if each length costs €25.00?

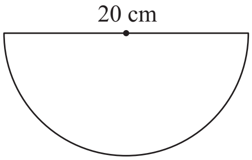
5 A car wheel has a radius of 35 cm.

a What is the circumference of the wheel?

b If the wheel rotates 100 000 times, how far does the car travel?

6 Find the perimeter of:

a



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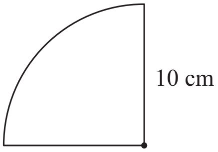

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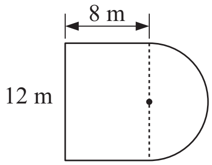

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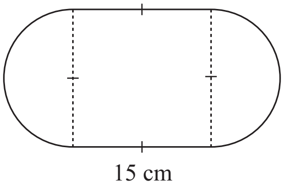

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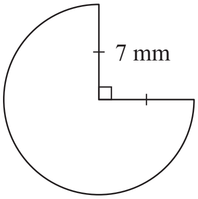

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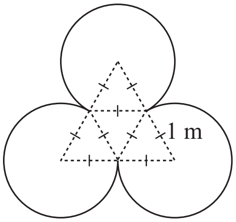


FINDING THE DIAMETER OR RADIUS

If we are given the circumference of a circle we can calculate its diameter and radius.

Example 3 **Self Tutor**

What is the diameter of a circular pond with circumference 100 m?

$$\begin{aligned} \pi d &= C \\ \therefore \quad \pi d &= 100 \\ \therefore \quad d &= \frac{100}{\pi} && \{\text{dividing both sides by } \pi\} \\ \therefore \quad d &\approx 31.8 && \{100 \ \boxed{\div} \ \boxed{\pi} \ \boxed{=} \ \} \end{aligned}$$

So, the diameter is about 31.8 m.

EXERCISE 18B.2

1 Find the diameter of a circle with circumference:

a 10 m **b** 50 cm **c** 5 km

2 Find the radius of a circle with circumference:

a 80 cm **b** 25.6 m **c** 3.842 km

3 A trundle wheel is used for measuring distances. The circumference of the wheel is exactly 1 m. Each time the wheel rotates through one complete turn a click sound is heard and a counter adds a metre to the total. What is the radius of this wheel?

4 A circular garden plot has circumference 12.65 m. Find its radius in metres, correct to the nearest centimetre.

C AREA OF A CIRCLE

We can show using geometry that the area of a circle with radius r is the same as the area of a rectangle with length πr and width r. You can view a demonstration of this by clicking on the icon.

So, the area of a circle, $A = \pi r \times r$

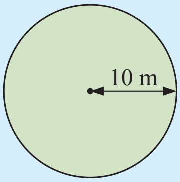
Hence $\boldsymbol{A = \pi r^2}$

DEMO

Example 4 **Self Tutor**

Find the area of a circle of radius 10 m.

$$A = \pi r^2$$
$$\therefore \quad A = \pi \times 10^2$$
$$\therefore \quad A \approx 314 \text{ m}^2$$

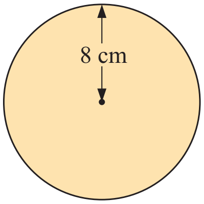


So, the area is about 314 m^2.

EXERCISE 18C

1 Find the area of:

a



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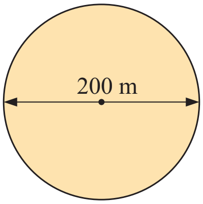

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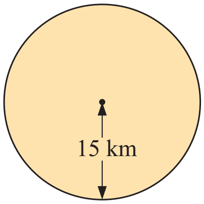


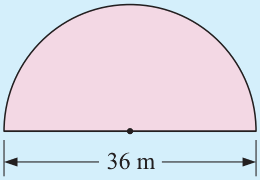
2 Find the area of a circular paint can lid with diameter 24 cm.

3 An irrigation sprinkler sprays water over a field. The radius of the spray is 12.6 m. What area of the field is being watered?

4 The rope connecting a goat to a pole is 8 m long. What area of grass can the goat eat?

Example 5 **Self Tutor**

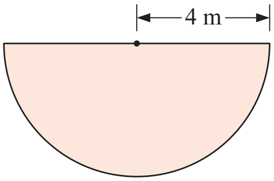
Find the area of:



$$\begin{aligned} \text{Area} &= \tfrac{1}{2} \text{ of the area of the whole circle} \\ &= \tfrac{1}{2} \times \pi r^2 \\ &= \tfrac{1}{2} \times \pi \times 18^2 \\ &\approx 509 \text{ m}^2 \end{aligned}$$

5 Find the area of:

a



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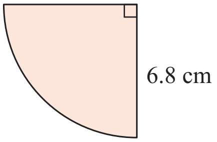

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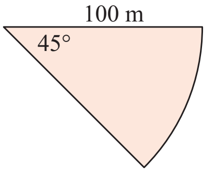
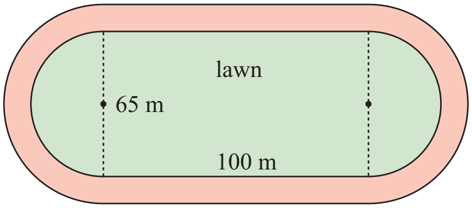


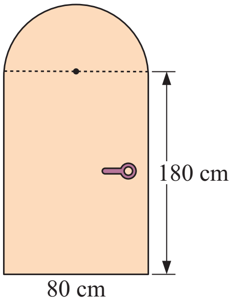
6 The inner part of an athletics track is lawn. Find the area of lawn.



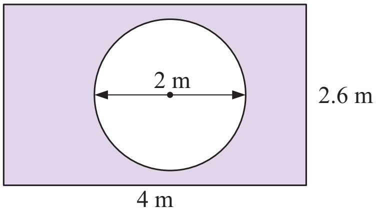
7 A door has the dimensions shown.

a How high is the door at its highest point?

b What is the area of the door in square metres?



8 Find the area of the shaded region.



9 A circular table top has a diameter of 1.6 m. A rectangular tablecloth 2 m by 2 m is placed over the table top. What area of the tablecloth overlaps the table?

10 The diagram shows plans for a garden which is 40 m by 40 m. It consists of 4 quarter circles of lawn with a flower bed in the middle as shown. Find:

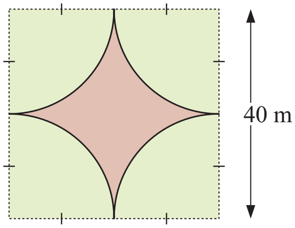
a the perimeter of the garden

b the total area of lawn

c the total area of the garden

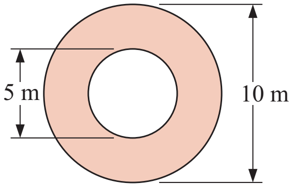
d the area of the flower bed

e the length of edging around the flower bed.



11 **a** Find the shaded area.

b Find the sum of the perimeters of the two circles.



12 **a** Circle A has radius 3 cm and circle B has radius 6 cm. Find these ratios:

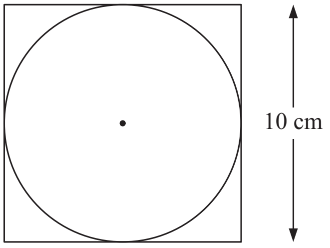
- **i** circumference of B : circumference of A
- **ii** area of B : area of A

b Circle C has radius 5 cm and circle D has radius 10 cm. Find these ratios:

- **i** circumference of C : circumference of D
- **ii** area of C : area of D

c Copy and complete:
If the radius of a circle is doubled, then its circumference is ...... and its area is ...... times greater.

13 Consider a circle within a square.

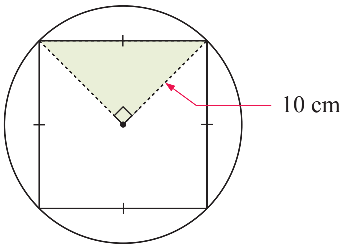


a Find the area of:

- **i** the square
- **ii** the circle.

b What percentage of the square is occupied by the circle?

14 Consider a square within a circle.

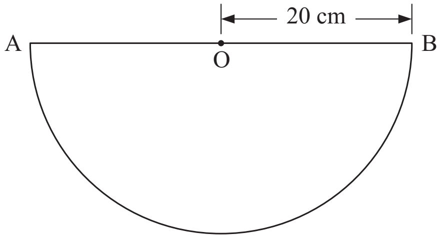


a Find the area of:

- **i** the circle
- **ii** the shaded triangle
- **iii** the square.

b What percentage of the circle is occupied by the square?

15 Consider the semi-circular piece of paper shown.
When sides [OA] and [OB] are put together, we form a cone. You can see this by clicking on the icon.



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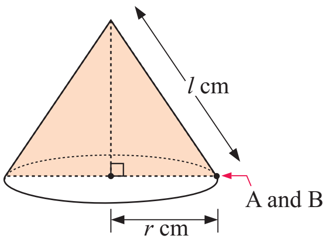


a Find the value of l.

b Explain why the length of the arc AB must equal the circumference of the base of the cone.

c Use **b** to find r.

d Find the area of the base of the cone.

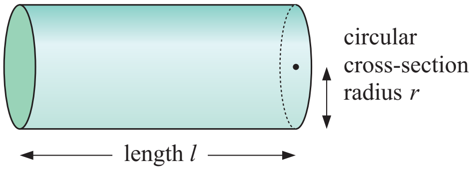
e Find the area of the curved surface of the cone.

f Find the total surface area of the cone.

D CYLINDERS

A **cylinder** is a solid of uniform cross-section for which the cross-section is a circle.

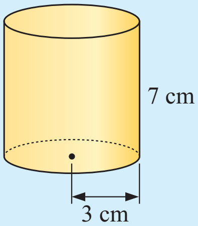
So, its volume can be calculated by multiplying its area of cross-section by its length.



$$\textbf{Volume} = \textbf{area of end} \times \textbf{length}$$
$$= \pi r^2 l$$

Example 6 **Self Tutor**

Find the volume of:



Volume

$=$ area of end $\times$ length

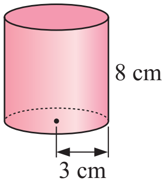
$= \pi \times 3^2 \times 7$

$\approx 198 \text{ cm}^3$

EXERCISE 18D

1 Find the volume of:

a



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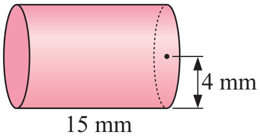

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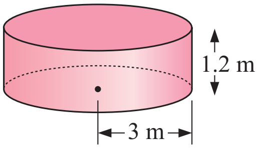

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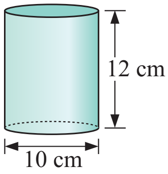

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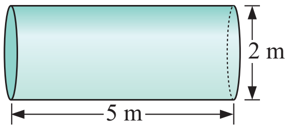

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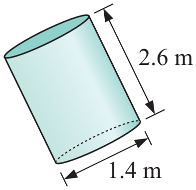
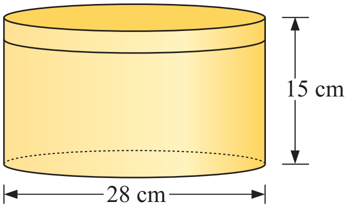


2 A steel bar is 2.2 m long and has a diameter of 5 cm. Find the volume of the bar in cm^3.

3 A stainless steel wine vat is cylindrical with base diameter 1.8 m and height 6 m. How much wine does it hold if it is 90% full?

4 A cylindrical biscuit barrel is 15 cm high and has a diameter of 28 cm.
Find the volume of the biscuit barrel.



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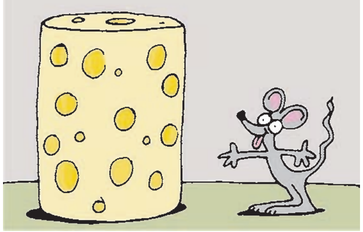
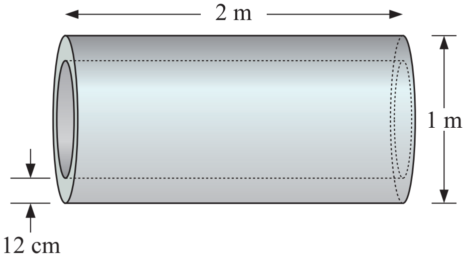


A round of cheese is 18 cm deep and has radius 12 cm.
Find the volume of cheese in this round.

6 The walls of a concrete pipe are 12 cm thick and the outer diameter is 1 m. Find:

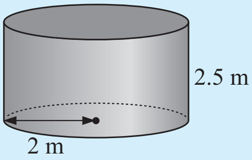
- **a** the area of the cross-section of the pipe
- **b** the volume of concrete needed to make one pipe
- **c** the volume of concrete needed to make 6000 of these pipes.



7 A medal with diameter 36 mm is made from 6200 mm^3 of metal.
How thick is the medal?

Example 7 **Self Tutor**

A cylindrical rainwater tank has a base radius of 2 m and a height of 2.5 m. Find the capacity of the tank in kL.



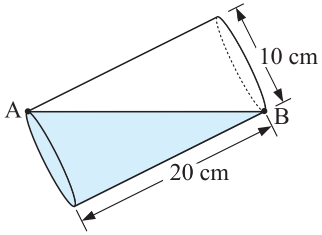
$$\begin{aligned} \text{Volume} &= \text{area of base} \times \text{height} \\ &= \pi r^2 \times \text{height} \\ &= \pi \times 2^2 \times 2.5\ \text{m}^3 \\ &\approx 31.4\ \text{m}^3 \\ \therefore\ \text{capacity} &\approx 31.4\ \text{kL} \end{aligned}$$

8 A cylindrical oil tank has a base area of 8 m^2 and a height of 3 m.
Find its capacity in kL.

9 A cylindrical rainwater tank has diameter 3 m and height 2 m.
Find the maximum number of kilolitres of water that it could hold.

10 A cylindrical paint can has base diameter 24 cm and height h cm.
Find h if the can holds 2 L of paint.
Hint: Begin by drawing a labelled diagram of the can.

11 Water is placed in a cylinder as shown.
The water level touches points A and B.
How much water is in the cylinder?



KEY WORDS USED IN THIS CHAPTER

- arc
- area
- chord
- circle
- circular disc
- circumference
- cylinder
- diameter
- exterior
- interior
- major arc
- minor arc
- radius
- sector
- segment
- semi-circle
- tangent
- volume

LINKS
click here

FLAG RATIOS

Areas of interaction:
Human ingenuity

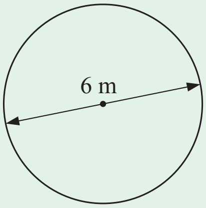
REVIEW SET 18A

1 Clearly define, with the aid of diagrams, the meaning of:

a an *arc* of a circle **b** a *sector* of a circle **c** a *chord* of a circle.

2 Find the circumference of:

a



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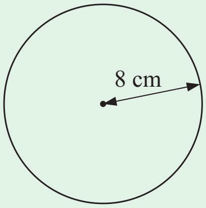

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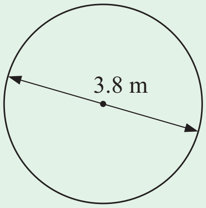


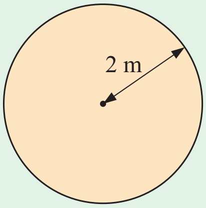
3 A circular hoop has a radius of 40 cm. Find the length of tubing needed to make the hoop.

4 Find the diameter of a circle with circumference 30 cm.

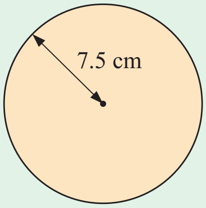
5 Eva walks 500 m to complete one lap of a circular walking trail. Find the radius of the trail.

6 Find the area of:

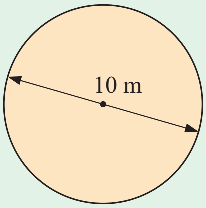
a



b



c



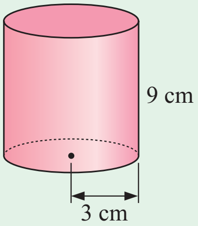
7 A gardener is making a path using 8 cylindrical concrete pavers. Each paver has a radius of 20 cm, and is 5 cm thick.

a Find the total area of the tops of the pavers.

b Find the total volume of the pavers.

8 Find the volume of:

a



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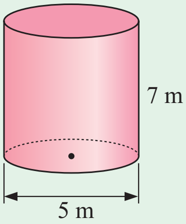


9 A coin is 20 mm in diameter and 2 mm high. Find the volume of the coin.

10 A cylindrical cooking pot has a radius of 15 cm and a height of 20 cm. How many litres of water can the pot hold?

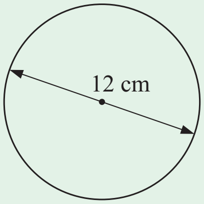
REVIEW SET 18B

1 Clearly define, with the aid of diagrams, the meaning of:

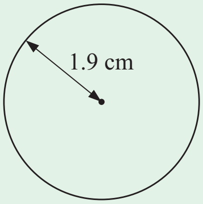
a a *circle* **b** a *semi-circle* **c** a *segment* of a circle.

2 Find the circumference of:

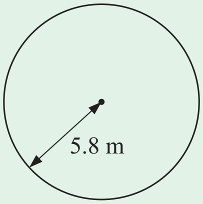
a



b

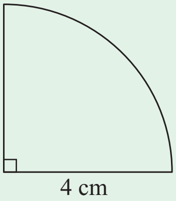


c



3 Find the perimeter of:

a



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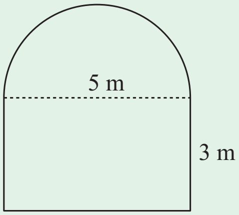
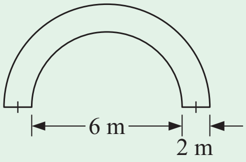


c



4 A circular disc has a diameter of 16 cm. Find the:

a circumference **b** area of the disc.

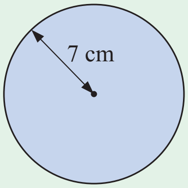
5 140 cm of metal is used to construct a circular basketball ring. Find the diameter of the ring.

6 A circle has a circumference of 90 cm. Find the:

a diameter **b** radius of the circle.

7 Find the area of:

a



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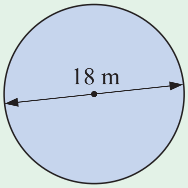
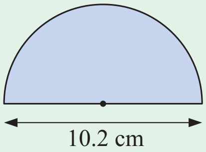


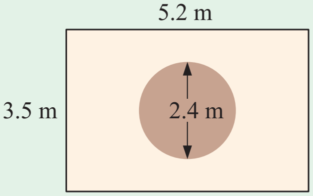
c



8 A circular rug is laid on a tiled floor. Find:

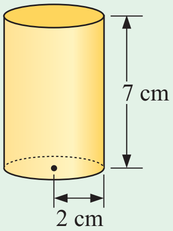
a the area of the rug

b the visible area of the tiled floor.

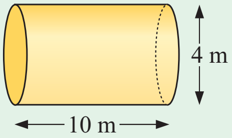


9 Find the volume of:

a



b

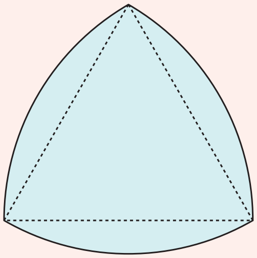


10 A cylindrical drinking glass has a radius of 4 cm. It is filled with juice to a height of 8 cm. Calculate the amount of juice in the glass in mL.

ACTIVITY — REULEAUX TRIANGLES

If we start with an equilateral triangle, we can draw 3 circular arcs centred at the vertices of the triangle.

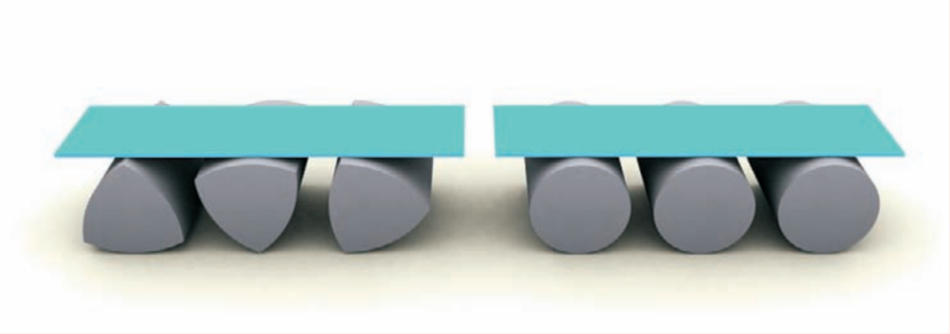
The resulting figure is known as a **Reuleaux Triangle**.



Like a circle, the Reuleaux triangle has constant width.

What to do:

1 To move a heavy piece of machinery, we can roll it along on cylindrical rollers. Could Reuleaux rollers be used instead of the cylindrical ones?



2 Research the properties and uses of the **Reuleaux triangle**.

Chapter 19

Chance

Contents:

OPENING PROBLEMS

There are many situations in life where we consider the *chance* or *likelihood* of something happening.

Things to think about:

1 A coin is tossed 5 times and *heads* results each time. On the next toss, is it more likely that the result will be *tails* than *heads*?

2 How can we use a die to generate three different outcomes, each of which has the same chance of occurring?

3 What is the chance that a family of *three* children will consist of all boys or all girls?

A DESCRIBING CHANCE

Nearly every day we hear statements involving chance. For example:

"We will probably buy a new television soon."
"I am almost certain that I passed the exam."
"It is likely that the storms will damage the crops."
"It is unlikely that our team will win today."

The key words in these statements are: *probably*, *unlikely*, *almost certain*, and *likely*.

Each of these words describes chance.

Chance is to do with the likelihood or probability of events occurring.

Many words are used to describe chance, including:

possible, likely, impossible, unlikely, maybe, certain, uncertain, no chance, little chance, good chance, highly probable, probable, improbable, doubtful, often, rarely, and '50 - 50' chance.

Example 1 **Self Tutor**

Describe using a word or phrase the chance of the following happening:

- **a** A woman will be playing netball at the age of 60.
- **b** Sam, who is now 13, will be alive in 12 months' time.
- **c** The next person to enter the room at a co-educational school will be female.

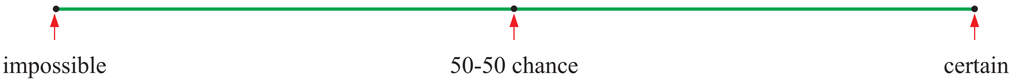
a highly unlikely **b** highly likely **c** a '50 - 50' chance

EXERCISE 19A

1 Describe using a word or phrase the chance of the following happening:

- **a** A person will live to the age of 100 years.
- **b** There will be a public holiday on the 1st day of January.
- **c** A gigantic meteorite will strike the earth in your lifetime.
- **d** You will win a prize in Cross-Lotto in your lifetime.
- **e** Your birthday in three years' time will fall on a weekend day.
- **f** You will get homework in at least one subject tonight.
- **g** You will be struck by lightning next January.
- **h** The sun will rise tomorrow.
- **i** You could do 10 laps around the school grounds in 24 hours.

2 Copy the line below, then add the following words using arrows in appropriate positions:



- **a** doubtful
- **b** very rarely
- **c** almost certain
- **d** highly likely
- **e** unlikely
- **f** a little more than even chance

3 A bag contains 100 marbles of which 99 are white and one is black. A marble is randomly chosen from the bag.

- **a** Describe in words how likely it is that the marble is white.
- **b** Is it certain that the marble is going to be white?
- **c** True or false: "There is a 1 in 99 chance it will be white."

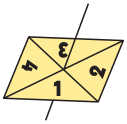
4 A tin contains 8 blue and 9 white discs. One disc is randomly selected from the tin.

- **a** Is it more likely that the disc is blue than it is white? Explain your answer.
- **b** What colour is more likely to be selected?
- **c** True or false: "There is a 9 in 17 chance that the disc is white."

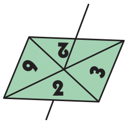
5 Describe the following events as either *certain*, *possible* or *impossible*:

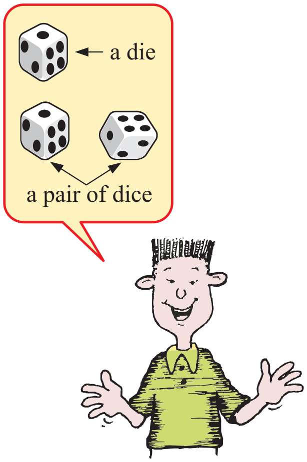
- **a** When tossing a coin it falls *heads* uppermost.
- **b** When tossing a coin it falls on its edge.
- **c** When tossing a coin ten times it falls heads every time.
- **d** When rolling a die a 4 results.
- **e** When rolling a die a 9 results.
- **f** When rolling a pair of dice a sum of 13 results.
- **g** When twirling the given square spinner a 4 results:

i



ii





B ASSIGNING NUMBERS TO CHANCE

When we talk about chance, we can pretend we are running an experiment.

The **outcomes** of the experiment are the possible results from running the experiment once.

An **event** occurs when we obtain an outcome with a particular property.

For example, when we roll a die, the possible outcomes are 1, 2, 3, 4, 5 and 6. The event *an even number* occurs if we get one of the outcomes 2, 4 or 6.

If an event **cannot occur** we assign it the number 0 or 0%.
If an event is **certain to occur** we assign it the number 1 or 100%.

The chance of any event occurring must appear between the two extremes of impossible and certain. So, the probability of any event occurring lies between 0 and 1, or 0% and 100% inclusive.

Events which may or may not occur with equal chance are assigned the probability 0.5 or $\frac{1}{2}$. These events both have a 1 in 2 chance of occurring.

Example 2 **Self Tutor**

Assign the probabilities 0, 0.5 or 1 to best describe:

- **a** the chance of a new-born baby being a boy
- **b** the chance of man being 4 m tall
- **c** the chance that the sun emits light tomorrow.

a There are approximately the same number of men as women.
$\therefore$ the chance of a boy is 0.5 .

b Historical records indicate that no human has ever reached anything like 4 m in height.
$\therefore$ the chance is 0.

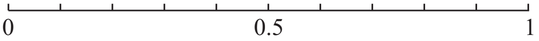
c The sun will emit light tomorrow.
$\therefore$ the chance of light from the sun is 1.

EXERCISE 19B

1 A container holds 6 red and 6 blue balls. One ball is randomly selected from it.

- **a** What is the probability it is a red ball?
- **b** If all of the blue balls are now removed and another ball is randomly selected, what is the probability that it is:
  - **i** a red ball
  - **ii** a blue ball?

2 Copy the probability line below and mark on it the approximate probabilities of:



a the sun not rising tomorrow
b being sick at least once during the year
c being born on a Monday
d being born on a weekend.

3 State whether the possible outcomes of these processes are equally likely:

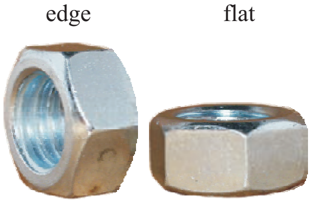
a selecting any card from a deck of 52 playing cards
b judging the winner in a figure skating competition
c getting a result of 1, 2, 3, 4, 5 or 6 when a die is rolled
d a particular team winning the netball competition.

C EXPERIMENTAL PROBABILITY

When we perform an experiment a number of times, we count the **frequency** with which a particular outcome or event occurs.

The **relative frequency** of the outcome or event is the frequency divided by the number of trials.

For example, suppose a metal nut was tossed in the air 50 times. It finished on a *flat* side 41 times and on an *edge* 9 times.



The frequency of the nut finishing on a flat side is 41, and the relative frequency is $\frac{41}{50}$.

The frequency of the nut finishing on an edge is 9, and the relative frequency is $\frac{9}{50}$.

We can summarise the results of the experiment in a table:

Outcome	*Frequency*	*Relative frequency*
flat	41	$\frac{41}{50} = 0.82$
edge	9	$\frac{9}{50} = 0.18$
Total	50	$\frac{50}{50} = 1.00$

DISCUSSION ESTIMATING CHANCE

Discuss the following:

- When tossing a coin it could be argued that the chance of throwing a *head* is 1 in 2 or 0.5. Could you use this sort of argument for finding the chance of getting a *flat* in the nut tossing example above?
- How can you use the results in the table above to estimate the chance of getting a *flat*?
- How can you get a more reliable estimate?

INVESTIGATION 1 TOSSING A COIN

When tossing a coin it could land showing *heads* or *tails*.

TOSSING COIN SIMULATION

What to do:

1 If you toss a coin, what do you think the chance of getting a *head* will be?

2 Toss a coin 2, 5, 10, 50 and 200 times and record your results in a table like this:

Sample size	*Frequency of heads*	*Frequency of tails*	*Relative freq. of heads*	*Relative freq. of tails*
2				
5				
10				
50				
200				

3 Using the simulation, toss the coin 10 000 times. Repeat this procedure a further 4 times. Record all of your results in a table.

4 Discuss and comment on your results. Do they agree with your answer in **1**?

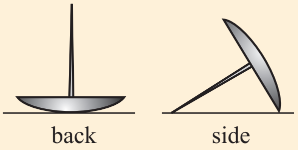
5 Do you agree with the statement:
"*The relative frequency for a large number of trials is approximately the probability that we expected*"?

Sometimes it is not possible to make a reasonable prediction of chance. The problem of tossing a nut is one such example, especially because nuts vary in shape and size.

INVESTIGATION 2 TOSSING DRAWING PINS

When a drawing pin falls to the floor, is it more likely to finish on its back or on its side?

To answer this question, you should perform this experiment using drawing *pins from the same box*. Be careful not to prick yourself.



What to do:

1 Form groups of 3 students. Each student should toss a drawing pin 60 times. Record your own results in a table like this:

Result	*Tally*	*Frequency*
Back		
Side		
	Total	60

2 Add a *Relative frequency* column to the table and fill this in for each outcome.

3 Compare your results with the other two in your group. Are they the same? Why or why not?

4 Estimate the chance of a drawing pin finishing on its back.

5 Combine the results of your group to make 180 tosses in total.

6 Combine the results of the entire class. Which set of data gives the best estimate of the chance of a drawing pin finishing on its back?

7 If you were to toss a drawing pin one million times, find your best estimate of how many times would you expect it to finish on its back.

From the **Investigations** above:

The **experimental probability** of an event occurring is its **relative frequency**.
The experimental probability can generally be improved or made more accurate by **using a larger sample** size.

Sometimes experimental data is the only way we can estimate the probability of an event occurring.

Example 3 **Self Tutor**

The population is not made up of exactly 50% males and 50% females.

The table shows percentages for several age groups for each sex in the Australian population. Estimate the chance that if you met a person in the 5 to 14 age group, the person would be:

a male **b** female.

Australian population: 2006 Sex by age group		
	Sex	
Age group	*% male*	*% female*
5 to 14	51.3	48.7
15 to 24	50.9	49.1
25 to 34	49.4	50.6
35 to 44	48.9	51.1
45 to 54	49.2	50.8
55 to 64	50.0	50.0

In the 5 to 14 age group, 51.3% are male, and 48.7% are female.

a chance of male
$= 51.3\%$
$= 0.513$

b chance of female
$= 48.7\%$
$= 0.487$

EXERCISE 19C

1 For the population of Australia data in **Example 3**:

- **a** In which age groups are there more females than males?
- **b** What age groups are not included in the data?
- **c** Estimate the chance that, if you met someone in the age group 15 to 24, the person would be: **i** male **ii** female.
- **d** Could the table be used to find the chance that the next person you meet would be in the age group 15 to 24?

2 Use the given table to estimate the probability that the next person you meet will be:

- **a** between 15 and 29 years of age
- **b** less than 30 years
- **c** over 44 years of age
- **d** between 30 and 74 years of age.

Australian population: 2006 Total by age group	
Age group	*Percentage of total*
0 to 14	19.8
15 to 29	20.1
30 to 44	21.8
45 to 59	20.1
60 to 74	11.7
75+	6.4

3

2006 cinema attendances for persons aged 15 and over	
Age group	*Percentage of total*
15 to 24	27.3
25 to 34	21.8
35 to 44	18.5
45 to 54	17.2
55 to 64	8.3
65+	6.9

Estimate the chance that if you met a person aged over 14 who attended the cinema in 2006, they would have been:

- **a** between 45 and 54 years of age
- **b** less than 35 years of age
- **c** over 54 years of age
- **d** between 25 and 54 years of age.

4 637 randomly chosen people were asked how many children were in their family. The results are shown in the table:

- **a** What was the sample size?
- **b** Copy the table and add to it a *Relative frequency* column.
- **c** Estimate the probability that a randomly selected family has:
  - **i** 2 children **ii** less than 3 children
  - **iii** 3 or more children.
- **d** If a city has 5630 families, estimate the number of families with 4 or more children.

Number of children	*Frequency*
0	217
1	218
2	124
3	52
4	17
5	5
6 or more	4

5 Fifty window-sized sheets of glass were carefully examined for observable flaws. The number of flaws per sheet was recorded below:

```
0  1  0  2  1  2  0  3  0  1  0
1  0  1  0  5  1  0  0  1  0  0
1  4  1  1  0  0  0  0  2  1  0
0  1  0  0  0  0  0  1  2  0  0
1  0  0  1  3  0
```

a Complete a table showing the frequency and relative frequency table for each outcome.

b Estimate the probability that a sheet of glass will have:

  i no flaws
  ii 1 flaw
  iii 8 flaws
  iv less than 2 flaws
  v more than 3 flaws.

D LISTING POSSIBLE OUTCOMES

A **sample space** is the set of possible outcomes of an experiment.

Three pieces of equipment which are commonly used in games of chance are coins, dice, and spinners. We use these items because there is an equal chance of their different outcomes occurring on each throw or spin.

COINS

When a **coin** is tossed there are two possible sides which could show upwards: the *head* which is usually the head of a monarch, president or leader, and the *tail* which is the other side of the coin. For simplicity we abbreviate a head as H and a tail as T.

head tail

So, when tossing one coin, the sample space is $\{H, T\}$.

DICE

The most commonly used dice are small cubes with the numbers 1, 2, 3, 4, 5 and 6 marked on them using dots. The numbers on the faces are arranged so that the sum of each pair of opposite faces is seven.

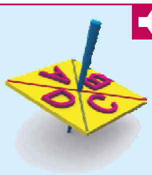
The sample space when rolling one die is $\{1, 2, 3, 4, 5, 6\}$.

SPINNERS

A spinner can be made from a regular polygon or a circle divided into equal sectors, with a needle spinning at the centre.

The sample space for this spinner is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Example 4 **Self Tutor**



a List the possible outcomes for the children in a two child family.

b List the possible outcomes for two spins of this spinner.

a We let B represent a boy and G represent a girl.
The sample space is {BB, BG, GB, GG}.

b We let AB represent a result of A with the first spin and B with the second spin.
The sample space is {AA, AB, AC, AD, BA, BB, BC, BD, CA, CB, CC, CD, DA, DB, DC, DD}.

EXERCISE 19D

1 List the sample spaces for the following:

- **a** flipping a disc with a crown on one side and a diamond on the other
- **b** twirling an equilateral triangle spinner with faces A, B and C
- **c** choosing a month of the year
- **d** tossing a 5-cent and a 10-cent coin
- **e** taking a marble at random from a bag containing red and yellow marbles
- **f** picking a letter of the English alphabet.

2 List the sample spaces for the following:

- **a** the different ways in which 3 students Anna, Barry and Catherine may line up
- **b** tossing 3 different coins simultaneously
- **c** the 8 different orders for the genders of 3 kittens in a litter
- **d** the results when two dice are rolled simultaneously
- **e** the different orders in which 4 alphabet blocks W, X, Y and Z may be placed in a line.

3 Consider a fly landing on the top edge of an open box. It could land with equal chance on any top edge. Explain how you could write down the sample space of possible outcomes.

E THEORETICAL PROBABILITY

We saw earlier in the chapter how the probabilities of some events can be estimated by experiment.

If the outcomes of an experiment are equally likely, we can use symmetry to generate a **mathematical** or **theoretical** probability for an event. This probability is based on what we theoretically expect to occur.

COIN TOSSING

When a coin is tossed there are two possible outcomes. From the symmetry of the coin we expect each of these results to occur 50% of the time, or 1 time in every 2.

We say that the probability of getting a *head* with one toss is $\frac{1}{2}$, and write $P(H) = \frac{1}{2}$.

We read this as "the probability of a head occurring is one half".

Likewise the probability of getting a *tail* is $\frac{1}{2}$, and we write $P(T) = \frac{1}{2}$.

INVESTIGATION 3 TOSSING TWO COINS

In this investigation we find experimental probabilities for the outcomes when two coins are tossed simultaneously.

The results will be recorded as the number of heads showing after each toss.

head

tail

What to do:

1 Copy the table alongside. Record the frequencies you expect for the 3 possible results.

2 Toss two coins 60 times and record your results in the table.

Results	*Expected frequency*	*Actual frequency*
2 heads		
1 head		
0 heads		
Total	60	

3 Did your predicted outcomes differ significantly from your experimental results? If they did differ significantly, explain why they did.

DEMO

4 A simulation of this experiment enables you to repeat this procedure over and over. You can try this by clicking on the icon.

From the **Investigation** you should have observed that there are roughly twice as many *one head* results as there are *no heads* or *two heads*.

The explanation for this is best seen using two different coins. When they are tossed simultaneously, we could get:

two heads

one head

one head

no heads

This explains why the ratio two heads : one head : no heads is 1 : 2 : 1.

We could list the sample space for tossing two coins as {HH, HT, TH, TT}.

So, the theoretical probabilities are:

$$P\{\text{no heads}\} = \tfrac{1}{4} \qquad P\{\text{one head}\} = \tfrac{2}{4} = \tfrac{1}{2} \qquad P\{\text{two heads}\} = \tfrac{1}{4}.$$

ROLLING A DIE

When a die is rolled there are 6 possible equally likely outcomes $\{1, 2, 3, 4, 5, 6\}$ that form our sample space.

The probability of each outcome occurring is $\frac{1}{6}$, and we write for example $P(\text{a } 5) = \frac{1}{6}$.

For some events there may be more than one outcome.

For example:

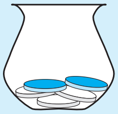
- $P(\text{a 5 or a 6}) = \frac{2}{6}$ since 2 of the 6 outcomes are in the event.
- P(result is *even*)
  $= P(\text{a 2, a 4 or a 6})$
  $= \frac{3}{6}$ since 3 of the 6 outcomes are in the event.

In general, when we are dealing with an event in a sample space containing a finite number of outcomes, then:

$$\textbf{P(an event)} = \frac{\textbf{number of outcomes in that event}}{\textbf{total number of possible outcomes}}.$$

Example 5 Self Tutor

Two blue and three white discs are placed in a bag and one disc is randomly selected from it.
What is the probability of selecting:



a a blue disc **b** a white disc?

There are 5 discs which could be selected with equal chance.

a 2 discs are blue, so there is a 2 in 5 chance of selecting a blue.
$\therefore \ P(\text{a blue}) = \frac{2}{5}$.

b 3 discs are white, so there is a 3 in 5 chance of selecting a white.
$\therefore \ P(\text{a white}) = \frac{3}{5}$.

For a truly random selection, each disc should have the same chance of being selected.

Example 6 Self Tutor

A die has 4 faces painted red and 2 faces painted green. When the die is rolled, what is the chance that the uppermost face is:

a a red **b** a green?

4 faces are red, 2 faces are green, and there are 6 faces in all.

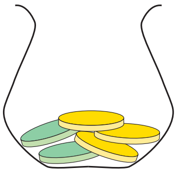
a $P(\text{a red}) = \frac{4}{6}$ ← total red faces ← total outcomes possible

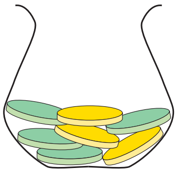
b $P(\text{a green}) = \frac{2}{6}$

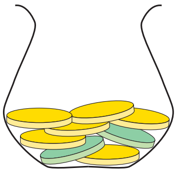
EXERCISE 19E

1 Green and yellow discs are placed in a bag and one disc is randomly selected from it. For the following bags of discs, answer these questions:

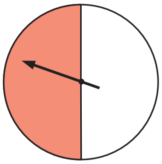
i How many of each disc are there in the bag?
ii What is the probability of selecting a green disc?
iii What is the probability of selecting a yellow disc?

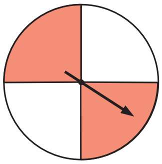
a


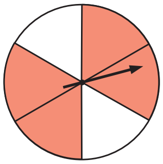
b


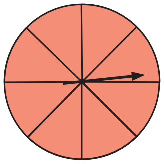
c


2 Determine the probability that the spinning needle will finish on orange:

a


b


c


d


3 The illustrated spinner is a regular octagon.
If the spinner is spun once, find the probability of getting:

a a 6 **b** a 3 or a 4 **c** a 1, 2 or 3
d a result less than 6 **e** a result more than 8.

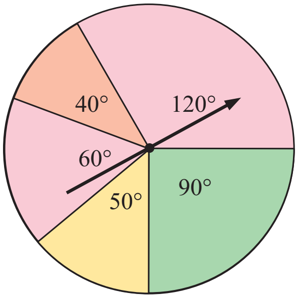
4 A hat contains 4 red, 3 white and 2 grey discs. One disc is randomly selected from it. Determine the likelihood that it is:

a red **b** white **c** grey
d green **e** not red **f** not white
g not grey **h** neither red nor grey **i** red, white or grey.

5 There are 52 cards in a pack of playing cards. They are divided into four suits: the red suits Hearts and Diamonds, and the black suits Spades and Clubs. In each suit there is an ace, the numbers 2 to 10, and three picture cards called the jack, queen and king.
Frank shuffles a pack of cards thoroughly, places them face down on the table, then picks one card at random. Determine the chance of getting:

a a heart ♥ **b** the 7 of ♥
c a club ♣ **d** a black 4
e a black ace **f** a 5 or a 6
g an ace **h** a picture card.

6



The given spinner has sector angles 120^o, 90^o, 50^o, 60^o, and 40^o as shown.

- **a** Are the outcomes of a spin equally likely?
- **b** What outcome do you expect to occur:
  - **i** most often
  - **ii** least often?
- **c** Since we know the angles on each sector, we can calculate theoretical probabilities for the spinner. What is the probability of getting a green?

7 List the sample space when a 5-cent and a 20-cent coin are tossed simultaneously. Let HT represent "a head with the 5-cent coin and a tail with the 20-cent coin". Determine the chance of getting:

- **a** two heads
- **b** two tails
- **c** exactly one head
- **d** at least one head.

8 List the possible two-child families, letting B represent a boy and G represent a girl. If these families occur with equal chance, determine the probability that a randomly selected two-child family consists of:

- **a** two boys
- **b** at least one boy
- **c** children of the same sex.

9 List the 8 possible 3-child families according to sex. One of them is GBB. Assuming that each of them is equally likely to occur, determine the probability that a randomly chosen 3-child family consists of:

- **a** all boys
- **b** all girls
- **c** boy, then girl, then girl
- **d** two girls and a boy
- **e** a girl for the eldest
- **f** at least one boy.

10 Three seats are placed in a row. Three children A, B and C enter the room and sit down randomly, one on each chair. List the sample space of all possible orderings. Hence, determine the probability that:

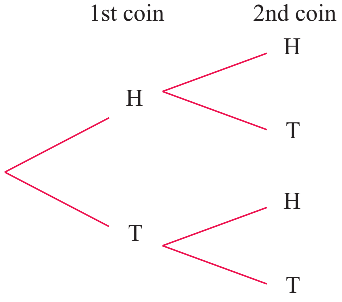
- **a** A sits on the leftmost chair
- **b** they sit in the order BCA from left to right
- **c** C sits in the middle
- **d** B does not sit in the middle.

F TREE DIAGRAMS

We have seen that the four possible results when tossing a pair of coins are: {HH, HT, TH, TT}. This sample space of possible results can also be represented on a tree diagram.

The first set of branches are the possible results from tossing the first coin.

For each of these results we have a second set of branches. These are the results from tossing the second coin.



In this chapter we will only consider situations in which the probability of obtaining each outcome is the same. This means we can determine probabilities using the total number of outcomes. For example, when tossing two coins there are four possible outcomes which are equally likely, so each has probability $\frac{1}{4}$.

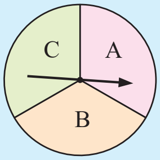
Example 7 **Self Tutor**

A coin is tossed and the spinner alongside is spun simultaneously.

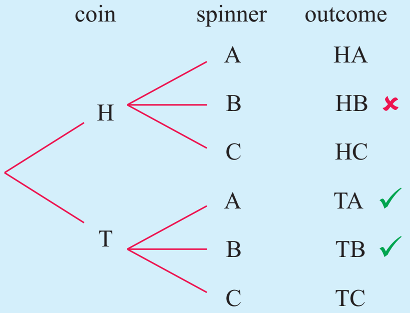
a Illustrate the possible outcomes on a tree diagram.

b Find the probability of getting:

- **i** a head and a B
- **ii** a tail and an A or B



a



b **i** P(H and B) $= \frac{1}{6}$ ✗

ii P(T and an A or B)

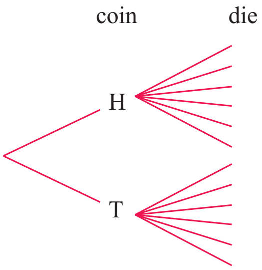
$= $ P(TA or TB) ✓

$= \frac{2}{6}$

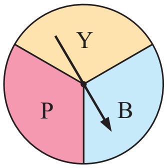
EXERCISE 19F

1 Draw a tree diagram to show the possible outcomes in these situations:

a A coin is tossed and a die is rolled at the same time.

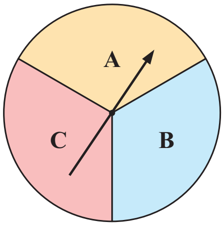


b This disc is spun twice.



c A bag contains some red, blue and green counters. One counter is selected, and the result recorded. The counter is then replaced and another counter is selected.

d Three coins are tossed at the same time.



2 This spinner is spun twice.

a Illustrate the possible outcomes on a tree diagram.

b Find the probability that:

- **i** you get an **A** followed by a **B**
- **ii** you get a **C** followed by a **C**
- **iii** you have at least one **C** in your outcomes
- **iv** you have a **B** and a **C**

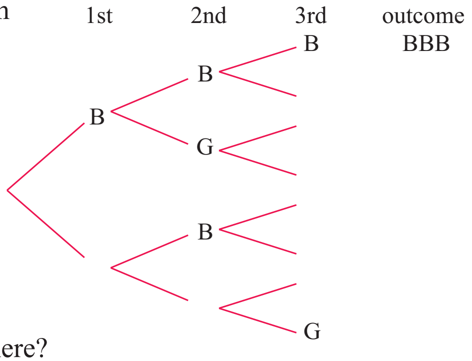
3 A coin is tossed and a spinner like the one in question **2** is twirled.

a Draw a tree diagram to list the possible outcomes.

b Find the probability of getting:

- **i** a head and an **A**
- **ii** a tail and a **C**
- **iii** a head and not a **B**.

4 **a** Copy and complete this tree diagram for the sexes of a 3-child family:



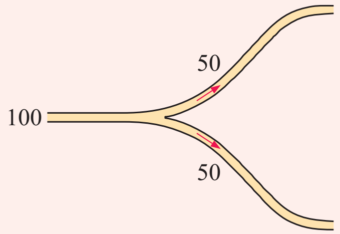
b How many different outcomes are there?

c Assuming the possible outcomes are equally likely, find the probabilities that a randomly selected three-child family consists of:

- **i** 3 boys
- **ii** two girls
- **iii** at least 1 girl
- **iv** no boys
- **v** no more than 2 girls.

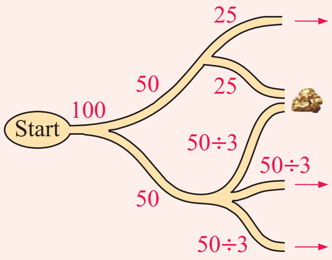
ACTIVITY — YOUR CHANCE OF BEING RICH

When walking along a path that branches in two different directions, we have a 50-50 choice of which branch to take.



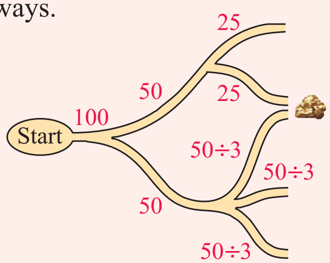
So, if we start with 100%, there is a 50% chance of taking either path. If we come to a junction with three pathways, we divide each probability by 3.

Consider the map alongside. From the start you move along the paths without backtracking. We seek the probability that by choosing paths at random, you will arrive at the gold mine.



We start with 100% and allocate percentages to the pathways.

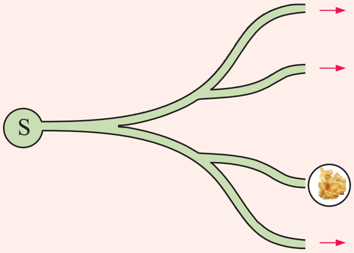
So, the chance of being rich is $(25 + 50 \div 3)\%$
$\approx 41.7\%$



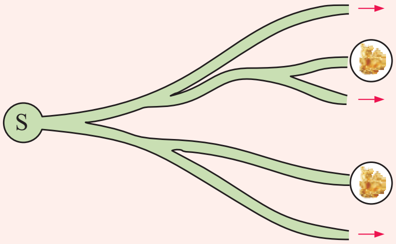
What to do:

1 Find the chance of reaching a gold mine:

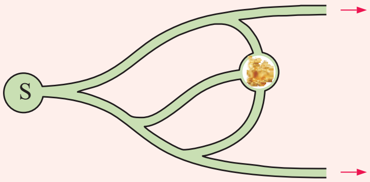
a



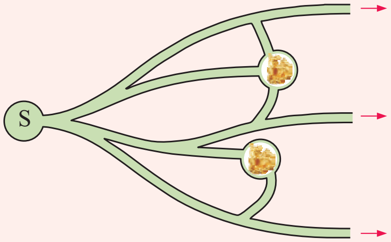
b



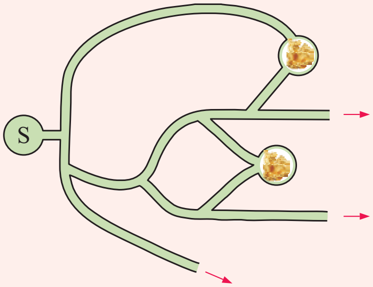
c



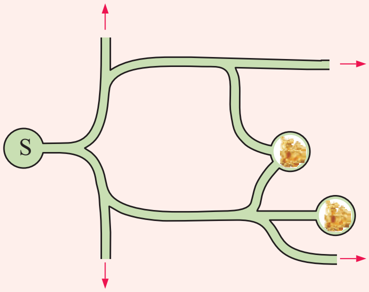
d



e



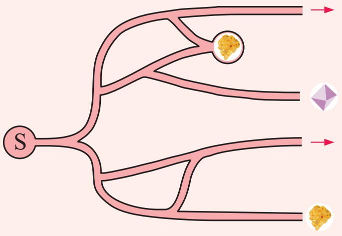
f



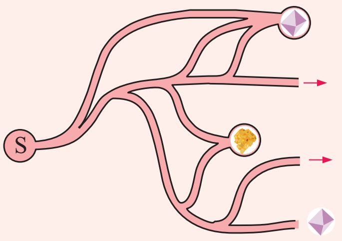
2 For the following pathways, find the probability of discovering:

i the gold mine **ii** the diamond mine **iii** any one of the mines.

a



b



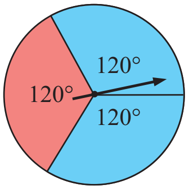
G MAKING PROBABILITY GENERATORS

Suppose you wish to make a device which generates probabilities of $\frac{2}{3}$ and $\frac{1}{3}$. In other words, one event should have $\frac{2}{3}$ chance of occurring and an alternative event should have $\frac{1}{3}$ chance of occurring.

For example, $P(\text{blue}) = \frac{2}{3}$ and $P(\text{red}) = \frac{1}{3}$.

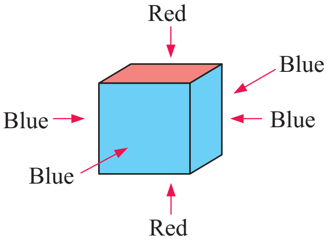
To achieve this we could use a **spinner** or a **die**.

We divide the spinner into three equal sectors. We colour two of them blue and the third red.



120^o is $\frac{1}{3}$ of 360^o.
240^o is $\frac{2}{3}$ of 360^o.

A die has 6 faces so we need to notice that $\frac{2}{3} = \frac{4}{6}$ and $\frac{1}{3} = \frac{2}{6}$.



4 faces are blue.
2 faces are red.

EXERCISE 19G

1. Design *two* devices which generate $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{2}$.
2. Design a device which generates $P(\text{red}) = \frac{2}{5}$ and $P(\text{blue}) = \frac{3}{5}$.
3. Design *two* devices which generate $P(A) = \frac{1}{6}$, $P(B) = \frac{2}{6}$ and $P(C) = \frac{3}{6}$.
4. Draw a circular dart board with three colours so that the chance of scoring:
   - **a** red is $\frac{1}{2}$ and white is $\frac{1}{4}$
   - **b** red is $\frac{1}{2}$ and white is $\frac{1}{3}$
   - **c** red is $\frac{1}{4}$ and white is $\frac{1}{3}$
   - **d** red is 25% and white is 30%
   - **e** red is 0 and white is $\frac{2}{3}$.
5. Suppose there are twenty marbles in a hat and one is to be drawn out. The marbles may be red, blue or white. Find how many red marbles must be in the hat so that the chance of drawing out a red marble is:
   - **a** 50%
   - **b** $\frac{3}{4}$
   - **c** $\frac{1}{20}$
   - **d** $\frac{7}{20}$
   - **e** 0
   - **f** 1

KEY WORDS USED IN THIS CHAPTER

- chance
- die
- probability line
- tree diagram
- consequence
- outcome
- random selection
- dice
- probability
- spinner

REVIEW SET 19A

1 Draw a probability line and mark on it the approximate probabilities of:

- **a** being born on a weekday
- **b** the next person you see being female
- **c** snow falling in Dubai.

2 The number of sweets in each of 25 packets was recorded below. The results were:

41 42 39 40 42 38 41 42 41 40 39 40 40
40 39 38 39 41 39 38 40 42 40 41 39

- **a** Complete a frequency and relative frequency table for this information.
- **b** What is the probability that a randomly selected packet will contain:
  - **i** 40 sweets **ii** less than 40 sweets **iii** at least 40 sweets?

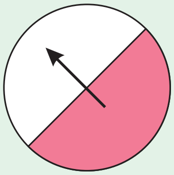
3 List the 8 different possible three-child families. Determine the probability that a randomly selected three-child family consists of 2 boys and a girl.

4 A bag contains 5 red, 4 blue and 3 yellow discs. If one disc is randomly selected from it, determine the chance that it is:

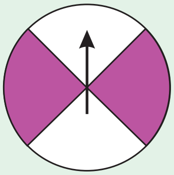
- **a** red **b** blue **c** not yellow
- **d** red or yellow **e** neither red nor blue.

5 Determine the probability that the spinning needle will finish on white:

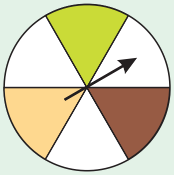
a



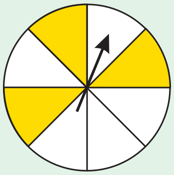
b



c



d



6 A pack of 52 cards is well shuffled and a card is selected at random. Determine the chance that this card is:

- **a** a spade **b** a red 3 **c** a picture card **d** a 7 or an 8

7 The numbers 1 to 50 are marked on separate cards and placed in a hat. Determine the probability that the number on a randomly chosen card is closer to 40 than it is to 15.

8 Design a device which would generate $\text{P(blue)} = \frac{1}{4}$ and $\text{P(yellow)} = \frac{3}{4}$.

9 Draw a circular dart board with three colours where the chance of scoring red is $\frac{1}{4}$ and of scoring white is $\frac{2}{3}$.

REVIEW SET 19B

1 State whether the following events are equally likely or not:

- **a** selecting a red card or a black card from a deck of 52 playing cards
- **b** getting an odd result or an even result when a die is rolled

c getting 1, 2 or 3 heads when three coins are tossed

d selecting the premiership team in a hockey competition.

2 A group of people are surveyed to determine the number of cups of coffee they consume per day. The results are listed below:

0 3 2 4 3 1 0 2 0 6 3 2 0 4 1 5 3 4 2 0
0 8 2 0 4 3 5 2 0 3 4 2 1 5 0 3 0 5 1 0

a Complete a frequency and relative frequency table for this information.

b What is the probability that a randomly selected person consumed:

i 0 cups **ii** 4 cups **iii** 8 cups?

3 The numbers 1 to 30 are marked on separate cards and placed in a hat. Determine the probability that a randomly chosen card is:

a a 7 **b** a multiple of 5 **c** an even number

4 List the sample space when tossing a 5-cent, a 10-cent, and a 20-cent coin simultaneously. Let HTH represent 5-cent head, 10-cent tail, 20-cent head. Hence determine the likelihood of tossing:

a 3 heads **b** 2 heads and 1 tail

c 1 head and 2 tails **d** 3 tails.

5 When a metal nut was tossed 500 times it finished on its edge 156 times and on a flat side for the rest.

Result	*Frequency*	*Rel. frequency*
Edge		
Flat		
Total	500	

a Copy and complete the table:

b Estimate the chance of a tossed nut finishing on a flat side.

c If the nut was tossed one million times, estimate of the number of times it would finish on an edge.

6 Suppose there are twenty marbles in a bag. They could be black, white or red. How many black marbles are in the bag if the chance of drawing out a black marble is:

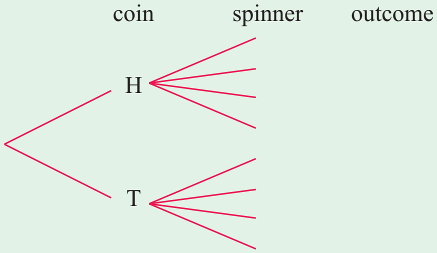
a 25% **b** $\frac{11}{20}$ **c** 1 **d** 0?

7 **a** Complete the tree diagram to show the possible outcomes when a coin is tossed and a spinner with equal sectors labelled A, B, C and D is spun.

b Find the probability of getting:

i a head and a D

ii a tail and a C.



8 Draw a circular dart board with three colours where the chance of scoring red is 40% and of scoring white is 25%.

9 Design a device which generates the probabilities: $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{6}$.

Chapter 20

Statistics

Contents:

We are bombarded with facts and figures each day.

For example:

- Josua averages 14.6 points per game.
- Vanessa scores 68% for her exam.
- If an election was held tomorrow the two major parties would get 89% of the vote.
- Yesterday was the coldest summer day since February 4th 1897.

Some statements, particularly those about weather, cannot be made without gathering a large amount of information over a long period of time. It is very important that this information is collected accurately.

The facts or pieces of information are called **data**. An individual piece of information is one **datum**.

Data may be collected by counting, measuring or asking questions.

If we collect the weights of students in our school to the nearest kg, we will generate a list of numbers such as 53, 57, 69, 63, 48, 56, 56, 43, 57, 57, ...

This number list is called a **data set**. It is not organised so it is called **raw data**.

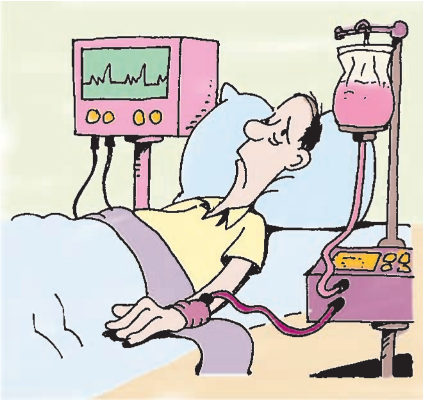
Statistics is the art of solving problems and answering questions by collecting and analysing data.

Statistics are used by governments, businesses, sports organisations, manufacturers, and scientific researchers.

For example, a medical researcher may believe that a newly discovered drug could prolong the life of heart attack patients.

To prove this, the drug would have to be given to a group of heart attack sufferers. Data would then be gathered to compare their quality of life and life expectancy with another group where the drug was not given. If the drug is found to improve life expectancy and quality of life with no bad side effects, it will likely become a legal drug.

In statistical work we use **tables**, **graphs** and **diagrams** to represent data.



The process of **statistical enquiry** or **investigation** includes the following steps:

Step 1: Examine a problem which may be solved using data. Pose the correct questions.
Step 2: Collect data.
Step 3: Organise the data.
Step 4: Summarise and display the data.
Step 5: Analyse the data and make a conclusion.
Step 6: Write a report.

HISTORICAL NOTE

- Before 3000 BC, the **Babylonians** recorded yields for their crops on small clay tablets.
- Pharoahs in ancient **Egypt** recorded their wealth on walls of stone.
- Censuses were conducted by the **Ancient Greeks** so that taxes could be collected.
- After **William the Conqueror** invaded and conquered England in 1066, his followers overtook estates previously occupied by Saxons. Confusion reigned over who owned what.

  In 1086 William ordered that a census be conducted to record population, wealth, and land ownership. A person's wealth was recorded in terms of land, animals, farm implements, and the number of peasants on the estate. All this information was collated and has become known as the **Domesday Book**. It is regarded as the greatest public record of Medieval Europe.

  The Domesday Book is displayed in the National Archives in Kew.

OPENING PROBLEM

A city school can be easily accessed by train or school bus, as well as by walking or riding in a private car. The school is interested in finding out how students travel to and from school as there are traffic problems in the area during these times.

An initial survey of 75 students was carried out. The results were:

T T B W W T W C W C C C B C B W T W C C W C C B B
B C B B W W B T B B W B C B C C W B W T T C B B T
B W T C B B C C C C W C W T B T T T C C C C C T W

where T = by train, B = by school bus, C = by car, and W = by walking.

Things to think about:

- 859 students attend the school. How were the students selected for the survey? Would the survey be a fair representation of the whole school if the students were only selected from Year 12?
- Is the sample large enough to reflect the method of travel for all students in the school?
- How could we best organise this raw data?
- How could we display this information in a graph?
- What calculations could we perform on the organised data to make it more meaningful?
- What conclusions could we make and report to the school?

A DATA COLLECTION

When a statistical investigation is to be conducted there is always a target **population** about which information is required.

The population might be the entire population of the country, all the students at a school, an entire animal species, or the complete output of a machine making a particular item.

CENSUS OR SAMPLE

One of the first decisions to be made is from whom or what we will collect data. We can collect data using either a census or a sample.

A **census** involves collecting data about every individual in the *whole population*.

The individuals may be people or objects. A census is detailed and accurate but is expensive, time consuming, and often impractical.

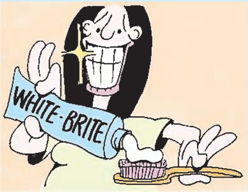
A **sample** involves collecting data about a *part of the population* only.

A sample is cheaper and quicker than a census but is not as detailed or as accurate. Conclusions drawn from samples always involve some error.

Example 1 **Self Tutor**

Would a census or sample be used to investigate:

- **a** the length of time an electric light globe will last
- **b** the causes of car accidents in a particular state
- **c** the number of people who use White-brite toothpaste?



- **a** *Sample*. It is impractical to test every light globe produced as there would be none left for sale!
- **b** *Census*. An accurate analysis of all accidents would be required.
- **c** *Sample*. It would be very time consuming to interview the whole population to find out who uses or does not use White-brite toothpaste.

BIAS IN SAMPLING

The most common way of collecting information is by using a sample. For the sample to be of use, it must accurately reflect the characteristics of the whole population. The challenge in selecting a sample is to make it as free from prejudice or **bias** as possible, and large enough to be representative of the whole population.

A **biased sample** is one in which the data has been unfairly influenced by the collection process. It is not truly representative of the whole population.

Example 2 **Self Tutor**

Suggest a possible bias in each of the following samples:

- **a** a phone survey during the day
- **b** a survey of people on a train station
- **c** a survey of a football crowd

a The sample would be biased towards people who are at home during the day. It would not include people who go out to work.

b The sample would be biased towards people who catch the train. It would not include people who use other forms of transport or work at home.

c The sample would be biased towards people who attend football matches. For example, there would probably be more males than females attending football matches.

Sometimes people use **biased samples** to enhance their claims for their products or to support a particular point of view.

For example, a person wanting the local council to upgrade its swimming pool might sample the views of swimmers who currently use the pool.

You would expect the people who use the pool to be biased very favourably towards the proposal, so the person taking the sample could be accused of producing an unfair or biased report.

EXERCISE 20A

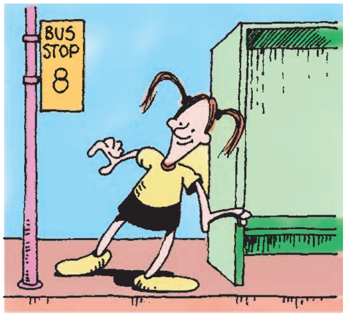
1 State whether a census or a sample would be used for each of these investigations:

- **a** the number of goals scored each week by a netball team
- **b** the heights of the members of a football team
- **c** the most popular radio station
- **d** the number of children in a family
- **e** the number of loaves of bread bought each week by a family
- **f** the pets owned by students in a given Year 7 class
- **g** the number of leaves on the stems of plants

2 Give three examples of data which would be collected using a:

- **a** census
- **b** sample

3 Explain and discuss any possible bias in the following samples:
- **a** a phone survey on a Saturday night
- **b** a survey of people at a bus stop
- **c** a survey of people in a supermarket carpark
- **d** a survey of people at the beach
- **e** a survey of people in your street



4 Comment on any possible bias in the following situations:
- **a** Year 7 students are interviewed about changes to the school uniform.
- **b** Motorists stopped in peak hour are interviewed about traffic problems.
- **c** Real estate agents are interviewed about the prices of houses.
- **d** Politicians are interviewed about the state of the country's economy.
- **e** People are asked to phone in to register their vote on an issue.
- **f** An opinion poll is conducted by posting a questionnaire to people.
- **g** An advertisement claims that "Dog breeders recommend Puppy Meats dog food."

B CATEGORICAL DATA

Categorical data is data which can be placed in categories.

For example, the data for the *method of transport* in the **Opening Problem** is categorical data.

The *method of transport* is the independent variable, and the categories are:

by train (T), by school bus (B), by car (C), walking (W).

We can organise categorical data using a **tally and frequency table**.

Method of transport	*Tally*	*Frequency*
by train	卌 卌 ||||	14
by school bus	卌 卌 卌 卌	20
by car	卌 卌 卌 卌 卌	25
by walking	卌 卌 卌 |	16
Total		75

From this table we can identify features of the data. For example:

- The most favoured method of travel is the car.
- $\frac{20}{75} \times 100\% \approx 27\%$ of students travel to school by bus.

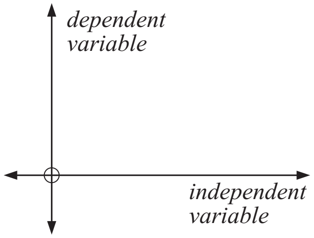
The **mode** is the most frequently occurring category.

GRAPHS TO DISPLAY CATEGORICAL DATA

We have seen in previous chapters that when we compare the two quantities or variables, one variable is *dependent* on the other.

Generally, when drawing **graphs** involving two variables, the *independent variable* is on the horizontal axis and the *dependent variable* is on the **vertical axis**.

An exception to this is when we draw a **horizontal bar chart**.



DISCUSSION

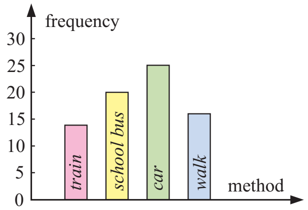
- Discuss the following sentences and identify the **dependent** and **independent** variables:
  'The number of hours worked by a plumber affects the total charge.'
  'The amount received by each person in a Lottery syndicate is linked to the number of people in the syndicate.'
  'The diameter of a circular table top determines its area.'
- Discuss and write down *two* sentences which contain variables. In each case identify the dependent variable and the independent variable.

Categorical data may be displayed using:

- a vertical bar chart or column graph
- a pie chart
- a horizontal bar chart
- a segmented bar chart.

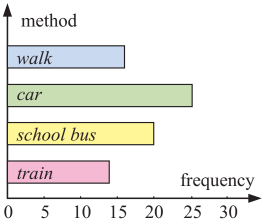
For the **Opening Problem** data of the method of transport to school, these graphs are:

Vertical column graph



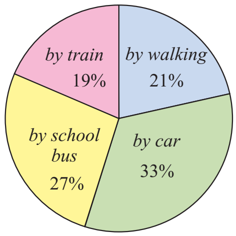
The heights of the columns indicate the frequencies.

Horizontal bar chart



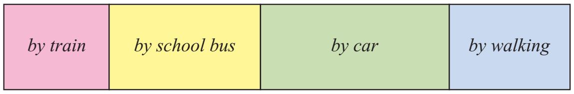
The lengths of the bars indicate the frequencies.

Pie chart



The angles at the centre indicate the frequencies.

Segmented bar chart



The lengths of the segments indicate the frequencies.

This segmented bar chart is 75 mm long because there were 75 students in the sample. Each student contributes 1 mm to the length of a segment.

USING HAESE & HARRIS SOFTWARE

Click on the icon to load a statistical package which can be used to draw a variety of statistical graphs.

STATISTICS PACKAGE

Change to a different graph by clicking on a different icon. See how easy it is to change the labels on the axes and the title of the graph.

Type into the correct cells the information given on horse colour.

Colour	*Frequency*
grey	15
bay	27
chestnut	8
palomino	2
brown	3

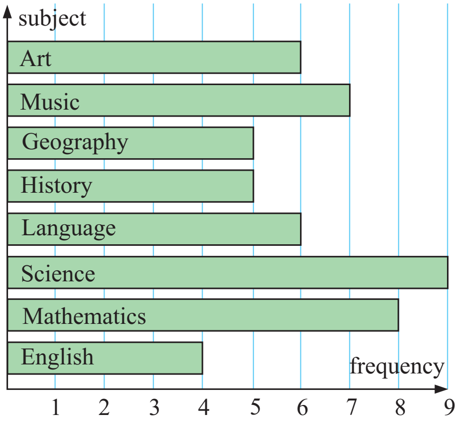
Print off graphs of the data including a column graph, a horizontal bar chart, a pie chart, and a segmented bar chart.

Use the software or a spreadsheet to reproduce some of the statistical graphs in the remaining part of this chapter. You can also use this software in any statistical project you may be required to do.

EXERCISE 20B

1 50 randomly selected students were asked to name their favourite subject at school. The results of the survey are displayed in the graph.

- **a** What sort of graph is being used?
- **b** Which was the most favoured subject?
- **c** How many students chose Art as their favourite subject?
- **d** What percentage of the students named Mathematics as their favourite subject?
- **e** What percentage of the students chose either History or English as their favourite subject?



2 In an English middle school, 80 students were asked to name their favourite fruit. The following data was collected:

- **a** Construct a vertical column graph to illustrate this data.
- **b** For this group of students, which was the most favoured fruit?
- **c** Can we make conclusions about the favourite fruit of all middle school students from this survey? Give a reason for your answer.

Type of fruit	*Frequency*
Apple	20
Banana	24
Grapes	3
Orange	11
Mandarin	10
Nectarine	7
Peach	2
Pear	3

3 A randomly selected sample of adults was asked to name the evening television news service that they watched. The following results were obtained:

News service	*Frequency*
BBC	40
CNN	45
NBC	64
Sky	25
ABC	23
None	3

a How many adults were surveyed?

b Which news service is the most popular?

c What percentage of those surveyed watched the most popular news service?

d What percentage of those surveyed watched CNN?

e Draw a horizontal bar chart to display the data.

4 100 randomly selected students at an international school were asked to indicate the *continent of origin of their father*. The data collected has been organised into the following frequency table.

a Display this data using a segmented bar chart. Start with a bar 10 cm long. Make sure you use a legend or mark the segments clearly. Include an appropriate heading for your chart.

b What percentage of the sample had fathers who were born outside Europe?

Continent of origin of father	*Frequency*
Africa	4
Asia	9
Australia	3
Europe	48
North America	25
South America	11
Total	100

5 A survey of eye colour in a class of 30 teenagers revealed the following results:

Eye colour	*Blue*	*Brown*	*Green*	*Grey*
Number of students	9	12	2	7

a Illustrate these results on a pie chart.

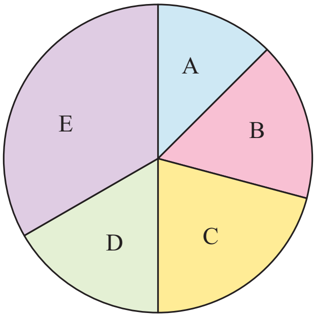
b What percentage of the group have:

  i green eyes **ii** blue or grey eyes?

6 A business has a major store in five cities. We will call the stores A, B, C, D and E. The pie chart shows the proportion of sales for each store.

a Use your protractor to measure the size of each sector angle.

b If the total sales from all stores is €72 million, find the sales made by each of the stores.

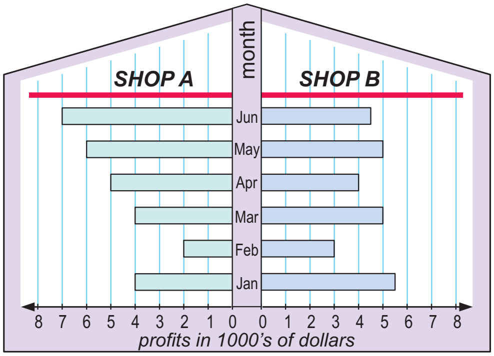


7 On a pie chart a sector has an angle of 30^o and this represents 171 people.

a How many people are represented by the whole chart?

b How many people would be represented by a sector of size 72^o?

8 The graph alongside compares the profits of two pizza shops from the same chain of stores. Shop A undertook extensive advertising during this six month period.



a In what month were the most profits for each shop?

b What were the profits for each shop during May?

c What features of the graph indicate the effectiveness of the advertising?

d Find the total profit for each shop over the 6 month period.

Back-to-back bar graphs like this are often used to compare two sets of data.

DISCUSSION WRITING REPORTS

So far for the **Opening Problem** we have seen how to:

- take an unbiased sample
- collect and organise the data
- summarise and display the data.

We are now left with the task of analysing the data and writing a report which states our conclusions.

When we compare the number or percentage in each category, we look for the **mode**, or category which occurs most frequently.

For the **Opening Problem**, we can write the methods of transport in order from most to least popular: by car (33%), by school bus (27%), by walking (21%), and by train (19%). The mode is '*travelling by car*'.

In writing a conclusion we must be careful not to overstate the situation.

Discuss which of these statements you think would be acceptable:

Statement 1: In conclusion, the students of schools in this city are highly likely to travel to school by car or bus.

Statement 2: In conclusion, the data suggests that students from this school are more likely to travel to school by car or bus than other modes of transport.

When you write a report of your findings, it is important to discuss how your data:

- was collected properly with *no bias*
- was in sufficient quantity to accurately reflect the whole population.

Discuss why it is important to include these things in your report.

RESEARCH A STATISTICAL STUDY: 'RED ALERT'

Your task is to test the theory that there are more red sweets produced than any of the other colours. You should choose a brand of coloured candy-coated chocolates or similar sweets, but make sure everyone in your class has a packet of the same brand.

What to do:

A complete statistical analysis of the given problem is required. Each member of the class is expected to hand up his or her own report which must include all *six* steps of the statistical method.

1 Describe exactly what the problem is in your own words.

2 **a** For your box of sweets, count the number of each colour and construct a frequency table of your results.

b Which colour occurs most frequently in your sample? Do your results support the theory that there are more red sweets produced?

c Combine your results with another person in your class and construct a frequency table of the combined results. Do these combined results support the theory?

3 **a** As a class exercise, combine the data of every student in the class. Do these results support the theory?

b For the combined class results, calculate the percentage of red sweets. What would you expect this percentage to be if equal numbers of all the colours are produced?

c Which of the previous results (yours, yours and your friend's, or the whole class) should give *the best* data to test the theory? Give a reason for your answer.

4 Use a **spreadsheet** to draw a vertical column graph of the data from:

a your results **b** your results and another person's

c the combined class.

5 What is the mode for:

a your results **b** your results and another person's

c the combined class results? Write out carefully your conclusion.

6 Submit your completed report to your teacher for assessment.

C NUMERICAL DATA

Numerical data is data which is given in number form.

Numerical data can be organised using a **stem-and-leaf plot** or a **frequency table**, and represented graphically using a **column graph**.

STEM-AND-LEAF PLOTS

A stem-and-leaf plot displays a set of data in order of size.

For example, the scores out of 150 for students in an exam were:

116 97 132 124 108 73 111 140 123 102 84 128 91 105 113
131 126 97 106 119 61 125 130 104 98 114 121 106 127 92

For each data value, the units digit is used as the **leaf**, and the digits before it determine the **stem** on which the leaf is placed.

So, the stem labels are 6, 7, 8, 9, 10, 11, 12, 13, 14, and they are written under one another in ascending order.

We now look at each data value in turn. We remove the last digit to find the stem, then write the last digit as a leaf in the appropriate row.

Once we have done this for all data values, we have an **unordered stem-and-leaf plot**. We can then **order** the stem-and-leaf plot by writing each set of leaves in ascending order.

Unordered stem-and-leaf plot of exam scores

6	1	
7	3	
8	4	
9	7 1 7 8 2	
10	8 2 5 6 4 6	
11	6 1 3 9 4	
12	4 3 8 6 5 1 7	
13	2 1 0	
14	0	unit = 1

Ordered stem-and-leaf display of exam scores

6	1
7	3
8	4
9	1 2 7 7 8
10	2 4 5 6 6 8
11	1 3 4 6 9
12	1 3 4 5 6 7 8
13	0 1 2
14	0

Notice that:

- 9|1 2 7 7 8 represents the scores 91, 92, 97, 97 and 98.
- The scale (unit = 1) tells us the place value of each leaf.
  If the scale was 'unit = 0.1' then 9|1 2 7 7 8 would represent 9.1, 9.2, 9.7, 9.7, 9.8.

Example 3 **Self Tutor**

The birth weights of 25 alpacas were recorded to the nearest 0.1 kg. The weights were:

6.1 9.8 6.7 8.1 5.6 6.4 7.5 8.6 8.5 7.2
9.5 6.8 8.9 7.3 6.8 7.7 9.3 9.0 8.4 7.6
8.2 8.5 7.9 8.3 9.5

a Draw a stem-and-leaf plot of the data.

b What was the
i lightest **ii** heaviest weight recorded?

a The whole number of kilograms range from 5 to 9 kg. These are the stem values.

Unordered stem-and-leaf plot of alpaca birth weight data

5	6
6	1 7 4 8 8
7	5 2 3 7 6 9
8	1 6 5 9 4 2 5 3
9	8 5 3 0 5

unit = 0.1 kg

Ordered stem-and-leaf plot of alpaca birth weight data

5	6
6	1 4 7 8 8
7	2 3 5 6 7 9
8	1 2 3 4 5 5 6 9
9	0 3 5 5 8

unit = 0.1 kg

b **i** The lightest weight was 5.6 kg. **ii** The heaviest weight was 9.8 kg.

EXERCISE 20C.1

1 Kelvin picked watermelons to take to market. To the nearest 0.1 kg they weighed:

7.7, 5.6, 4.7, 9.1, 6.9, 5.5, 7.6, 8.0, 6.4, 5.3, 4.8, 7.2, 7.3, 8.2, 7.8,
4.5, 8.7, 8.2, 6.9, 6.1, 5.6, 8.0, 5.0, 7.7, 5.8, 7.3, 6.4, 7.0, 6.5, 6.9

- **a** Construct an unordered stem-and-leaf plot of the data. Make sure you have a title and a scale.
- **b** Construct an ordered stem-and-leaf plot of the data.
- **c** How many watermelons went to market?
- **d** Find the weight of the **i** lightest **ii** heaviest watermelon taken to market.

2 Piotr performed a reaction test several times. The results were then used to construct the stem-and-leaf plot shown alongside.

1	9
2	2 4 6 7 8
3	3 4 5 5 7 8 9 9 9
4	1 5 7 9
5	2

unit = 0.01 seconds

- **a** What was his fastest reaction time?
- **b** What was his slowest reaction time?
- **c** How many times was the test performed?
- **d** What reaction time occurred most often?

3 The weights of a flock of newborn lambs were recorded to the nearest 0.1 kg as follows:

3.9, 4.9, 5.0, 2.9, 3.1, 3.6, 4.2, 4.8, 5.5, 2.8, 3.3, 4.7, 5.1, 2.6, 3.6,
4.9, 4.7, 3.5, 3.3, 5.4, 4.8, 3.0, 3.3, 4.1, 4.4, 5.0, 4.2, 3.1, 5.2, 3.8

- **a** Construct a stem-and-leaf plot of the data.
- **b** What scale did you use?
- **c** What percentage of weights were less than 3.2 kg?
- **d** What percentage of weights were at least 4 kg?

4 At practice a golfer hit 39 balls with a 7-iron. The distances recorded in metres were:

140, 163, 129, 138, 144, 144, 158, 167, 173, 141, 139, 152, 167,
170, 132, 138, 165, 134, 153, 152, 127, 176, 156, 155, 165, 159,
149, 145, 159, 148, 144, 152, 141, 137, 147, 146, 143, 145, 149

- **a** Construct a stem-and-leaf plot of the data.
- **b** What was the shortest distance hit?
- **c** What percentage of shots were less than 150 m long?
- **d** What percentage of shots were between 140 m and 160 m?

5 Sonya's best 24 long jump results for the athletics season were, in metres:

6.19, 6.32, 6.52, 6.35, 6.41, 6.49, 6.49, 6.39, 6.23, 6.55, 6.27, 6.46,
6.30, 6.38, 6.40, 6.47, 6.61, 6.21, 6.41, 6.49, 6.35, 6.44, 6.48, 6.38.

- **a** Construct an ordered stem-and-leaf plot of the data.
- **b** What percentage of her jumps were beyond 6.4 m?

FREQUENCY TABLES AND COLUMN GRAPHS

A common method of counting data is to use a **tally-and-frequency** or **frequency table**.

Example 4 **Self Tutor**

A tennis player has won the following number of matches in tournaments during the last 18 months:

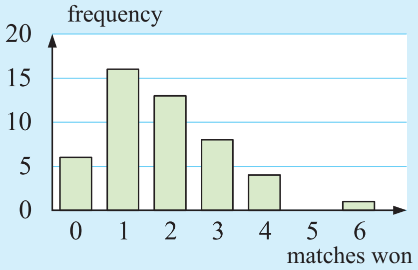
1 2 0 1 3 1 4 2 1 2 3 4 0 0 1 2 2 3 2 1 6 3 2 1
1 1 1 2 2 0 3 4 1 1 2 3 0 2 3 1 4 1 2 0 3 1 2 1

- **a** Organise the data to form a frequency table.
- **b** Draw a frequency column graph of the data.
- **c** How many times did the player advance past the second match of a tournament?
- **d** On what percentage of occasions did the player win less than 2 matches?

a

Wins	*Tally*	*Frequency*
0	𝍸 |	6
1	𝍸 𝍸 𝍸 |	16
2	𝍸 𝍸 |||	13
3	𝍸 |||	8
4	||||	4
5		0
6	|	1
	Total	48

b **Tennis matches won**



c $13 + 8 + 4 + 1 = 26$ times

d The player won less than 2 matches on $6 + 16 = 22$ occasions.

$$\therefore \quad \text{percentage} = \tfrac{22}{48} \times 100\% \approx 45.8\%$$

EXERCISE 20C.2

1 Jed breeds cats. The number of kittens in each litter last year were:

4 3 5 6 2 4 5 6 3 4 2 5 4 6 4 5 5 4 3 3 4 5 4 2 6 4 4 5 4 5

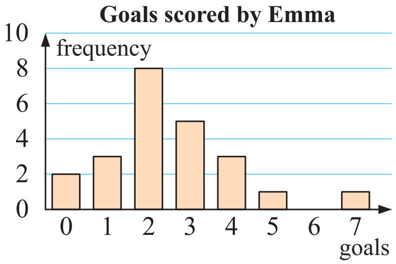
- **a** Construct a frequency table for the kitten data.
- **b** Draw a frequency column graph to illustrate the data.
- **c** How many litters contained 4 or more kittens?
- **d** What percentage of litters had fewer than 4 kittens?

2 The members of a girls' debating club had ages:

15, 14, 12, 13, 11, 13, 15, 12, 15, 13, 14, 16, 13, 12, 14, 15, 13, 17, 12, 13.

- **a** Draw a frequency column graph of the data.
- **b** What percentage of the girls were aged:
  - **i** 14
  - **ii** less than 16
  - **iii** more than 13
  - **iv** between 13 and 16?

3 During the 2008 season Emma was very successful at scoring goals for her field hockey team. In fact on one occasion she scored 7 goals in a match. Her goal scoring results are illustrated on the graph.



- **a** How many games did Emma play?
- **b** On how many occasions did she score 4 or more goals?
- **c** In what percentage of games did she score:
  - **i** less than 2 goals
  - **ii** between 2 and 5 goals inclusive
  - **iii** more than 4 goals?

4 The tail lengths of a sample of lobster were measured to the nearest centimetre. The results in centimetres were:

15 19 18 14 21 17 19 17 18 15 17 16 18 16 17
17 17 18 15 17 17 20 15 19 17 15 16 19 15 17

- **a** Construct a tally-frequency table of the data.
- **b** Construct a frequency column graph of the data.
- **c** What was the sample size?
- **d** What percentage of lengths were:
  - **i** 18 or more centimetres
  - **ii** between 16 cm and 19 cm inclusive?

D THE MEAN, MEDIAN AND MODE

There are three different numbers which are commonly used to measure the **middle** or **centre** of a set of numerical data. These are the **mean** or **average**, the **median**, and the **mode**.

THE MEAN

The **mean** or **average** is the total of all data values divided by the number of data values.

We use the symbol $\overline{x}$ to represent the mean.

$$\overline{x} = \frac{\textbf{sum of data values}}{\textbf{number of data values}}$$

THE MEDIAN

The **median** of a set of data is the middle value of the ordered set.

For an *odd number* of data values there is one middle value which is the median.

For an *even number* of data values there are two middle values. The median is the average of these two values.

THE MODE

The **mode** is the score which occurs most often in a data set.

For example, the mode of the data set 0, 2, 3, 3, 4, 5, 5, 5, 6, 7, 9 is 5 since 5 occurs the most frequently.

Example 5 **Self Tutor**

In the last 7 matches, a basketballer has scored 19, 23, 16, 11, 22, 27 and 29 points.

a Find the mean score of these matches.

b Find the median score for these matches.

c In the next game the basketballer scores 45 points.
Find his new **i** mean **ii** median.

a The mean,
$$\begin{aligned}\overline{x} &= \frac{\text{total of all data values}}{\text{number of data values}}\\ &= \frac{19+23+16+11+22+27+29}{7}\\ &= \frac{147}{7}\\ &= 21 \text{ points}\end{aligned}$$

b In order, the scores are: ~~11 16 19~~ 22 ~~23 27 29~~

$\therefore$ the median $= 22$ points {the middle score of the ordered set}

c **i** The new mean $= \dfrac{147+45}{8} = \dfrac{192}{8} = 24$ points.

ii The ordered set is now: ~~11 16 19~~ 22 23 ~~27 29 45~~

$$\begin{aligned}\therefore \text{ the median } &= \frac{22+23}{2}\\ &= 22.5 \text{ points}\end{aligned}$$ {average of the middle scores}

DISCUSSION

In the basketball example above:

- Which of the mean or median is more affected by the addition of the large score of 45?
- Why is the mode useless as a measure of the centre of this set of data?

Example 6 **Self Tutor**

An exceptional footballer scores the following numbers of goals for her school during a season:

1 3 2 0 4 2 1 4 2 3 0 3 3 2 2 5 2 3 1 2

Find the: **a** mean **b** median **c** mode for the number of goals she scored.

a $$\text{mean} = \frac{\text{sum of all scores}}{\text{number of matches}} = \frac{45}{20} = 2.25 \text{ goals}$$

b The ordered data set is:

$$0\,0\,1\,1\,1\,2\,2\,2\,2\,\underbrace{2\,2}\,2\,3\,3\,3\,3\,3\,4\,4\,5$$

$$\therefore \quad \text{median} = \frac{2+2}{2} = 2 \text{ goals}$$

c mode = 2 goals {2 occurs most often}

EXERCISE 20D

1 Find the mean of:

a 1, 2, 3, 4, 5, 6, 7 **b** 7, 8, 0, 3, 0, 6, 0, 11, 1

2 Find the median of:

a 3, 2, 2, 5, 4, 4, 3, 2, 6, 4, 5, 4, 1 **b** 7, 11, 4, 8, 6, 9, 8, 8, 13

3 Consider the data set: 7, 8, 0, 3, 0, 6, 0, 11, 1.

a Find the **i** mean **ii** median **iii** mode of this data.

b Which is the best measure of the 'middle' of this data set? Explain your answer.

4 Hemi snorkels for abalone each day for a fortnight.
His total catches for each day were:

1 2 0 1 3 2 3 0 2 0 1 4 1 2

For these catches find the:

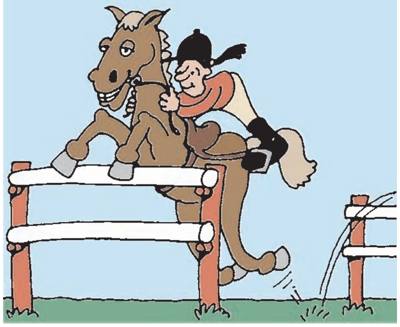
a mean **b** median **c** mode

5 During the showjumping season, Edwina and her horse Lightning had the following numbers of penalties in successive competitions:

0 5 0 8 4 0 0 4 4 4 12 8 4 4 5 0

For these results find the:

a mean **b** median **c** mode.



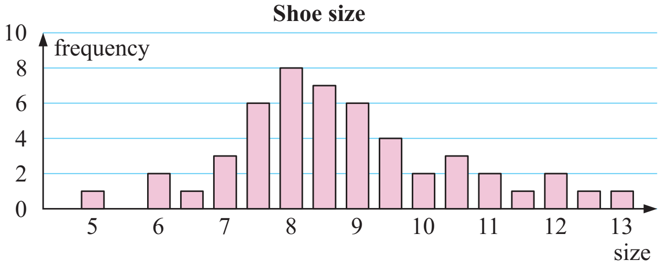
6 Surefix paper clips come in small boxes. The manufacturer claims that the average contents is 80. When 50 boxes were sampled, the following numbers of clips were counted in each box:

80 78 79 79 77 80 82 79 80 78 79 80 80 79 81 79 79
80 81 80 78 78 80 79 80 80 81 79 79 79 80 78 80 79
80 78 81 79 80 79 79 77 79 79 80 78 79 80 80 79

- **a** Find the mean contents of the boxes.
- **b** Do you think there is sufficient evidence from this data to prove that the manufacturer's claim is false? Explain your answer.

7 For this column graph on shoe sizes, determine the:

- **a** mode
- **b** median shoe size.



8 Consider the performances of two groups of students in the same mental arithmetic test out of 10 marks.

Group X: 7, 6, 6, 8, 6, 9, 7, 5, 4, 7 *Group Y:* 9, 6, 7, 6, 8, 10, 3, 9, 9, 8, 9

- **a** Calculate the mean mark for each group.
- **b** There are 10 students in *group X* and 11 in *group Y*. Is it unfair to compare the mean scores for these groups?
- **c** Which group performed better at the test?

KEY WORDS USED IN THIS CHAPTER

- biased sample
- column graph
- dependent variable
- independent variable
- mode
- raw data
- stem-and-leaf plot
- vertical bar chart
- categorical data
- data
- horizontal axis
- mean
- numerical data
- sample
- tally and frequency table
- census
- data set
- horizontal bar chart
- median
- pie chart
- segmented bar chart
- vertical axis

REVIEW SET 20A

1 State whether a census or a sample would be used to investigate:

- **a** the number of goals scored each week by a lacrosse team
- **b** the weight of a 6-week old calf
- **c** the most popular make of cars
- **d** the favourite foods of students in Year 7 classes
- **e** the height of oak trees.

2 Comment on the following statements:

a To find out the students' opinions on whether 'fast food' should be available from the school canteen, the principal asked one student from each year group for his or her views.

b To find out the students' views on a possible change to the school uniform, the principal asked 60 students from Year 12 to give their views.

3 In a survey of school girls to determine their favourite sport, the following data was obtained:

Sport	Netball	Basketball	Tennis	Softball	Hockey
Number of girls	56	43	47	31	23

a How many girls were surveyed?

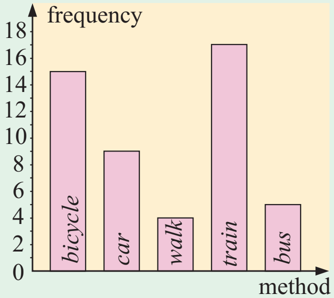
b What percentage of those surveyed favoured netball?

c For this group of girls, which was the least favoured sport?

d Construct a horizontal bar chart to display this information.

e Can conclusions be made from this survey about the favourite sport of all school girls? Give a reason for your answer.

4 50 randomly selected students were asked about their method of transport to school. The results of the survey are displayed in the graph opposite.



a What sort of graph is this?

b Which was the most common method of transport to school?

c How many students travelled to school by bus?

d What percentage of students travelled to school by car?

e What percentage of students travelled to school either by bicycle or walking?

5 During the 38 game season, Franki played in all games and scored 122 goals. Due to injury, Hugo played only 25 games in the season, and scored a total of 85 goals. Which player received the trophy for the 'greatest average number of goals scored per match'?

6 The following data are the ages of students in a chess club:

13 14 15 12 17 13 16 12 14 15 14 16 13 15 13 14 14 14
14 14 16 13 15 14 13 15 15 12 15 15 12 17 15 14 13

a Find the **i** mean **ii** median **iii** mode of the ages.

b Construct a frequency column graph for the data.

REVIEW SET 20B

1 State whether each of the following is a census or a sample:

a All 430 students of a high school were surveyed to determine their preferred school uniform.

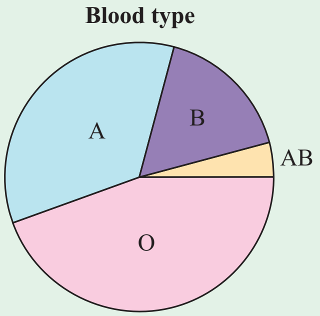
b Last Saturday some spectators entering the Entertainment Centre were surveyed on whether the car parking was adequate at the Centre.

2 Explain any possible bias in the following samples:

a a survey of people at the cinema **b** a questionnaire in a newspaper

c a survey of people watching their children playing football.

3 A random sample of people are surveyed about their blood type. The results are displayed in the pie chart opposite.



a Use your protractor to measure the size of each sector angle.

b What percentage of people surveyed have type A blood?

c If 7 of the people surveyed have type AB blood, how many people were surveyed in total?

4 A survey of hair colour in a class of 40 students revealed the following results:

Hair colour	*Frequency*
Red	4
Brown	17
Black	11
Blonde	8

a Construct a segmented bar chart to display this data.

b For this group of students, which was the least common hair colour?

c Could conclusions be made from this survey about the colour of hair of all students? Give a reason for your answer.

5 **a** Copy and complete this tally-frequency table for the ages of debaters in a university debating club.

b Draw a frequency column graph for this data.

c What is the most common age of debaters in the club?

d What percentage of club members have not yet reached the age of 20?

Age	*Tally*	*Frequency*
18	||	
19	||||	
20	𝍸 ||	
21	𝍸 𝍸 |	
22	𝍸 ||||	
23	𝍸 𝍸 |	
24	𝍸 ||	
25	𝍸 ||	
26	|||	
	Total	

6 To compare the spelling abilities of two groups of students, a special test was given to both groups. The results were:

Group A 8, 8, 5, 6, 8, 10, 7, 9, 7, 7
Group B 10, 9, 10, 10, 4, 10, 9, 7, 7, 7, 10

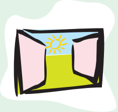
a Find the mean score for each group. **b** Which group performed better?

Chapter 21

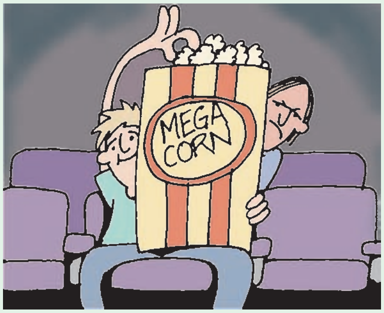
Sets

Contents:

OPENING PROBLEM



There were 50 people in a cinema. 20 people had bought popcorn, 35 had bought a drink, and 15 had bought both popcorn and a drink.



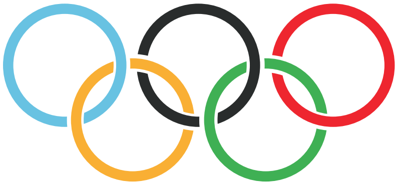
How many people in the cinema had bought:

- popcorn, but not a drink
- a drink, but not popcorn
- neither popcorn nor a drink?

A SETS

A **set** is a collection of objects or things.

Consider the colours of the Olympic rings. They are blue, black, red, yellow and green.



We can write these colours as the set $C = \{$blue, black, red, yellow, green$\}$.

We say:
"C is the set of colours of the Olympic rings."

The objects or things in a set are called the **elements** or **members** of the set.

In the example above, *blue* is an element of the set C.

The elements of a set can take almost any form, including colours, numbers, letters, and symbols. For example, the set of all multiples of 4 which are less than 30 can be written as the set $M = \{4, 8, 12, 16, 20, 24, 28\}$.

SET NOTATION

- $\in$ means: '*is a member of*' or '*is an element of*' or '*is in*' or '*belongs to*'
- $\notin$ means: '*is not a member of*' or '*is not an element of*' or '*is not in*' or '*does not belong to*'.
- $n(A)$ means: '*the number of elements in set A*'.

For example, if $M = \{4, 8, 12, 16, 20, 24, 28\}$ then $12 \in M$, $19 \notin M$, and $n(M) = 7$.

EQUAL SETS

Two sets are **equal** if they contain exactly the same elements.

SUBSETS

Set P is a **subset** of set Q if every element of P is also an element of Q. We write $P \subseteq Q$.

For example, if $A = \{1, 3, 6\}$ and $B = \{1, 2, 3, 5, 6, 7\}$, then every element of A is also an element of B. We say that A is a subset of B, and write $A \subseteq B$.

EMPTY SET

The **empty set** $\varnothing$ or $\{\ \}$ is a set which contains no elements.

An example of an empty set is the set of multiples of 5 between 1 and 4.

The empty set is a subset of all other sets.

Example 1 **Self Tutor**

Let P be the set of all multiples of 6 less than 20, and Q be the set of all even numbers less than 20.

a List the elements of P and Q.

b True or false: **i** $10 \in P$ **ii** $10 \notin Q$ **iii** $12 \in P$?

c Find: **i** $n(P)$ **ii** $n(Q)$.

d Is $P \subseteq Q$?

a $P = \{6, 12, 18\}$, $Q = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

b **i** 10 is not an element of P, so $10 \in P$ is false.
ii 10 is an element of Q, so $10 \notin Q$ is false.
iii 12 is an element of P, so $12 \in P$ is true.

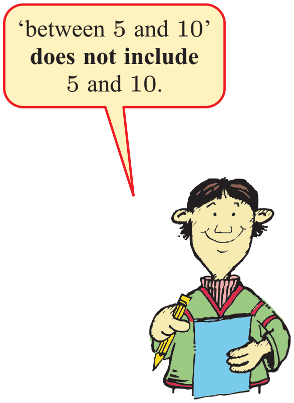
c **i** $n(P) = 3$ $\{P$ has 3 elements$\}$
ii $n(Q) = 9$ $\{Q$ has 9 elements$\}$

d Every element of P is also an element of Q, so $P \subseteq Q$.

EXERCISE 21A

1 List the elements of the set A, which is the set of all:

- **a** positive whole numbers between 5 and 10
- **b** odd numbers between 10 and 20
- **c** months of the year
- **d** factors of 30
- **e** letters on the bottom row of a computer keyboard
- **f** planets of the solar system
- **g** letters which make up the word BASEBALL
- **h** colours of keys on a piano
- **i** square numbers between 50 and 60.



2 Find $n(A)$ for each of the sets in **1**.

3 Let $P = \{1, 5, 7, 8, 10\}$ and $Q = \{1, 4, 5, 8, 9, 10\}$.

  a Find **i** $n(P)$ **ii** $n(Q)$.

  b True or false:

    i $8 \in P$ **ii** $8 \notin Q$ **iii** $9 \in P$ **iv** $3 \notin Q$?

  c Is $P \subseteq Q$?

4 Suppose $R = \{\text{even numbers less than } 10\}$, $S = \{\text{factors of } 24\}$, and $T = \{\text{prime numbers less than } 15\}$.

  a List the elements of R, S and T.

  b Find: **i** $n(R)$ **ii** $n(S)$ **iii** $n(T)$.

  c Is $T \subseteq S$?

  d Is $R \subseteq S$?

5 List all the subsets of:

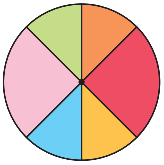
  a $\{5, 7\}$

  b $\{c, l, p\}$

6 Let A be the set of colours on spinner 1, and B be the set of colours on spinner 2.

Spinner 1

Spinner 2



  a List the elements of A and B.

  b Is green $\in B$?

  c Find: **i** $n(A)$ **ii** $n(B)$.

  d Is $A \subseteq B$?

7 Suppose $P = \{1, 6, 3, x, 9, 8, 12, 2\}$, $Q = \{3, 6, 7, 12\}$ and $Q \subseteq P$. Find x.

8 Are the following statements true or false?

  a If $A \subseteq B$ then $n(A) \leqslant n(B)$.

  b If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

9 List *all* the subsets of $\{w, x, y, z\}$.

B COMPLEMENT OF A SET

When we are dealing with sets:

The universal set U is the set of all elements we are considering.

For example, if we are considering the letters of the English alphabet, the universal set is:

$U = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

From this universal set we can define subsets of U, such as $V = \{\text{vowels}\} = \{a, e, i, o, u\}$ and $C = \{\text{consonants}\} = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$.

The **complement** of a set A is the set of all elements of U that are not elements of A.

The complement of A is written A'.

For example:

- if $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{2, 3, 5, 7\}$, then $A' = \{1, 4, 6, 8, 9\}$
- if $U = \{\text{letters of the English alphabet}\}$, $V = \{\text{vowels}\}$ and $C = \{\text{consonants}\}$ then V is the complement of C, and C is the complement of V.

Example 2 **Self Tutor**

Let $U = \{0, 1, 2, 3, 4, 5, 6\}$. Find A' if A is:

a $\{0, 2, 5, 6\}$ **b** $\{\text{factors of } 6\}$ **c** $\{\text{whole numbers which are} < 3\}$

a $A = \{0, 2, 5, 6\}$
$\therefore \quad A' = \{1, 3, 4\}$

b $A = \{1, 2, 3, 6\}$
$\therefore \quad A' = \{0, 4, 5\}$

c $A = \{0, 1, 2\}$
$\therefore \quad A' = \{3, 4, 5, 6\}$

Example 3 **Self Tutor**

If $U = \{\text{whole numbers between 1 and 20}\}$, find the complement of:

a $A = \{\text{even numbers}\}$ **b** $B = \{\text{prime numbers}\}$

a $A' = \{\text{odd numbers in } U\}$
$= \{3, 5, 7, 9, 11, 13, 15, 17, 19\}$

b $B' = \{\text{non-prime numbers in } U\}$
$= \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

The set of whole numbers *between* 1 and 20 does not include 1 or 20.

EXERCISE 21B

1 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Find the complement of:

a $A = \{1, 4, 7\}$ **b** $B = \{2, 3, 4, 5, 9\}$

c $C = \{5, 1, 9, 7\}$ **d** $D = \{2, 4, 6, 8\}$

2 Suppose $U = \{\text{whole numbers between 0 and 15}\}$, $P = \{\text{factors of 10}\}$ and $Q = \{\text{prime numbers less than 15}\}$. List the elements of:

a P **b** Q **c** P' **d** Q'

3 Consider the list of sports at a school sports day. Let B be the set of sports that are played with a ball.

a List the elements of:

i U **ii** B **iii** B'.

b What do the elements of B' represent?

SPORTS

Sprinting
Basketball
High Jump
Hurdles
Shot Put
Tennis
Long Jump

4 Consider the letters of the English alphabet. Let A be the set of letters which make up the word "MATHEMATICS", and B be the set of letters which make up the word "GEOGRAPHY". List the elements of:

a A **b** B **c** A' **d** B'

5 Let $U = \{$whole numbers between 0 and 10$\}$,
$A = \{$multiples of 3 which are less than 10$\}$, and $B = \{$odd numbers less than 10$\}$.

a List the elements of:

i U **ii** A **iii** A' **iv** B **v** B'

b Find:

i $n(U)$ **ii** $n(A)$ **iii** $n(A')$ **iv** $n(B)$
v $n(B')$ **vi** $n(A) + n(A')$ **vii** $n(B) + n(B')$.

c Copy and complete:
For any set A within a universal set U, $n(A) + n(A') = n(....)$.

6 Find A' if $A = U$.

C INTERSECTION AND UNION

Friends Jacinta and Kaitlin work at the same coffee shop. Jacinta works from Tuesday to Friday, and Kaitlin works from Thursday to Saturday.

Let $J = \{$Tuesday, Wednesday, Thursday, Friday$\}$ be the set of days Jacinta works,
and $K = \{$Thursday, Friday, Saturday$\}$ be the set of days Kaitlin works.

By inspecting the sets we can see that Thursday and Friday are in both sets, which means Jacinta and Kaitlin both work on Thursday and Friday.

We say the set $\{$Thursday, Friday$\}$ is the *intersection* of sets J and K.

> The **intersection** of two sets A and B is the set of elements that are in both set A and set B.
>
> The intersection of sets A and B is written $A \cap B$.

Suppose you want to know the days that at least one of the girls work. By inspecting the sets, we can see that either Jacinta or Kaitlin or both work on the days Tuesday, Wednesday, Thursday, Friday and Saturday.

We say the set {Tuesday, Wednesday, Thursday, Friday, Saturday} is the *union* of sets J and K.

The **union** of two sets A and B is the set of elements that are in either set A or set B. Elements in both A and B **are included** in the union of A and B.

The union of sets A and B is written $A \cup B$.

Example 4 **Self Tutor**

Let $A = \{1, 2, 4, 5, 7\}$ and $B = \{2, 3, 5, 8\}$. Find: **a** $A \cap B$ **b** $A \cup B$.

a $A \cap B = \{2, 5\}$ {2 and 5 are elements of both sets}

b $A \cup B = \{1, 2, 3, 4, 5, 7, 8\}$ {elements of either A or B or both}

Example 5 **Self Tutor**

Let $P = \{b, c, f, g, h\}$ and $Q = \{c, d, g, i\}$. Find:

a $P \cap Q$ **b** $P \cup Q$

a $P \cap Q = \{c, g\}$ {c and g are elements of both sets}

b $P \cup Q = \{b, c, d, f, g, h, i\}$ {elements of either P or Q}

EXERCISE 21C

1 $A = \{10, 12, 14, 16\}$ and $B = \{11, 12, 13, 14\}$. Find:

a $A \cap B$ **b** $A \cup B$

2 Find $C \cap D$ and $C \cup D$ for:

a $C = \{5, 6, 9, 11, 13\}$, $D = \{4, 6, 7, 11, 13, 15\}$

b $C = \{a, c, d, m, p, r\}$, $D = \{d, f, l, p, t\}$

c $C =$ {Rome, Tokyo, London, New York}, $D =$ {Tokyo, Paris, Rome, Cairo}.

3 Suppose $X =$ {square numbers less than 20} and $Y =$ {factors of 20}. Find:

a $X \cap Y$ **b** $n(X \cap Y)$ **c** $X \cup Y$ **d** $n(X \cup Y)$

4 Betty has roses and daisies in her garden. Her roses flower every year from March to August, and her daisies flower from May to November. Let R be the set of months that the roses flower, and D be the set of months that the daisies flower.

a State the universal set U in this case.

b List the elements of R and D.

c Find $R \cap D$. What does this set represent?

d Find $R \cup D$. What does this set represent?

e Find $(R \cup D)'$. What does this set represent?

5 Suppose $P = \{\text{multiples of 3 which are less than 10}\}$ and $Q = \{\text{factors of 18}\}$.

- **a** List the elements of P and Q.
- **b** Is $P \subseteq Q$?
- **c** Find: **i** $P \cap Q$ **ii** $P \cup Q$.
- **d** Copy and complete: If $P \subseteq Q$, then $P \cap Q = ...$ and $P \cup Q = ...$.

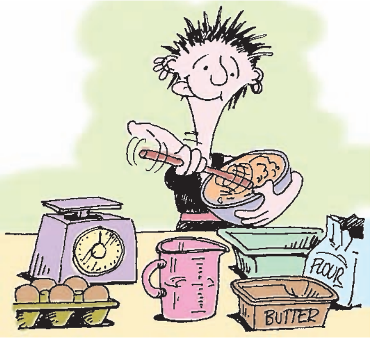
6 Rosanne is baking biscuits and cake for a special afternoon tea. The ingredients needed for her recipes are shown alongside. Let B be the set of ingredients needed to make the biscuits, and C be the set of ingredients needed to make the cake.

BISCUITS

1 cup flour
150 g butter
1 tbsp vanilla
1 egg
1 tbsp cinnamon

CAKE

100 g butter
150 g sugar
150 g flour
2 eggs
4 tbsp milk
2 tbsp lemon juice



- **a** List the elements of B and C.
- **b** Find $B \cap C$. What does this set represent?
- **c** Find $B \cup C$.
- **d** How many different ingredients will Rosanne use in her baking? How does your answer relate to $B \cup C$?

D DISJOINT SETS

Lisa works at the same coffee shop as Jacinta and Kaitlin. Lisa works from Monday to Wednesday and we represent this by the set $L = \{\text{Monday, Tuesday, Wednesday}\}$. We remember that the days Kaitlin works are represented by $K = \{\text{Thursday, Friday, Saturday}\}$.

Looking at the sets, we can see there are no days that both Lisa and Kaitlin work. Since there are no days that are elements of both K and L, $K \cap L = \varnothing$. We say that K and L are *disjoint* sets.

Two sets A and B are **disjoint** if they have no elements in common. If A and B are disjoint then $A \cap B = \varnothing$.

For example, if $A = \{1, 4, 7, 9\}$ and $B = \{2, 3, 8\}$, then $A \cap B = \varnothing$ and A and B are disjoint.

Example 6 **Self Tutor**

$A = \{2, 4, 7, 10\}$, $B = \{\text{prime numbers less than 10}\}$ and $C = \{\text{factors of 15}\}$.

- **a** Find: **i** $A \cap B$ **ii** $A \cap C$ **iii** $B \cap C$.
- **b** Which two sets are disjoint?

$A = \{2, 4, 7, 10\}$, $B = \{2, 3, 5, 7\}$, $C = \{1, 3, 5, 15\}$

a **i** $A \cap B = \{2, 7\}$ $\{$2 and 7 are common to A and $B\}$

ii $A \cap C = \varnothing$ $\{$no elements are common to A and $C\}$

iii $B \cap C = \{3, 5\}$ $\{$3 and 5 are common to B and $C\}$

b A and C have no elements in common, so A and C are disjoint.

EXERCISE 21D

1 True or false:

- **a** $\{1, 4, 7, 10\}$ and $\{2, 5, 8, 11\}$ are disjoint
- **b** $\{k, m, p, x, z\}$ and $\{h, p, r, t, u\}$ are disjoint
- **c** $\{\#, !, \blacklozenge, *\}$ and $\{\bullet, \uparrow, \%\}$ are not disjoint
- **d** $\{8, 10, 15, 16, 20\}$ and $\{1, 3, 4, 8\}$ are not disjoint?

2 Suppose $A = \{$multiples of 4 between 0 and 25$\}$, $B = \{$square numbers less than 20$\}$, $C = \{$factors of 21$\}$ and $D = \{$prime numbers less than 20$\}$.

- **a** List the elements of A, B, C and D.
- **b** Which pairs of sets are disjoint?

3 At an international convention, most of the delegates can speak multiple languages. Eduardo can speak Portugese, Spanish and Japanese. Fernando can speak Spanish and French. Gordon can speak English and French. Huan can speak Chinese, Japanese and English.

Let the sets E, F, G and H be the sets of languages spoken by Eduardo, Fernando, Gordon and Huan respectively.

- **a** List the elements of E, F, G and H.
- **b** Find: **i** $E \cap F$ **ii** $E \cap G$ **iii** $E \cap H$ **iv** $F \cap G$ **v** $F \cap H$ **vi** $G \cap H$.
- **c** Which language would be spoken between:
  - **i** Eduardo and Huan
  - **ii** Gordon and Fernando?
- **d** Explain why delegates cannot communicate with each other if their language sets are disjoint.
- **e** Which delegates cannot communicate with each other?

DISCUSSION DISJOINT SETS

Explain why the following statements are true:

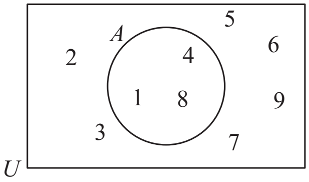
- A and A' are disjoint sets
- If A and B are disjoint sets, then $n(A \cup B) = n(A) + n(B)$.

E VENN DIAGRAMS

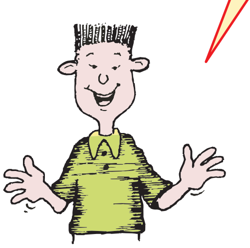
We can represents sets visually using Venn diagrams.

A **Venn diagram** consists of a universal set U represented by a rectangle, and sets within it that are generally represented by circles.

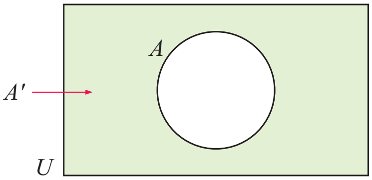
For $A = \{1, 4, 8\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the Venn diagram is:



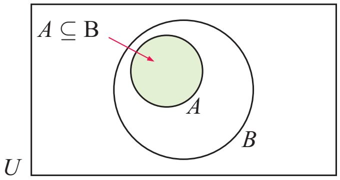
Start by drawing a rectangle for U and a circle for A. Place the elements of A inside the circle and all other elements outside it.



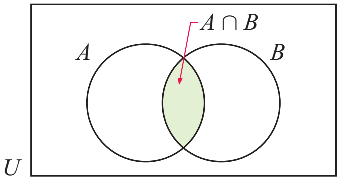
The **complement** of a set A is represented by the region outside the circle which represents A.



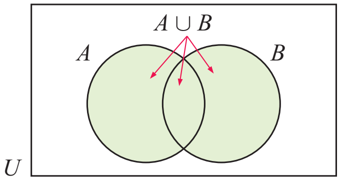
If A is a **subset** of B, we place a circle representing A completely within the circle representing B. Every element of A is also an element of B.



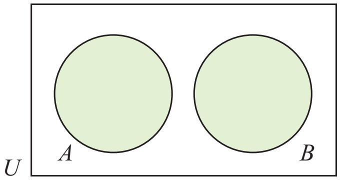
The **intersection** of two sets A and B is represented by the region where the circles representing A and B overlap. Elements in this region are elements of both A and B.



The **union** of two sets A and B is represented by the region inside one or both of the circles representing A and B. Elements in this region are elements of either A or B.



Disjoint sets A and B are represented by two circles which do not overlap. There are no elements that are members of both A and B.



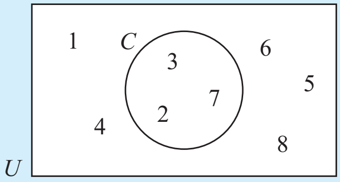
Example 7 Self Tutor

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Draw a Venn diagram to represent:

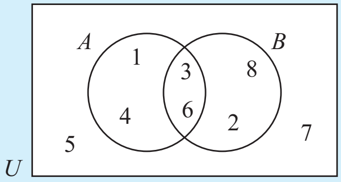
a $C = \{2, 3, 7\}$

b $A = \{1, 3, 4, 6\}$, $B = \{2, 3, 6, 8\}$

a $C = \{2, 3, 7\}$
$C' = \{1, 4, 5, 6, 8\}$



b $A \cap B = \{3, 6\}$
$A \cup B = \{1, 2, 3, 4, 6, 8\}$



It is a good idea to put elements in the intersection on the Venn diagram first.

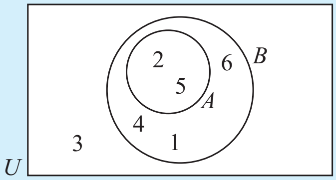
Example 8 Self Tutor

Let $U = \{1, 2, 3, 4, 5, 6\}$. Draw a Venn diagram to represent:

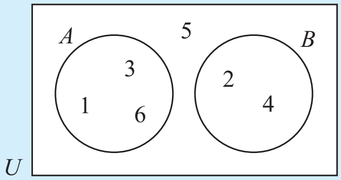
a $A = \{2, 5\}$, $B = \{1, 2, 4, 5, 6\}$

b $A = \{1, 3, 6\}$, $B = \{2, 4\}$

a Every element in A is also in B, so $A \subseteq B$.



b $A \cap B = \varnothing$, so A and B are disjoint sets.



EXERCISE 21E

1 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Draw a Venn diagram to represent:

a $A = \{1, 5, 6, 9\}$

b $A = \{2, 4, 5, 8\}$, $B = \{3, 4, 6, 9\}$

c $A = \{3, 4, 5, 6, 7\}$, $B = \{5, 6, 7, 8, 9\}$

d $A = \{1, 5, 9\}$, $B = \{2, 3, 7\}$.

2 Let $U = \{a, b, c, d, e, f, g, h, i, j\}$. Draw a Venn diagram to represent:

a $P = \{f, a, c, e\}$, $Q = \{h, e, a, d\}$

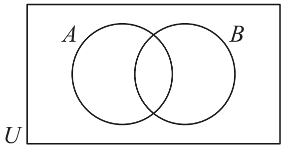
b $P = \{c, d, f, g, i, j\}$, $Q = \{d, g, j\}$.

3 Consider the coffee shop workers Jacinta and Kaitlin from page **412**.
$J = \{\text{Tuesday, Wednesday, Thursday, Friday}\}$ and $K = \{\text{Thursday, Friday, Saturday}\}$.

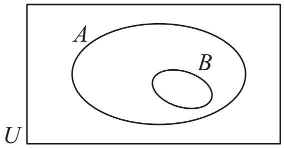
- **a** State the universal set U.
- **b** Represent the sets J and K on a Venn diagram.

4 Determine which form of Venn diagram would be most appropriate to represent the following sets A and B.

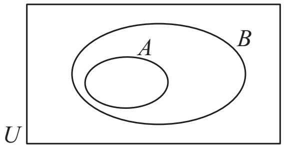
A



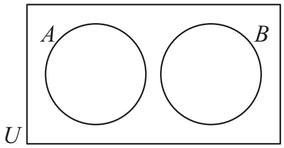
B



C



D



- **a** $A = \{\text{whole numbers from 1 to 10}\}$, $B = \{\text{whole numbers from 5 to 15}\}$
- **b** $A = \{\text{multiples of 6}\}$, $B = \{\text{multiples of 3}\}$
- **c** $A = \{\text{people aged under 30}\}$, $B = \{\text{people aged over 50}\}$
- **d** $A = \{\text{people aged over 30}\}$, $B = \{\text{people aged over 50}\}$
- **e** $A = \{\text{people aged over 30}\}$, $B = \{\text{people aged under 50}\}$
- **f** $A = \{\text{odd numbers}\}$, $B = \{\text{even numbers}\}$
- **g** $A = \{\text{multiples of 3}\}$, $B = \{\text{multiples of 5}\}$
- **h** $A = \{\text{people born in China}\}$, $B = \{\text{people born in France}\}$
- **i** $A = \{\text{people born in China}\}$, $B = \{\text{people born in Asia}\}$
- **j** $A = \{\text{factors of 100}\}$, $B = \{\text{factors of 50}\}$.

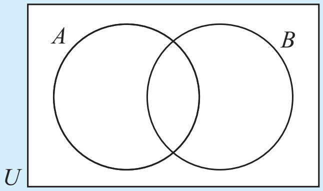
5 Suppose $U = \{\text{whole numbers from 1 to 15}\}$, $A = \{\text{factors of 12}\}$ and $B = \{\text{even numbers less than 15}\}$.

- **a** List the elements of A and B.
- **b** Find: **i** $A \cap B$ **ii** $A \cup B$.
- **c** Draw a Venn diagram to illustrate the sets.
- **d** How many elements are in: **i** A but not B **ii** B **iii** either A or B?

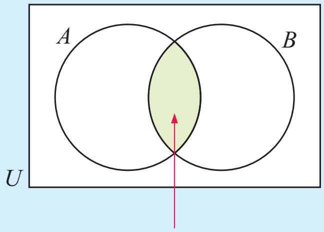
Example 9 **Self Tutor**

Shade the region of a Venn diagram representing:

a in A and B **b** in A but not in B.

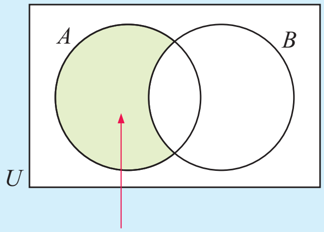


a



Elements in this region are in both A and B.

b

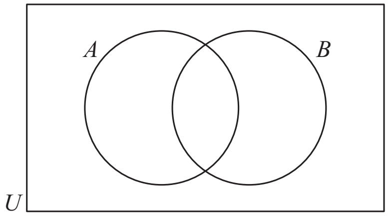


Elements in this region are in A but not in B.

6 On separate Venn diagrams like the one illustrated, shade the region representing:

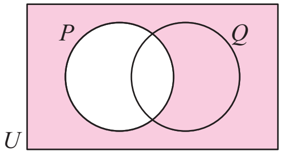
- **a** in A
- **b** in B but not in A
- **c** in A or B
- **d** not in B.

PRINTABLE PAGES OF VENN DIAGRAMS

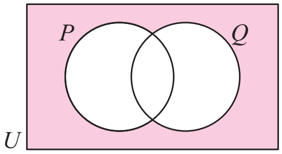


7 Describe in words the shaded region:

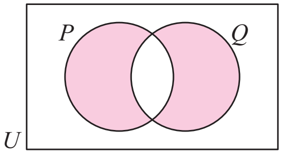
a



b



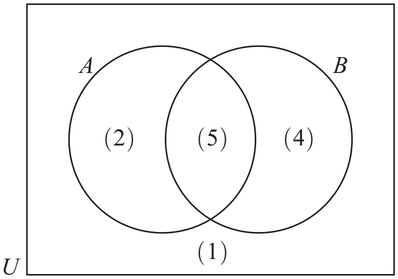
c



F PROBLEM SOLVING WITH VENN DIAGRAMS

We can use Venn diagrams to solve problems. This is done by finding the **number of elements** in each region.

To indicate how many elements are in each region of a Venn diagram, we place brackets around the numbers.



For example, in the Venn diagram alongside there are 5 elements in both A and B, 2 elements in A but not B, 4 elements in B but not A, and 1 element in neither A nor B.

In total there are $2 + 5 = 7$ elements in A, and $5 + 4 = 9$ elements in B.

Example 10 **Self Tutor**

There are 30 houses on a street. 16 of the houses have a burglar alarm, 22 houses have a smoke alarm, and 10 houses have both a burglar alarm and a smoke alarm.

a Place this information on a Venn diagram.

b How many houses on the street have:

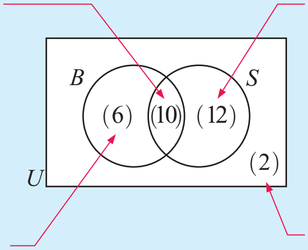
  i a burglar alarm but not a smoke alarm

  ii either a burglar alarm or a smoke alarm?

a Let B represent the set of houses with burglar alarms, and S represent the set of houses with smoke alarms.

① We first place a (10) in the intersection to indicate 10 houses have both alarms.

② Since 16 houses have burglar alarms, there must be 6 houses in this region so that $6 + 10 = 16$.



③ Since 22 houses have smoke alarms, there must be 12 houses in this region so that $12 + 10 = 22$.

④ Since $6 + 10 + 12 = 28$, and there are 30 houses in total, there must be 2 houses in neither B nor S.

b **i** There are 6 elements in B but not in S.

$\therefore$ 6 houses have a burglar alarm but not a smoke alarm.

ii There are $6 + 10 + 12 = 28$ elements in either B or S.

$\therefore$ 28 houses have either a burglar alarm or a smoke alarm.

EXERCISE 21F

1 A group of 15 friends are discussing which foods they like. 10 of them like chocolate, 9 of them like ice cream, and 6 like both these foods.

  a Place this information on a Venn diagram.

  b How many people in the group like:

    i ice cream but not chocolate

    ii chocolate but not ice cream

    iii neither food

    iv chocolate or ice cream?

2 Police conducted a safety check of 80 cars. They found that 15 cars had faulty brakes and 25 cars had faulty tyres. 10 of the cars had both faulty tyres and faulty brakes.

  a Place this information on a Venn diagram.

  b How many cars had:

    i faulty tyres but not faulty brakes

    ii neither faulty tyres nor faulty brakes

    iii either faulty tyres or faulty brakes?

3 In a class of 24 students, 13 own a bike, 9 own a skateboard, and 5 own both items.

a Place this information on a Venn diagram.

b How many students in the class own:

i a bicycle but not a skateboard

ii neither a bicycle nor a skateboard

iii a bicycle or a skateboard?

4 Jane went to the library and borrowed a book which was 40 pages long. Unfortunately her dog ripped 13 of the pages, and a leaking pen stained some of the pages. 6 pages were both ripped and stained, and 22 pages were not damaged at all.

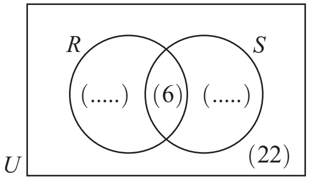
a Copy and complete the Venn diagram alongside.

b How many of the pages were:

i stained

ii ripped but not stained

iii ripped or stained?



5 Troy went on a holiday to England for 21 days. It rained on 15 of the days and was windy on 9 of the days. There were only 4 days when it was neither rainy nor windy.

a Present this information on a Venn diagram.

b On how many days was it:

i both rainy and windy

ii windy but not rainy

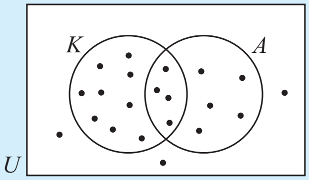
iii either rainy or windy, but not both?

G FINDING PROBABILITIES FROM VENN DIAGRAMS

Venn diagrams can also be used to calculate probabilities.

Example 11 **Self Tutor**

The Venn diagram shows the injuries sustained by a team of footballers in their careers. Each dot represents one footballer. K represents the footballers who have had an injured knee, and A represents the footballers who have had an injured ankle.



a How many footballers are there in the team?

b If a footballer from the team is selected at random, find the probability that the footballer has had:

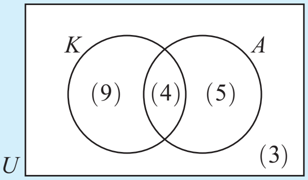
i an injured knee

ii an injured ankle, but not an injured knee

iii either injury

iv both injuries.

The number of elements in each region are:



a There are $9+4+5+3=21$ footballers in the team.

b **i** Of the 21 footballers in the team, $9+4=13$ have had an injured knee.

$\therefore$ P(randomly selected player has had an injured knee) $= \frac{13}{21}$

ii 5 players have had an injured ankle but not an injured knee.

$\therefore$ P(random player has had an injured ankle but not an injured knee) $= \frac{5}{21}$

iii $9+4+5=18$ players have had either an injured knee or an injured ankle.

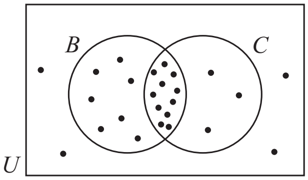
$\therefore$ P(randomly selected player has had either injury) $= \frac{18}{21}$

iv 4 players have had both an injured knee and an injured ankle.

$\therefore$ P(randomly selected player has had both injuries) $= \frac{4}{21}$.

EXERCISE 21G

1 The Venn diagram alongside shows the items collected by members of a collectors club. Each dot represents one member. B represents the members who collect baseball cards, and C represents the members who collect coins.



a How many members does the club have?

b If a member of the club is chosen at random, find the probability that the member collects:

i baseball cards but not coins **ii** coins **iii** exactly one of the items.

2 Wen has put 38 photographs into an album. His friends are in 28 of the photos, and his pets are in 15 of the photos. 8 of the photos contain both his friends and his pets.

a Place this information on a Venn diagram.

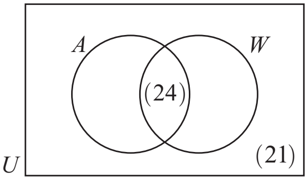
b Find the probability that a randomly selected photograph contains:

i his friends but not his pets

ii either his friends or his pets

iii neither his friends nor his pets.

3 A gym has 75 members. 42 members attended the aerobics classes, 24 members attended both the aerobics and weights classes, and 21 members do not attend either class.



a Copy and complete the Venn diagram alongside.

b Find the probability that a randomly selected gym member attends:

i weights classes **ii** weights classes but not aerobics classes

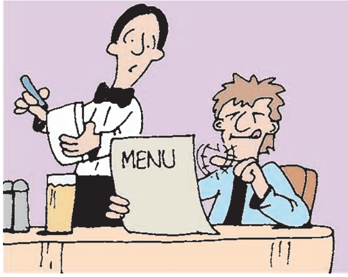
iii aerobics classes but not weights classes.

4 30 students took part in a school concert. 23 students had their mother in the audience, 18 students had their father in the audience, and 15 students had both parents in the audience.

- **a** Place this information on a Venn diagram.
- **b** Find the probability that a randomly selected student had:
  - **i** their mother but not their father in the audience
  - **ii** neither parent in the audience
  - **iii** exactly one parent in the audience.

5 There are 18 items on a restaurant menu. 6 of these contain peanuts, 7 contain egg, and 2 contain both peanuts and egg.

- **a** Place this information on a Venn diagram.
- **b** Charles will have an allergic reaction if he eats either peanuts or egg. If he randomly selects an item from the menu to eat, find the probability that he will have an allergic reaction.



KEY WORDS USED IN THIS CHAPTER

- complement
- disjoint sets
- element
- empty set
- equal sets
- intersection
- member
- set
- subset
- union
- universal set
- Venn diagram

REVIEW SET 21A

1 List the elements of the set of all multiples of 6 between 20 and 50.

2 Suppose $P = \{\text{factors of } 36\}$ and $Q = \{\text{square numbers less than } 20\}$.

- **a** List the elements of P and Q.
- **b** Find: **i** $n(P)$ **ii** $n(Q)$.
- **c** Is $Q \subseteq P$?
- **d** True or false: **i** $12 \in P$ **ii** $25 \in Q$ **iii** $24 \notin P$?

3 Consider the set of uppercase letters in the English alphabet. Let A be the set of letters which consist of straight edges only. Let V be the set of vowels.

- **a** List the elements of:
  - **i** A **ii** A' **iii** V.
- **b** Find:
  - **i** $n(A)$ **ii** $n(A')$ **iii** $A \cap V$ **iv** $A' \cap V$.

4 Suppose $S = \{\text{multiples of 3 between 8 and 28}\}$ and $T = \{\text{odd numbers between 8 and 28}\}$.

- **a** List the elements of: **i** S **ii** T.
- **b** Find:
  - **i** $S \cap T$ **ii** $S \cup T$ **iii** $n(S \cap T)$ **iv** $n(S \cup T)$.

c True or false:

i $18 \in S$ **ii** $25 \in T$ **iii** $24 \in S \cap T$ **iv** $16 \in S \cup T$?

5 True or false?
If A and B are disjoint sets, and B and C are disjoint sets, then A and C are also disjoint sets.

6 Suppose $A = \{1, 2, 7, 9\}$, $B = \{2, 5, 6, 7\}$, $C = \{3, 4, 5, 8\}$, $D = \{1, 3, 4, 9\}$, and $E = \{2, 6, 7, 9\}$. Which pairs of sets are disjoint?

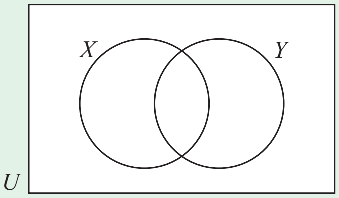
7 Consider the set of whole numbers from 1 to 9.
Let $A = \{\text{odd numbers between 0 and 10}\}$ and
$B = \{\text{prime numbers between 0 and 10}\}$.

a Find: **i** $A \cup B$ **ii** $A \cap B$.

b Illustrate the sets A, B and U on a Venn diagram.

8 On separate Venn diagrams, shade the regions representing:

a not in X **b** in X but not in Y.



9 90 students from one school took part in a Mathematics competition and a Science competition. 12 of the students won a prize in the Mathematics competition, and 17 won a prize in the Science competition. 6 students won prizes in both competitions.

a Display this information on a Venn diagram.

b The students who won a prize all attended an awards presentation. How many students attended the presentation?

10 There are 67 paintings hanging in the local art gallery. 18 of these are self-portraits. 32 are paintings from Europe. 10 of the paintings are self-portraits from Europe.

a Display this information on a Venn diagram.

b Determine the probability that a randomly selected painting is:

i a self-portrait but not European

ii neither a self-portrait nor European.

REVIEW SET 21B

1 List the elements of the set of all:

a multiples of 7 between 0 and 50 **b** common factors of 50 and 75.

2 For the following sets, determine whether $P \subseteq Q$:

a $P = \{1, 5, 7\}$, $Q = \{1, 3, 5, 6, 7, 8\}$

b $P = \varnothing$, $Q = \{e, f, g\}$

c $P = \{\#, !, \blacklozenge\}$, $Q = \{@, \#, =, \blacklozenge, \$\}$

d $P = \{\text{red, blue, yellow, green}\}$, $Q = \{\text{yellow, red, green, blue}\}$

e $P = \{\text{multiples of 4 less than 100}\}$, $Q = \{\text{multiples of 8 less than 100}\}$.

3 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 9\}$ and $B = \{1, 2, 8, 9\}$.

a Find the complement of: **i** A **ii** B.

b Is $A \subseteq B$? **c** Is $B' \subseteq A'$?

4 Suppose $P = \{\text{multiples of 6 less than 40}\}$ and $Q = \{\text{factors of 48}\}$. Find $P \cap Q$ and $P \cup Q$.

5 Twins Roger and Stephen are organising a joint birthday party. Each makes a list of people they want to invite to the party.

Let R be the set of people Roger wants to invite, and S be the set of people Stephen wants to invite.

a List the elements of R and S.

b Find $R \cap S$. What does this set represent?

c Find $R \cup S$ and $n(R \cup S)$. How many guests will be invited to Roger and Stephen's party?

ROGER

Michael
Bradley
Craig
Sally
Alistair
Kylie
Emma
Nigel

STEPHEN

William
Nigel
Kylie
David
Sam
Craig
Luke

6 Which of the following sets are disjoint?

a $A = \{1, 5, 7, 12\}$, $B = \{2, 6, 7, 11, 13\}$

b $A = \{\text{multiples of 7 which are less than 50}\}$,
$B = \{\text{multiples of 8 which are less than 50}\}$

c $A = \{\text{people who own a car}\}$, $B = \{\text{people who own a bicycle}\}$

d $A = \{\text{people whose favourite drink is milk}\}$,
$B = \{\text{people whose favourite drink is water}\}$.

7 Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Draw Venn diagrams to illustrate the following sets:

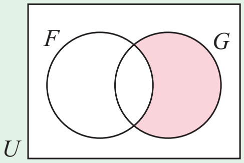
a $P = \{1, 5, 6, 7, 9\}$, $Q = \{2, 4, 5, 8, 9\}$

b $P = \{\text{multiples of 4 which are less than 10}\}$,
$Q = \{\text{even numbers less than 10}\}$

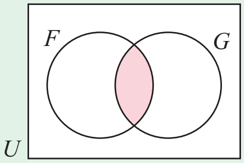
c $P = \{\text{multiples of 3 which are less than 10}\}$, $Q = \{\text{factors of 10}\}$.

8 Describe in words the shaded regions:

a



b



9 Of the last 30 days when Maria has driven to work, she has been delayed at a railway crossing on 8 of these days, and been late to work on 13 days. Every time she was delayed at the railway crossing, she was late for work.

- **a** Display this information on a Venn diagram.
- **b** On how many days did Maria:
  - **i** arrive late even though she was not delayed at the railway crossing
  - **ii** get to work on time?

10 40 people were sunbathing on a beach. 28 people were wearing a hat, 20 people were wearing a hat and sunscreen, and 3 people were wearing neither a hat nor sunscreen.

- **a** Place this information on a Venn diagram.
- **b** Find the probability that a randomly selected person on the beach was wearing:
  - **i** a hat but not sunscreen
  - **ii** sunscreen but not a hat
  - **iii** sunscreen.

Chapter 22

Rates

Contents:

OPENING PROBLEM

1 A greyhound runs 515 m in 29.5 seconds while a horse gallops 1650 m in 1 minute 45 seconds. Which animal moves faster?

2 Which is heavier, lead or gold?
How can we make this comparison?

A RATES

We have seen that a **ratio** is an ordered comparison of quantities of the **same** kind. For example, we can have a ratio of lengths or a ratio of times.

A **rate** is an ordered comparison of quantities of **different** kinds.

One of the most common rates we use is **speed**, which is a comparison between the *distance travelled* and the *time taken*.

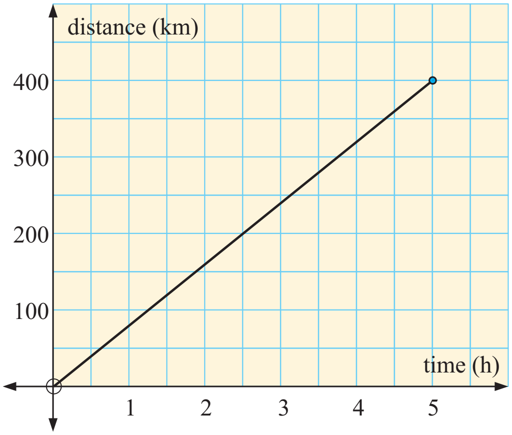
Because we are comparing quantities of different kinds, the units cannot be omitted as they are with ratios. In the case of speed, the units we use are either kilometres per hour, or metres per second.

The graph alongside shows that a car has travelled a distance of 400 km in 5 hours.

We can write the speed of the car as the rate $\dfrac{400 \text{ km}}{5 \text{ hours}} = 80$ km per hour.

The **slope** of the line can be measured by the *rise* : *run* ratio.

Note that this can also be written as $\dfrac{400 \text{ km}}{5 \text{ hours}}$ which is the car's speed.



The slope of a graph gives the rate of change of one variable with respect to the other.

Other common examples of rates are:

	Examples of units
Rates of pay	dollars per hour or euros per hour
Petrol consumption	litres per 100 km or km per litre
Annual rainfall	mm per year
Unit cost	dollars per kilogram or pounds per kg
Population density	people per square kilometre

Example 1 **Self Tutor**

A tap fills a 9 litre bucket in 3 minutes. Express this as a rate in simplest form.

$$\text{rate} = \frac{9 \text{ L}}{3 \text{ minutes}}$$

$$\therefore \quad \text{rate} = \tfrac{9}{3} \text{ L per minute}$$

$$\therefore \quad \text{rate} = 3 \text{ L per minute}$$

EXERCISE 22A

1 Write down the meaning of the following rates:

- **a** 8 m per s
- **b** 13 L per s
- **c** 4 km per h
- **d** 2.5 kL per min
- **e** 32 g per h
- **f** 2.7 kg per min
- **g** 74 $ per day
- **h** 23 cents per s

2 Copy and complete:

- **a** 35 apples bought for €9.45 is at a rate of ...... cents per apple.
- **b** If 24 kg of peas are sold for $66.24, they earn me $...... per kilogram.
- **c** My car uses 18 L of petrol every 261 km. The rate of petrol consumption is ...... km per litre.
- **d** 675 litres of water are pumped into a tank in 25 minutes. This is a rate of ...... litres per minute.
- **e** A cyclist travels 120 km in 5 hours. This is a rate of ...... km per hour.
- **f** A cleaner works for 4 hours and receives £48. His rate of pay is £..... per hour.

3 Trevor's car uses 16 litres of fuel to travel 240 km. Jane's car uses 12 litres to travel 204 km.

- **a** Find the rate of fuel consumption in km per L for each car.
- **b** Which car is more economical to run?

4 Mandy earns $144 for 8 hours' work. Dzung earns $210 for 14 hours' work.

- **a** Find the rate of pay in $ per hour for each person.
- **b** Who gets paid the better hourly rate?

B COMPARING PRICES

When shopping, it is important to get good value for money. It is often not obvious which item represents the best value for money, however, because the same item can come in several different sized packages.

To properly compare prices we need to convert the cost of an item into a rate called a **unit cost**. This could be the cost per gram, the cost per kilogram, the cost per litre or similar. We then compare the unit costs for packages of different sizes.

Example 2 **Self Tutor**

Which box of Sudso detergent gives better value for money?

A 500 g costing \$3.50 *or* **B** 750 g costing \$5.40

A unit cost $= \dfrac{350 \text{ cents}}{500 \text{ grams}}$

$= 0.7$ cents per gram

B unit cost $= \dfrac{540 \text{ cents}}{750 \text{ grams}}$

$= 0.72$ cents per gram

So, **A** is better value.

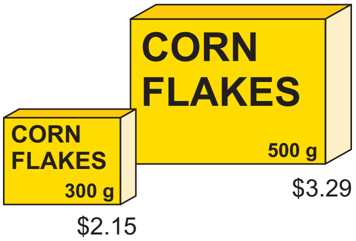
EXERCISE 22B

1 Use your calculator to find unit costs for the following items. Express your answer in the units in brackets.

a	250 g chocolate for \$3.75	(cents per gram)
b	2 L juice for £2.80	(£ per L)
c	5 kg potatoes for €4.45	(cents per kg)
d	packet of 3 toothbrushes for RM 15.39	(RM per toothbrush)
e	5 m of fabric for €16.50	(€ per metre)
f	40 L of petrol for £37.60	(pence per L)

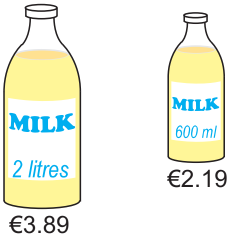
2 Consider the following grocery items and decide which is the better value for money:

a

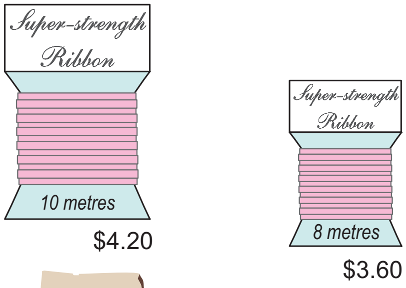


b

c



d



e

f

C USING RATES

Some rates are constant whereas others vary with time. For example, we know the speed of a car varies as we accelerate or brake.

If a rate remains constant over time then we can use this to solve problems.

Example 3 **Self Tutor**

Henry eats peanuts at a constant rate of 24 every 3 minutes. How many peanuts will he eat in 10 minutes?

$$\begin{aligned}\text{Henry's rate of eating peanuts} &= \frac{24 \text{ peanuts}}{3 \text{ minutes}}\\ &= 8 \text{ peanuts per minute}\end{aligned}$$

So, in 10 minutes Henry will eat $8 \times 10 = 80$ peanuts.

EXERCISE 22C

1 Water consumption for a person living in Sydney is approximately 2100 litres per week. At this rate, how much water would be used by a person in two days?

2 Bill is a part time curator at the local cricket ground. He earned £169.60 for working 8 hours last week. This week he has worked 21 hours. How much will he earn this week if he is paid at the same hourly rate?

3 A petrol tanker takes 18 minutes to discharge 8100 litres of fuel.

- **a** At this rate, how much fuel would the tanker discharge in 14 minutes?
- **b** How long would it take the tanker to discharge 4950 litres of fuel at the same rate?

4 A motor vehicle can travel 84 km on 7 litres of petrol.

- **a** How far could the vehicle travel on 57 litres of petrol?
- **b** How many litres of petrol would be required to travel a distance of 420 km?

5 It costs $2340 to lay a drain pipe of length 36 m.

- **a** Find the cost of laying a drain pipe of length 63 m.
- **b** Find the length of pipe that could be laid for a cost of $7540.

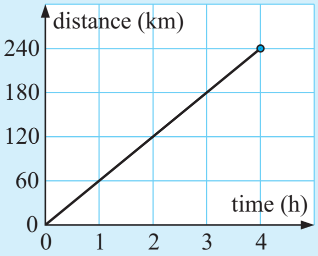
6 To travel 484 km a car uses 55 litres of petrol.

- **a** Find the rate at which the petrol is used in:
  - **i** km per litre
  - **ii** litres per 100 km.
- **b** At this rate, how many litres of fuel would be needed to travel 300 km?
- **c** If petrol costs \$1.85 per litre, how much would the journey in **b** cost?

Example 4 **Self Tutor**

The graph shows the progress of a train travelling between cities.

- **a** How far does the train travel in the 4 hour journey?
- **b** Write down a rate which compares the distance travelled with the time taken.
- **c** What was the speed of the train for this journey?



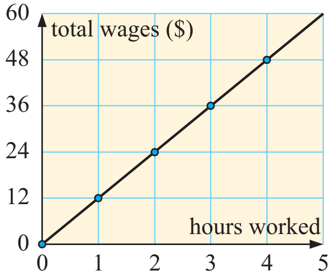
a The train travels 240 km in the 4 hours.

b Rate is $\dfrac{\text{distance travelled}}{\text{time taken}} = \dfrac{240\text{ km}}{4\text{ hours}} = 60$ km per h.

c The speed of the train was 60 km per h.

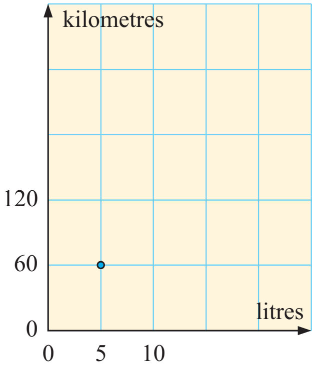
7 The graph alongside shows how much a student earns for working at a local newsagent.

- **a** How much does the student receive for working 4 hours?
- **b** How many hours does the student have to work to earn \$18?
- **c** What is the rate of pay?

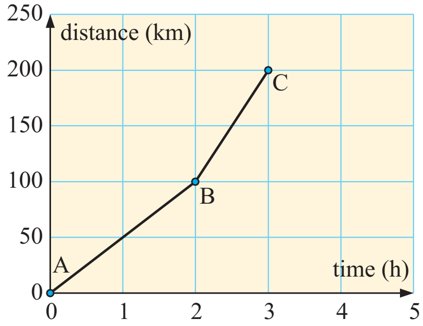


8 For each 60 km travelled a rally car uses five litres of fuel.

- **a** Copy and complete the graph opposite.
- **b** Use the graph to find the distance the car would travel on 20 L of fuel.
- **c** How many litres of fuel would be needed to travel 180 km?
- **d** Join the points on your graph. What do you notice?
- **e** Use the graph to estimate how far the rally driver could travel on 18 L of petrol.
- **f** What is the rate of petrol consumption?



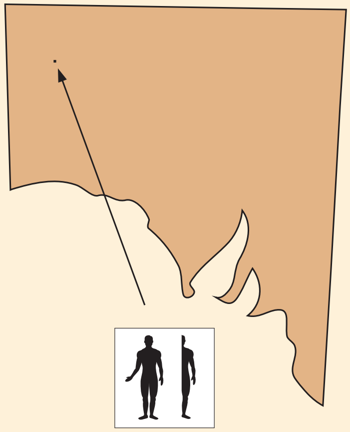
9 Consider the following distance-time graph showing the progress of a car travelling from town A to B to C.



- **a** How *far* is it from A to B?
- **b** How *long* did it take to get from A to B?
- **c** What was the *speed* of the car while travelling from A to B?
- **d** How *far* is it from B to C?
- **e** How *long* did it take to get from B to C?
- **f** What was the *speed* of the car while travelling from B to C?
- **g** What feature of the graph shows the change in speed of the car?

INVESTIGATION — POPULATION DENSITY

Since most of South Australia is desert, its population density is $1\frac{1}{2}$ people per km^2.



What to do:

1. Explain what the statement above means.
2. How is the population density calculated?
3. Explain why the population density is important.
4. Find, using the library or the internet, the population density of:
   - **a** Ireland
   - **b** Sri Lanka
   - **c** Canada
   - **d** China
   - **e** your country.
5. What would you need to know to be able to compare the population densities of two cities such as Barcelona and Istanbul?

D AVERAGE SPEED

We have already seen how speed is a rate, and that it may vary over time. In real world situations the speed of a car is rarely constant for long, and a graph of distance against time will not be a straight line.

However, we can still find the **average speed** of the car over an interval of time. This is the constant speed we would need to travel at to travel the same distance in the same time.

Average speed can be found using the formula: $$\textbf{average speed} = \frac{\textbf{total distance travelled}}{\textbf{time taken}}$$

For example, if a car travels 344 km in 4 hours, its average speed is

$$\frac{344 \text{ km}}{4 \text{ hours}} = 86 \text{ km per hour.}$$

We can rearrange the formula for average speed to obtain:

$$\textbf{total distance travelled} = \textbf{average speed} \times \textbf{time taken}$$

$$\textbf{time taken} = \frac{\textbf{total distance travelled}}{\textbf{average speed}}$$

We can verify these formulae since $86 \text{ km per hour} \times 4 \text{ hours} = 344 \text{ km}$

and $\dfrac{344 \text{ km}}{86 \text{ km per hour}} = 4 \text{ hours.}$

Example 5 **Self Tutor**

A cyclist travels 92 km in 4 hours.

a What was the cyclist's average speed?

b How far would the cyclist ride in 7 hours at the same rate?

c How long would it take the cyclist to ride 253 km at the same rate?

a The cyclist rode at $\dfrac{92 \text{ km}}{4 \text{ hours}} = 23 \text{ km per hour}$

b In 7 hours the cyclist would ride $7 \times 23 = 161$ km.

c The cyclist rides at 23 km per hour

$\therefore$ to ride 253 km it would take $\dfrac{253}{23} = 11$ hours.

EXERCISE 22D

1 Find the average speed of:

- **a** a cyclist who travels 80 km in 4 hours
- **b** an athlete who runs 10 km in 40 minutes
- **c** a truck which takes 4 hours to travel 300 km
- **d** an aeroplane which takes 50 minutes to fly 750 km.

2 The speed limit through a city is 50 km per hour. If Jason drives 25 km through the city in 20 minutes, has he broken the law?

3 Answer the **Opening Problem** question **1** on page **428**.

4 A motorist drives 400 km in 6 hours. How far would she drive in 3 hours at the same rate?

5 How far will:

- **a** a cyclist travel in 3 hours if his average speed is 35 km per h
- **b** a runner travel in 20 minutes if her average speed is 14 km per h
- **c** a walker travel in 1 hour 20 minutes if her average speed is 4.2 km per h?

6 Find the time required for:

- **a** a car to travel 120 km at a speed of 90 km per hour
- **b** a cyclist to travel 50 km at a speed of 30 km per hour
- **c** a jogger to run 8 km at a speed of 10 km per hour.

E DENSITY

In order to compare samples of metals or rocks, we use a rate called **density**. This is a measure of how *compact* the material is, or how much of it can be put into a given volume.

The **density** of an object is its mass per unit of volume.

For example, the density of pure gold is 19.3 grams per cm^3. This means that every cubic centimetre of pure gold weighs 19.3 grams.

Density can be found using the formula:

$$\textbf{density} = \frac{\textbf{mass}}{\textbf{volume}}$$

We can rearrange this formula to obtain:

$$\textbf{mass} = \textbf{density} \times \textbf{volume} \quad \text{and} \quad \textbf{volume} = \frac{\textbf{mass}}{\textbf{density}}.$$

DISCUSSION

- If you had a gold nugget, how could you find its volume without melting it?
- What other measurements would you need to determine its density?
- If your density calculation was 18.9 grams per cm^3, does this mean that the metal is not gold?

We know that 1 mL or 1 cm^3 of pure water weighs 1 gram.

The density of pure water is 1 gram per cm^3.

If an object has density less than 1 gram per cm^3 then it will float on water.
If its density is greater than 1 gram per cm^3 then it will sink.

This table lists some common densities in g per cm^3.

Material	*Density*	*Material*	*Density*
carbon dioxide	0.002	aluminium	2.7
petrol	0.70	iron	7.8
ice	0.92	lead	11.3
water	1.00	gold	19.3
milk	1.03	platinum	21.4

Example 6 **Self Tutor**

Find the density of a piece of timber which is 60 cm by 10 cm by 3 cm and weighs 1.62 kg.

$$\begin{aligned} 1.62 \text{ kg} &= 1.62 \times 1000 \text{ g} \\ &= 1620 \text{ g} \\ \text{Density} &= \frac{\text{mass}}{\text{volume}} \\ &= \frac{1620 \text{ g}}{60 \times 10 \times 3 \text{ cm}^3} \\ &= 0.9 \text{ g per cm}^3 \end{aligned}$$

Example 7 **Self Tutor**

How many times heavier is gold than iron?

The ratio of densities $\dfrac{\text{density of gold}}{\text{density of iron}} = \dfrac{19.3}{7.8} \approx 2.47$

$\therefore$ gold is 2.47 times heavier than iron.

EXERCISE 22E

1 Find the density of:

- **a** an object with a volume of 2.5 cm^3 which weighs 10 g
- **b** a piece of metal which weighs 2.4 kg and has a volume of 300 cm^3
- **c** a piece of timber which is 3.2 m by 20 cm by 5 cm and weighs 28.8 kg.

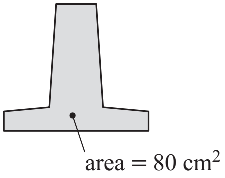
2 How many times heavier is:

- **a** gold than water
- **b** iron than aluminium
- **c** water than petrol?

3 Answer the **Opening Problem** question **2** on page **428**.

4 Find the mass of:

- **a** an aluminium block which is 30 cm by 4 cm by 2 cm
- **b** an iron bar which is 1 m by 5 cm by 3 cm
- **c** a length of railway track which is 10 m long and has a cross-sectional area of 80 cm^2.



5 Petrol and water do not mix. If the two liquids are poured into a container, they will separate into two layers. Which is the upper layer?

6 Find the volume of:
  a an ingot of lead which weighs 300 kg
  b a lump of ice which weighs 25.6 kg.

7

A jeweller buys a 4 cm by 2 cm by 1 cm bar of platinum for 42 pounds per gram. How much did the jeweller pay for the bar?

F CONVERTING RATES

Rates often need to be converted to different units. We do this so the rate is easier to understand for the situation we are dealing with.

DISCUSSION

2 metres per second is the same rate as 7.2 kilometres per hour. With which rate is it easier to understand just how fast you would be travelling?

Example 8 **Self Tutor**

A petrol bowser pumps petrol at the rate of 600 L per hour. Write this rate in L per minute.

In 1 hour, the bowser pumps 600 L.

There are 60 minutes in 1 hour, so in 1 minute the bowser pumps $\frac{600}{60} = 10$ L

This is a rate of 10 L per minute.

EXERCISE 22F.1

1 A petrol tanker discharges petrol at the rate of 200 L per min. Write this rate in L per hour.

2 Ricki's heart rate is 72 beats per min. Write her heart rate in:
  a beats per day
  b beats per year
  c beats per second.

3 A tap drips at 50 mL per minute. Write this rate in:

a mL per h **b** L per h.

4 Maya's hair grew 8 cm in 3 months. Write how fast her hair is growing in:

a cm per year **b** cm per day **c** mm per day.

5 Lleyton sleeps 8 hours per day. Write his rate of sleep in days per year.

6 A brass rod weighs 0.08 kg per m.

a Convert this rate into g per cm.

b Find the weight of a 27 cm length of this rod.

SPEED CONVERSIONS

Roger rides his bicycle at 36 km per hour. To write his speed in m per s, consider this conversion:

$$\begin{aligned} 36 \text{ km per h} &= \frac{36 \text{ km}}{1 \text{ hour}} \\ &= \frac{36\,000 \text{ metres}}{3600 \text{ seconds}} \qquad \{1 \text{ h} = 60 \text{ min} = 60 \times 60 \text{ s}\} \\ &= 10 \text{ m per s} \end{aligned}$$

So, travelling at 10 m per s is the same as travelling at 36 km per h.

We say that these are **equivalent** rates.

Notice that travelling at 1 m per s is the same as travelling at 3.6 km per h.

So, to convert:

- m per s into km per h we **multiply by 3.6**
- km per h into m per s we **divide by 3.6**

EXERCISE 22F.2

1 Using 10 m per s = 36 km per h, mentally convert to km per hour:

a 20 m per s **b** 50 m per s **c** 5 m per s

2 Mentally convert to m per s:

a 72 km per h **b** 144 km per h **c** 9 km per h

Example 9 **Self Tutor**

a A sprinter runs at 11 m per s. Convert this to km per h.

b An aeroplane travels at 900 km per h. Convert this to m per s.

a
$$\begin{aligned} &11 \text{ m per s} \\ &= (11 \times 3.6) \text{ km per h} \\ &= 39.6 \text{ km per h} \end{aligned}$$

b
$$\begin{aligned} &900 \text{ km per h} \\ &= (900 \div 3.6) \text{ m per s} \\ &= 250 \text{ m per s} \end{aligned}$$

3 Convert to km per h:

a 100 m per s **b** 15 m per s **c** 27.6 m per s **d** 500 m per s

4 Convert to m per s:

a 60 km per h **b** 100 km per h **c** 82.8 km per h **d** 630 km per h

Example 10 **Self Tutor**

A 400 m sprinter finishes a race in 45 seconds.

Find his speed in km per h.

$$\begin{aligned}\text{speed} &= \frac{400\text{ m}}{45\text{ sec}}\\ &= 8.888\,8\ldots\ldots \text{ m per s}\\ &= (8.888\,8\ldots\ldots \times 3.6) \text{ km per h}\\ &= 32 \text{ km per h}\end{aligned}$$

5 Find the following speeds in km per h:

a A sprinter runs 200 m in 24 seconds.

b A greyhound races 500 m in 29 seconds.

c A horse gallops 3200 m in 3 minutes and 20 seconds.

d A swimmer covers 1500 m in 15 minutes.

KEY WORDS USED IN THIS CHAPTER

- average speed
- rate
- time
- density
- ratio
- unit cost
- distance
- slope
- volume
- mass
- speed

LINKS
click here

HOW MUCH OXYGEN DOES A PERSON NEED?

Areas of interaction:
Environments, Health and social education

REVIEW SET 22A

1 A petrol pump delivers 45 litres of petrol into a car in 3 minutes. Write this rate in litres per minute.

2 Convert:

a 80 beats per minute into beats per day **b** 18 m per s to km per h

3 Which of the blocks of chocolate is the better value for money?

$2.19

$1.45

4 At a local market it costs €6.50 to buy 2.6 kg of apples.

- **a** Find the price per kilogram of these apples.
- **b** How much would it cost to buy 3.4 kg of these apples?

5 A German train travels 925 km in 6 hours, while a car on the autobahn travels 85 km in 65 minutes. Which travels at the faster rate?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6 Ben drives his motorboat for 4 hours. In this time he covers a distance of 76 km, and uses 15 L of petrol. Find:

- **a** Ben's average speed in km per h
- **b** the petrol consumption of the boat in km per L.

7 Dennis lives 27 km from his work. If he leaves home at 8:20 am and drives at an average speed of 36 km per hour, will he get to work by 9 am?

8 Find the density of a block of wood which is 2 m × 12 cm × 5 cm and has mass 10.56 kg.

9 Given that lead has a density of 11.3 g per cm^3, find the volume of a bar of lead with mass 16.5 kg.

10 A cyclist travels 800 metres in 1 minute and 35 seconds. Find the speed of the cyclist in km per h.

REVIEW SET 22B

1 Trevor receives €108 for 8 hours of work. What is his hourly rate of pay?

2 Which of these bottles is the better value for money?

3 Convert:

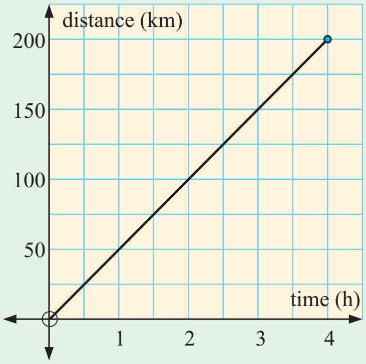
- **a** 50 km per h to m per s
- **b** 5 cm per day to m per year

4 Aslan made a 6 minute call on his mobile phone which cost RM 7.24. Li made a 13 minute call on her mobile which cost RM 15.52. Who is charged at the cheaper rate?

5 Water from a hose will fill a 2 L bucket in 10 seconds. How long will it take to fill a 12 L tank?

6 The graph shows the progress of a car as it travels between cities.

- **a** How far does the car travel in 3 hours?
- **b** How long does it take for the car to travel 100 km?
- **c** Find the speed of the car.



7 **a** How far will a runner travel in 2 hours and 30 minutes if her average speed is 13 km per h?

- **b** How long will it take for a bus to travel 60 km at an average speed of 48 km per h?

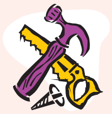
8 Find the density of an 32 g sinker with a volume of 6.8 cm^3.

9 Use the table of densities on page **436** to answer the following:

- **a** Which is heavier: a cube of iron with sides 8 cm or a cube of gold with sides 6 cm?
- **b** What is the volume of a 220 kg bar of aluminium?

10 Emily watches 3 hours of television per day. Write her rate of television watching in days per year.

ACTIVITY FINDING YOUR DAY OF BIRTH



Most people can remember their date of birth but have no idea on which day of the week they were born. The following method enables you to find the day of birth for anyone born in the **20th century**.

What to do:

1. Record the last two digits of the year in which you were born. Call this number A.
2. Divide A by 4 and if there is a remainder ignore it. Call the whole number part B.

3 Choose a third number C according to the month of your birth and the table below:

January	*February*	*March*	*April*	*May*	*June*
1	4	4	0	2	5
July	*August*	*September*	*October*	*November*	*December*
0	3	6	1	4	6

Note: If the year of your birth was a leap year, use 0 for January and 3 for February.

4 Call the date in the month of your birthday D.

5 Divide $A+B+C+D$ by 7. Find the remainder in the table below and hence determine the day of the week on which you were born.

Day	*Sunday*	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*	*Saturday*
Remainder	1	2	3	4	5	6	0

For example: Suppose a person was born on the 20th November 1944.
In this case $A = 44$, $B = 11$, $C = 4$, and $D = 20$
$\therefore$ $A+B+C+D = 79$, which has remainder 2 when divided by 7.
So, the person was born on a *Monday*.

6 See if you can find a set of instructions for finding the **day of birth** for anyone born in the **21st century**.

ACTIVITY THE GUESSING GAME

This is a game for **two players**. All you need is a pencil and paper.

What to do:

1 Player A thinks of any three digit number from 0 0 0 to 9 9 9 where *all three digits are different*, and secretly records this number on paper. For example, suppose player A writes down 2 0 5.

2 Player B tries to guess player A's secret number and tells A the guess.

3 Player A records the guess in a table, along with information about the guess which player B can then use.
The column marked ✓ shows the number of correct digits in the correct places.
The column marked ? shows the number of correct digits in wrong places.

Number	✓	?
2 3 1	1	0
⋮		

4 Player B makes another guess which is then recorded, and so on.

Number	✓	?
2 3 1	1	0
0 2 1	0	2

5 When B has correctly guessed the three digit number the players reverse roles.

The winner is the player who guesses the correct number in the **smallest number of guesses**.

Chapter 23

Algebraic fractions

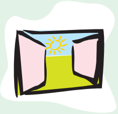
Contents:

Algebraic fractions are fractions which contain at least one variable or unknown.

The variable may be in the numerator, the denominator, or in both of these places.

For example, $\frac{x}{2}$, $\frac{4}{1-a}$ and $\frac{2p-1}{3q}$ are all algebraic fractions.

OPENING PROBLEM



$\frac{3}{4}+\frac{2}{3}$ can be simplified to $\frac{17}{12}$ or $1\frac{5}{12}$.

Can $\frac{x+1}{4}+\frac{2x-1}{3}$ be simplified to a single fraction?

The skills you have developed for dealing with number fractions will also allow you to work with **algebraic fractions**.

We simplify, add, subtract, multiply and divide algebraic fractions in the same way as we do ordinary fractions.

For example:

- $\frac{2}{7}+\frac{4}{7}=\frac{2+4}{7}$ leads to $\frac{a}{d}+\frac{b}{d}=\frac{a+b}{d}$
- $\frac{5}{7}-\frac{2}{7}=\frac{5-2}{7}$ leads to $\frac{a}{d}-\frac{b}{d}=\frac{a-b}{d}$
- $\frac{2}{3}\times\frac{5}{7}=\frac{2\times 5}{3\times 7}$ leads to $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$

A SIMPLIFYING ALGEBRAIC FRACTIONS

We have seen previously that number fractions can be simplified by cancelling common factors.

For example: $\frac{6}{9}=\frac{2\times\cancel{3}^{1}}{3\times\cancel{3}_{1}}=\frac{2}{3}$ where we cancel the common factor of 3 in the numerator and denominator.

If the numerator and denominator of an algebraic fraction are both written in factored form and common factors are found, we can simplify by **cancelling the common factors**.

For example:

$$\begin{aligned}\frac{2xy}{4x} &= \frac{{}^{1}\cancel{2}\times{}^{1}\cancel{x}\times y}{{}_{1}\cancel{2}\times 2\times\cancel{x}_{1}} && \{\text{fully factorised}\}\\ &= \frac{y}{2} && \{\text{after cancellation}\}\end{aligned}$$

Example 1 **Self Tutor**

Simplify: **a** $\frac{ad}{2a}$ **b** $\frac{ab}{ac}$ **c** $\frac{3a^2b}{6ab}$

a $\dfrac{ad}{2a}$

$= \dfrac{{}^{1}\cancel{a} \times d}{2 \times \cancel{a}_{1}}$

$= \dfrac{d}{2}$

b $\dfrac{ab}{ac}$

$= \dfrac{{}^{1}\cancel{a} \times b}{{}_{1}\cancel{a} \times c}$

$= \dfrac{b}{c}$

c $\dfrac{3a^2b}{6ab}$

$= \dfrac{{}^{1}\cancel{3} \times a \times \overset{1}{\cancel{a}} \times \cancel{b}^{1}}{2 \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{a}} \times \cancel{b}_{1}}$

$= \dfrac{a}{2}$

EXERCISE 23A

1 Simplify:

a $\dfrac{15}{3x}$ **b** $\dfrac{8a}{4}$ **c** $\dfrac{2x}{x}$ **d** $\dfrac{3c}{6c}$

2 Simplify, if possible:

a $\dfrac{bc}{b}$ **b** $\dfrac{b}{bd}$ **c** $\dfrac{ab}{an}$ **d** $\dfrac{a^2}{a}$

e $\dfrac{a}{a^2}$ **f** $\dfrac{2a}{a^2}$ **g** $\dfrac{n^2}{n^2}$ **h** $\dfrac{t^3}{t}$

i $\dfrac{10a^2}{2a}$ **j** $\dfrac{4ab^2}{2ab}$ **k** $\dfrac{4a^2b}{8a}$ **l** $\dfrac{4abc}{8a^2}$

B MULTIPLYING ALGEBRAIC FRACTIONS

To **multiply** two or more fractions, we multiply the numerators to form the new numerator, and we multiply the denominators to form the new denominator.

$$\frac{\boldsymbol{a}}{\boldsymbol{b}} \times \frac{\boldsymbol{c}}{\boldsymbol{d}} = \frac{\boldsymbol{a} \times \boldsymbol{c}}{\boldsymbol{b} \times \boldsymbol{d}} = \frac{\boldsymbol{ac}}{\boldsymbol{bd}}$$

Having multiplied the numerators and denominators, we look to cancel common factors and hence simplify the answer.

Example 2 **Self Tutor**

Find: **a** $\dfrac{a}{b} \times \dfrac{b}{2}$ **b** $\dfrac{n}{3} \times 6$

a $\dfrac{a}{b} \times \dfrac{b}{2}$

$= \dfrac{a \times \cancel{b}^{1}}{{}_{1}\cancel{b} \times 2}$

$= \dfrac{a}{2}$

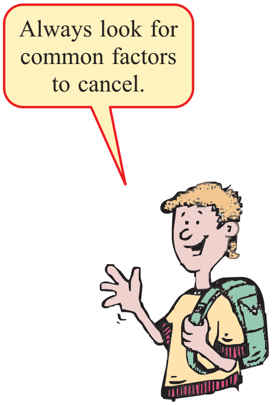
b $\dfrac{n}{3} \times 6$

$= \dfrac{n}{3} \times \dfrac{6}{1}$

$= \dfrac{n \times \cancel{6}^{2}}{{}_{1}\cancel{3} \times 1}$

$= \dfrac{2n}{1}$

$= 2n$


Always look for common factors to cancel.

EXERCISE 23B

1 Find:

a $\frac{1}{2} \times \frac{x}{2}$
b $\frac{1}{2} \times \frac{2}{x}$
c $\frac{x}{2} \times \frac{2}{x}$
d $\frac{x}{2} \times \frac{x}{2}$
e $\frac{a}{2} \times \frac{2}{3}$
f $\frac{2}{a} \times \frac{3}{2}$
g $\frac{a}{2} \times \frac{a}{3}$
h $\frac{a}{2} \times \frac{3}{a}$

2 Find:

a $\frac{a}{2} \times \frac{4}{a}$
b $\frac{a}{2} \times \frac{a}{4}$
c $\frac{n}{2} \times 8$
d $\frac{2}{n} \times 8$
e $\frac{a}{2} \times 3$
f $4 \times \frac{m}{2}$
g $\frac{b}{10} \times 5$
h $6 \times \frac{1}{a}$
i $\frac{c}{d} \times \frac{2}{3}$
j $\frac{m}{3} \times \frac{3}{m}$
k $\frac{a}{b} \times \frac{2b}{a}$
l $\frac{3a}{2} \times \frac{b}{3}$
m $\left(\frac{2}{m}\right)^2$
n $\frac{a}{b} \times \frac{1}{b}$
o $p \times \frac{1}{p}$
p $\frac{a^2}{2} \times \frac{4}{a^3}$

3 Find:

a $\left(\frac{c}{3}\right)^2$
b $2 \times \frac{1}{b}$
c $\frac{1}{x} \times 5$
d $2 \times \frac{2}{n}$
e $k \times \frac{1}{k}$
f $\frac{1}{k} \times \frac{1}{k}$
g $\frac{m}{3} \times \frac{6}{m}$
h $\frac{m}{3} \times \frac{m}{6}$
i $\frac{3}{m} \times \frac{m}{6}$
j $\frac{3}{m} \times \frac{6}{m}$
k $\frac{2a}{3} \times \frac{6}{a}$
l $\frac{21}{b} \times \frac{b}{7}$
m $\frac{a}{3} \times \frac{6}{a}$
n $\frac{a}{3} \times \frac{a}{6}$
o $\frac{5}{a} \times \frac{a}{15}$
p $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a}$

C DIVIDING ALGEBRAIC FRACTIONS

To **divide** by a fraction we multiply by its reciprocal.

$$\boldsymbol{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}}$$

So, to divide by a fraction we follow these steps:

Step 1: Write the second fraction as a reciprocal and change the divide to multiply.
Step 2: Multiply numerators and multiply denominators.
Step 3: Cancel any common factors.
Step 4: Write your answer in simplest form.

Example 3 **Self Tutor**

Find: **a** $\dfrac{a}{b} \div 3$ **b** $4 \div \dfrac{2}{x}$

a
$$\begin{aligned} &\frac{a}{b} \div 3 \\ &= \frac{a}{b} \div \frac{3}{1} \\ &= \frac{a}{b} \times \frac{1}{3} \\ &= \frac{a}{3b} \end{aligned}$$

b
$$\begin{aligned} &4 \div \frac{2}{x} \\ &= \frac{4}{1} \times \frac{x}{2} \\ &= \frac{{}^{2}\cancel{4} \times x}{1 \times \cancel{2}_{1}} \\ &= \frac{2x}{1} \\ &= 2x \end{aligned}$$

EXERCISE 23C

1 Find:

a $\dfrac{a}{b} \div \dfrac{1}{b}$ **b** $\dfrac{a}{b} \div b$ **c** $b \div \dfrac{a}{b}$ **d** $b \div \dfrac{b}{a}$

e $\dfrac{a}{2} \div \dfrac{a}{3}$ **f** $\dfrac{a}{2} \div \dfrac{3}{a}$ **g** $\dfrac{2}{a} \div \dfrac{3}{a}$ **h** $\dfrac{2}{a} \div \dfrac{a}{3}$

i $a \div \dfrac{1}{a}$ **j** $2 \div \dfrac{1}{x}$ **k** $\dfrac{1}{x} \div 5$ **l** $\dfrac{1}{b} \div b$

2 Find:

a $\dfrac{3a}{4} \div \dfrac{a^2}{2}$ **b** $\dfrac{t}{8} \div \dfrac{t^2}{12}$ **c** $\dfrac{a}{b} \div \dfrac{ab}{c}$ **d** $\dfrac{2a^2}{b} \div \dfrac{a}{6}$

D ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

The rules for the addition and subtraction of algebraic fractions are identical to those used with numerical fractions. We identify the **lowest common denominator (LCD)**, then write both fractions with this denominator.

To **add** two or more fractions we obtain the *lowest common denominator* then add the resulting numerators.

$$\boldsymbol{\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}}$$

To **subtract** two or more fractions we obtain the *lowest common denominator* then subtract the resulting numerators.

$$\boldsymbol{\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}}$$

We have previously seen how the lowest common denominator of number fractions is the **lowest common multiple of the denominators**.

For example, when adding $\frac{1}{2}+\frac{1}{5}$, the lowest common denominator is 10.

We use the same method when dealing with algebraic fractions.

Example 4 Self Tutor

Write as a single fraction: **a** $\dfrac{a}{6}+\dfrac{2a}{3}$ **b** $\dfrac{2x}{3}-\dfrac{x}{2}$

a
$$\begin{aligned} &\frac{a}{6}+\frac{2a}{3} \qquad \{\text{LCD} = 6\}\\ &= \frac{a}{6}+\frac{2a\times 2}{3\times 2}\\ &= \frac{a}{6}+\frac{4a}{6}\\ &= \frac{a+4a}{6}\\ &= \frac{5a}{6} \end{aligned}$$

b
$$\begin{aligned} &\frac{2x}{3}-\frac{x}{2} \qquad \{\text{LCD} = 6\}\\ &= \frac{2x\times 2}{3\times 2}-\frac{x\times 3}{2\times 3}\\ &= \frac{4x}{6}-\frac{3x}{6}\\ &= \frac{4x-3x}{6}\\ &= \frac{x}{6} \end{aligned}$$

Example 5 Self Tutor

Write as a single fraction: **a** $3+\dfrac{x}{7}$ **b** $\dfrac{c}{3}-2$

a
$$\begin{aligned} &3+\frac{x}{7}\\ &= \frac{3}{1}+\frac{x}{7} \qquad \{\text{LCD} = 7\}\\ &= \frac{3\times 7}{1\times 7}+\frac{x}{7}\\ &= \frac{21+x}{7} \end{aligned}$$

b
$$\begin{aligned} &\frac{c}{3}-2\\ &= \frac{c}{3}-\frac{2}{1} \qquad \{\text{LCD} = 3\}\\ &= \frac{c}{3}-\frac{2\times 3}{1\times 3}\\ &= \frac{c-6}{3} \end{aligned}$$

EXERCISE 23D.1

1 Write as a single fraction:

a $\dfrac{3b}{5}+\dfrac{b}{5}$ **b** $\dfrac{a}{4}+\dfrac{3}{4}$ **c** $\dfrac{2x}{3}+\dfrac{1}{3}$ **d** $\dfrac{b}{2}+\dfrac{b}{3}$

e $\dfrac{a}{2}-\dfrac{a}{4}$ **f** $\dfrac{x}{3}+\dfrac{x}{6}$ **g** $\dfrac{5b}{6}-\dfrac{b}{9}$ **h** $\dfrac{a}{4}-\dfrac{b}{3}$

2 Write as a single fraction:

a $\frac{n}{4}+1$ **b** $2+\frac{y}{5}$ **c** $3-\frac{c}{2}$ **d** $\frac{n}{3}+n$

e $x+\frac{x}{2}$ **f** $\frac{m}{3}+1$ **g** $1-\frac{3x}{4}$ **h** $\frac{x}{5}+x$

Example 6 **Self Tutor**

Write as a single fraction: $\frac{x}{2}+\frac{x-1}{5}$

$$\frac{x}{2}+\frac{(x-1)}{5}$$

$$=\frac{x\times 5}{2\times 5}+\frac{2\times(x-1)}{2\times 5} \quad \{\text{LCD} = 10\}$$

$$=\frac{5x+2(x-1)}{10} \quad \{\text{simplifying}\}$$

$$=\frac{5x+2x-2}{10} \quad \{\text{expanding the brackets}\}$$

$$=\frac{7x-2}{10} \quad \{\text{simplifying}\}$$

If we have a more complex numerator then we place brackets around it. This helps to make sure we get our signs correct.

Example 7 **Self Tutor**

Write as a single fraction: $\frac{x+2}{4}+\frac{2x-1}{7}$

$$\frac{(x+2)}{4}+\frac{(2x-1)}{7} \quad \{\text{place brackets around numerators}\}$$

$$=\frac{7\times(x+2)}{7\times 4}+\frac{4\times(2x-1)}{4\times 7} \quad \{\text{LCD} = 28\}$$

$$=\frac{7(x+2)+4(2x-1)}{28} \quad \{\text{combining the fractions}\}$$

$$=\frac{7x+14+8x-4}{28} \quad \{\text{expanding the brackets}\}$$

$$=\frac{15x+10}{28} \quad \{\text{collecting like terms}\}$$

3 Write as a single fraction:

a $\frac{x}{2}+\frac{x+1}{3}$ **b** $\frac{x}{5}+\frac{x+2}{4}$ **c** $\frac{2x}{3}+\frac{2x+1}{6}$

d $\frac{x+1}{3}+\frac{x+4}{9}$ **e** $\frac{x+3}{10}+\frac{x}{5}$ **f** $\frac{x+3}{7}+\frac{x+6}{14}$

g $\frac{x+1}{3}+2$ **h** $3+\frac{x-1}{4}$ **i** $\frac{2x+1}{6}+2$

j $\frac{2x+3}{4}+\frac{x-1}{5}$ **k** $\frac{3x+2}{4}+\frac{2x+1}{2}$ **l** $\frac{2x-1}{3}+\frac{2x+5}{9}$

m $\frac{x+1}{10}+\frac{2x-1}{5}$ **n** $\frac{x-8}{3}+\frac{2x-3}{5}$ **o** $\frac{4x+7}{6}+\frac{5x-1}{3}$

p $\frac{1+3x}{7}+\frac{2x-3}{14}$ **q** $\frac{5x+6}{4}+\frac{3x+2}{3}$ **r** $\frac{7x-2}{8}+\frac{6x-1}{4}$

s $\frac{8x-5}{2}+\frac{9x+7}{10}$ **t** $\frac{9x-5}{6}+\frac{4x-5}{9}$ **u** $\frac{5x-8}{7}+\frac{2x+3}{4}$

SUBTRACTING MORE COMPLICATED FRACTIONS

INVESTIGATION SUBTRACTING ALGEBRAIC FRACTIONS

Jon said to Su that subtracting complicated algebraic fractions is just as easy as adding them. He wrote out the following example:

$$\begin{aligned}
&\frac{2x-1}{3}-\frac{x-1}{4}\\
&=\frac{(2x-1)}{3}-\frac{(x-1)}{4}\\
&=\frac{4\times(2x-1)}{4\times 3}-\frac{3\times(x-1)}{3\times 4}\\
&=\frac{4(2x-1)}{12}-\frac{3(x-1)}{12}\\
&=\frac{8x-4}{12}-\frac{3x-3}{12}\\
&=\frac{8x-4-3x-3}{12}\\
&=\frac{5x-7}{12}
\end{aligned}$$

Su said to Jon that his working must contain at least one error since, when $x=1$,

$$\begin{aligned}
&\frac{2x-1}{3}-\frac{x-1}{4}\\
&=\frac{2-1}{3}-\frac{1-1}{4}\\
&=\tfrac{1}{3}-\tfrac{0}{4}\\
&=\tfrac{1}{3}
\end{aligned}
\qquad \text{whereas} \qquad
\begin{aligned}
&\frac{5x-7}{12}\\
&=\frac{5-7}{12}\\
&=\tfrac{-2}{12}\\
&=-\tfrac{1}{6}
\end{aligned}$$

What to do: Find the error in Jon's working.

From the **Investigation** you should have discovered the importance of brackets when working with fractions. They help to make sure we get the signs correct. In particular, when we combine fractions in a subtraction, the numerator of the second fraction should be placed in brackets. In Jon's solution, he should have written:

$$\begin{aligned}\frac{8x-4}{12}-\frac{3(x-1)}{12}&=\frac{(8x-4)-3(x-1)}{12}\\&=\frac{8x-4-3x+3}{12}\\&=\frac{5x-1}{12}\end{aligned}$$

Example 8

Self Tutor

Write as a single fraction: $2-\dfrac{x+1}{3}$

$$\begin{aligned}&\frac{2}{1}-\frac{(x+1)}{3} && \{\text{place brackets around non-simple numerators}\}\\=\;&\frac{2\times 3}{1\times 3}-\frac{(x+1)}{3} && \{\text{LCD}=3\}\\=\;&\frac{6-(x+1)}{3} && \{\text{combining the fractions}\}\\=\;&\frac{6-x-1}{3} && \{\text{expanding the brackets}\}\\=\;&\frac{5-x}{3} && \{\text{simplifying}\}\end{aligned}$$

Example 9

Self Tutor

Write as a single fraction: $\dfrac{3x+2}{5}-\dfrac{x-4}{6}$

$$\begin{aligned}&\frac{3x+2}{5}-\frac{x-4}{6}\\=\;&\frac{(3x+2)}{5}-\frac{(x-4)}{6} && \{\text{place brackets around non-simple numerators}\}\\=\;&\frac{6(3x+2)}{6\times 5}-\frac{5(x-4)}{5\times 6} && \{\text{LCD}=30\}\\=\;&\frac{6(3x+2)-5(x-4)}{30} && \{\text{simplifying}\}\\=\;&\frac{18x+12-5x+20}{30} && \{\text{expanding the brackets}\}\\=\;&\frac{13x+32}{30} && \{\text{simplifying}\}\end{aligned}$$

EXERCISE 23D.2

1 Write as a single fraction:

a $3 - \frac{x-1}{4}$ **b** $5 - \frac{x+1}{7}$ **c** $4 - \frac{1-x}{6}$

d $x - \frac{x+2}{3}$ **e** $2x - \frac{x-1}{2}$ **f** $3x - \frac{2-x}{4}$

2 Write as a single fraction:

a $\frac{x}{2} - \frac{x+1}{3}$ **b** $\frac{x}{5} - \frac{x+2}{4}$ **c** $\frac{2x}{3} - \frac{2x+1}{6}$

d $\frac{x+1}{3} - \frac{x+4}{9}$ **e** $\frac{x+3}{10} - \frac{x}{5}$ **f** $\frac{x+3}{7} - \frac{x+6}{14}$

g $\frac{x}{7} - \frac{1-2x}{3}$ **h** $\frac{x+3}{4} - \frac{1-x}{5}$ **i** $\frac{3-2x}{3} - \frac{x+1}{6}$

j $\frac{4x+3}{10} - \frac{x-5}{5}$ **k** $\frac{3-x}{2} - \frac{x-4}{6}$ **l** $\frac{x+6}{2} - \frac{1-3x}{7}$

m $\frac{2x+7}{3} - \frac{2x+3}{9}$ **n** $\frac{3x-1}{3} - \frac{2x-5}{8}$ **o** $\frac{3x+7}{6} - \frac{5-2x}{5}$

KEY WORDS USED IN THIS CHAPTER

- algebraic fraction
- lowest common denominator
- common factor
- numerator
- denominator
- reciprocal

REVIEW SET 23A

1 Simplify, if possible:

a $\frac{3c^2}{c}$ **b** $\frac{2a^2}{5b}$ **c** $\frac{k}{3k^3}$ **d** $\frac{3a^2b}{6ab}$

2 Find:

a $\frac{k}{8} \times \frac{2}{k}$ **b** $\frac{m}{2} \div \frac{6}{m}$ **c** $\frac{c}{4} \times \frac{d}{5}$

d $\frac{2a}{3} \div \frac{6}{c}$ **e** $\frac{p}{4} \times \frac{p}{4}$ **f** $2t \times \frac{3t}{4}$

3 Find:

a $\frac{d}{3} \div 2$ **b** $\frac{f}{4} \times \frac{2}{f}$ **c** $\frac{5}{k} \div \frac{k}{2j}$

d $\frac{3m^2}{n} \div \frac{mn}{2}$ **e** $\frac{2}{c^2} \div \frac{d}{c}$ **f** $5t \div \frac{1}{t}$

4 Write as a single fraction:

a $\frac{x}{3}+\frac{x}{4}$ **b** $\frac{5y}{4}-\frac{y}{2}$ **c** $\frac{n}{3}+4$

d $\frac{2k}{3}-\frac{k}{4}$ **e** $2-\frac{2t}{5}$ **f** $2s-\frac{3s}{4}$

5 Write as a single fraction:

a $\frac{x}{3}+\frac{x+5}{2}$ **b** $\frac{x+1}{2}-\frac{x}{4}$ **c** $\frac{x-2}{6}+\frac{2x+3}{12}$

d $\frac{2x+1}{3}-\frac{x+2}{4}$ **e** $\frac{3x-2}{6}-\frac{x-3}{3}$ **f** $\frac{2x-5}{5}-\frac{2-x}{3}$

REVIEW SET 23B

1 Simplify, if possible:

a $\frac{2ab}{6b}$ **b** $\frac{5k^2}{10k}$ **c** $\frac{2m^2n}{5n^2}$ **d** $\frac{3t}{9t^3}$

2 Find:

a $\frac{4}{p}\times\frac{p}{2}$ **b** $\left(\frac{m}{3}\right)^2$ **c** $\frac{x}{6}\div 4$

d $\frac{3t}{s}\div\frac{2s}{t}$ **e** $\frac{d}{12}\times\frac{6d}{5}$ **f** $\frac{2x}{y}\times\frac{x}{6}$

3 Find:

a $\frac{m}{4}\div\frac{m}{3}$ **b** $\frac{3a}{b}\times\frac{ab}{6}$ **c** $\frac{d^2}{3}\div\frac{1}{d}$

d $\frac{2l}{5}\times\frac{ml}{4}$ **e** $\frac{3k^2}{4}\div\frac{k}{2}$ **f** $5t\div\frac{10}{t^2}$

4 Write as a single fraction:

a $\frac{x}{4}+\frac{x}{6}$ **b** $5-\frac{3l}{2}$ **c** $\frac{m}{4}-m$

d $\frac{2c}{3}+\frac{c}{5}$ **e** $\frac{p}{4}-2p$ **f** $\frac{k}{9}-\frac{2k}{15}$

5 Write as a single fraction:

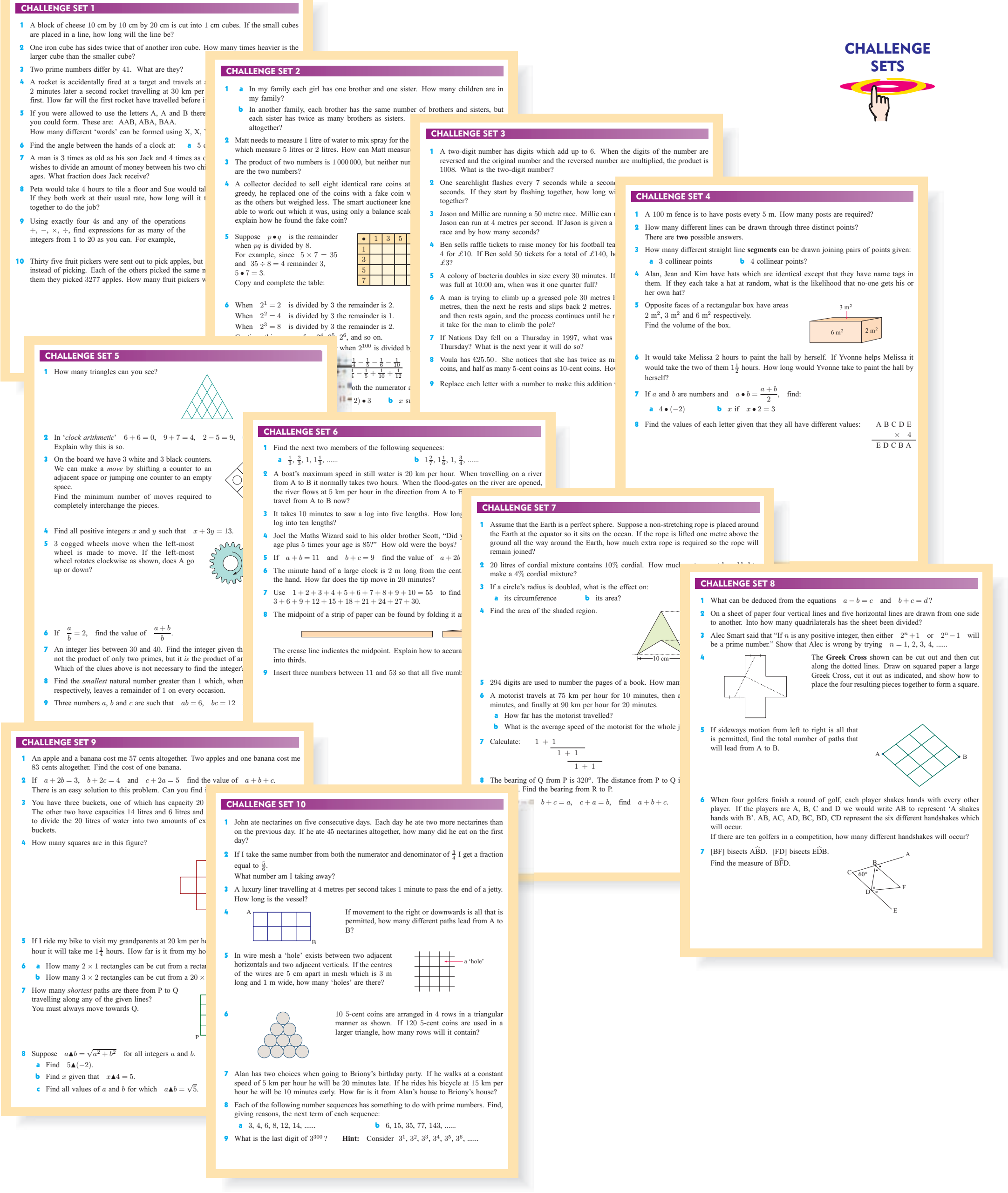
a $\frac{x}{4}-\frac{x+1}{2}$ **b** $\frac{x+2}{5}+\frac{2x+1}{10}$ **c** $\frac{2x-1}{9}-\frac{x-2}{3}$

d $\frac{2x+1}{3}+\frac{x-3}{4}$ **e** $\frac{1-x}{5}-\frac{x-3}{2}$ **f** $\frac{3x-2}{8}-\frac{1-4x}{6}$

CHALLENGE SETS

Click on the icon to access printable Challenge Sets.

CHALLENGE SETS



ANSWERS

EXERCISE 1A

1 **a** 47 **b** 648 **c** 701 **d** 3448
e 625 990 **f** 3 600 973

2 **a** ninety one dollars **b** three hundred and sixty two euros
c four thousand and fifty six dollars
d nine thousand, eight hundred and seven pounds
e forty three thousand six hundred and seventy dollars
f five hundred and seven thousand, eight hundred euros

3 **a** 7 **b** 7 **c** 70 **d** 700 **e** 700 **f** 7000 **g** 7
h 70 000 **i** 70 000 **j** 700 000 **k** 7 000 000 **l** 70

4 **a** 8 **b** 12 **c** 199 **d** 3002 **e** 99

5 **a** Sarah 49 kg, Kylie 57 kg, Lindy 60 kg, Amanda 75 kg
b Hao 138 cm, Xiang 148 cm, Wan 173 cm, Gan 174 cm
c \$1004, one thousand and forty dollars, \$1100
d Barina 708 kg, Excel 808 kg, Laser 880 kg, Corolla 890 kg
e fourteen dollars, forty dollars, forty four dollars,
forty five dollars, fifty four dollars

6 **a** 97 **b** 436 **c** 8004 **d** 5608 **e** 70 065
f 4 000 908

7 **a** $700 + 30$ **b** $4000 + 800 + 70 + 1$
c $60\,000 + 8000 + 900 + 4$
d $700\,000 + 60\,000 + 300 + 90 + 1$

8 **a** 971 **b** 754 310

EXERCISE 1B

1 **a** 40 **b** 70 **c** 100 **d** 150 **e** 200
f 450 **g** 800 **h** 10 000

2 **a** 100 **b** 400 **c** 400 **d** 700 **e** 1000
f 1400 **g** 11 800 **h** 34 000

3 **a** 1000 **b** 6000 **c** 7000 **d** 10 000 **e** 12 000
f 23 000 **g** 53 000 **h** 671 000

4 **a** 70 **b** 200 **c** 300 **d** 300 **e** 1000
f 3000 **g** 7000 **h** 40 000

5 **a** 890 **b** 170 **c** 750 **d** 240 **e** 560
f 5600 **g** 9800 **h** 24 000

6 **a** €35 000 **b** 3700 km **c** \$400 **d** £240 000
e 17 000 **f** 35 000

EXERCISE 1C

1 **a** 24 000 **b** 100 000 **c** 2 400 000 **d** 800 000
e 6 000 000 **f** 21 000 000

2 **a** 10 **b** 40 **c** 200 **d** 75 **e** 300 **f** 50

3 **a** 20 000 papers **b** 200 chairs **c** 100 km per h
d 80 000 passengers **e** 1600 jelly beans **f** €12 000

EXERCISE 1D.1

1 **a** 3 **b** 3 **c** 5 **d** 512 **e** 1 **f** 70 **g** 12 **h** 100

2 **a** 359 **b** 469 **c** 1387 **d** 1163 **e** 767
f 1779 **g** 1777 **h** 2899 **i** 200

3 **a** 20 **b** 37 **c** 55 **d** 156

4 458 **5** 4379 **6** 4398 m
7 \$225 700, \$350 850 each **8** €2743 **9** 62 kg

EXERCISE 1D.2

1 **a** 72 **b** 720 **c** 7200 **d** 30 **e** 300
f 30 000 **g** 91 **h** 9100 **i** 910 000

2 **a** 2 **b** 20 **c** 200 **d** 4 **e** 4 **f** 400
g 7 **h** 7 **i** 70

3 **a** 410 **b** 9100 **c** 11 300 **d** 190 000 **e** 57 000
f 7 890 000 **g** 1 000 000 **h** 9 600 000 **i** 57 000 000

4 **a** 0 **b** undefined **c** 0 **d** 0 **e** 0 **f** 0 **g** 0
h 0 **i** 0 **j** 0 **k** 0 **l** undefined

5 **a** 507 **b** 963 **c** 1989 **d** 14 **e** 39 **f** 57

6 **a** 544 **b** 23 **c** 120

7 **a** 55 **b** 317 **c** 100 000 cabbages
d 90 laps **e** €9384 **f** £17 316 **g** 17 buses

8 **a** 8 floors **b** 200 rooms **c** 84 chairs
d 396 rooms **e** 33 cleaners **f** €2112

EXERCISE 1E

1 **a** $2^2 \times 3^3$ **b** 2×5^2 **c** $2 \times 3^3 \times 5$ **d** $5^2 \times 7^2$
e $2^2 \times 5^3 \times 7$ **f** $3^2 \times 7^2 \times 11^2$ **g** $2^3 \times 3^4$ **h** $5^6 \times 7^3$

2 **a** 30 **b** 20 **c** 56 **d** 270 **e** 396 **f** 24 200

3 **a** 69 984 **b** 222 264 **c** 1 054 152 **d** 2 156 000
e 1 711 125 **f** 21 600 000

4 **a** 2^1 **b** 2^2 **c** 2^4 **d** 2^6

5 **a** 3^1 **b** 3^3 **c** 3^4 **d** 3^6

6 **a** 10^2 **b** 10^3 **c** 10^5 **d** 10^6

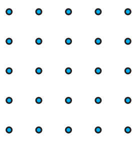
7 **a** 5^2 **b** 6^2 **c** 5^3 **d** 7^3

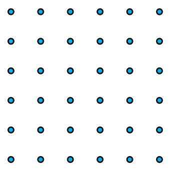
PUZZLE

^{1}3	6	^{2}1	■	^{3}1	6
6	■	^{4}2	5	6	■
■	^{5}2	5	■	^{6}8	1
^{7}2	7	■	^{8}8	1	■
■	^{9}4	8	4	■	^{10}4
^{11}6	4	■	^{12}1	6	9

EXERCISE 1F

1 **a**





b $5^2 = 25$, $6^2 = 36$

2 **a** $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$
b $17^2 = 289$, $20^2 = 400$, $50^2 = 2500$

3 **a** 9, 25 **b** 64, 100

4 **a**
$1^2 = 1$
$11^2 = 121$
$111^2 = 12\,321$
$1111^2 = 1\,234\,321$

b **i** $11\,111^2 = 123\,454\,321$
ii $111\,111^2 = 12\,345\,654\,321$

5 **a**
$$1 = 1 = 1^2$$
$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

b **i** $1 + 3 + 5 + 7 + 9 + 11 = 6^2 = 36$

ii $1+3+5+7+9+11+13+15+17+19$
$=10^2=100$

iii n^2

6 216 cubes **7** 8 high, wide and deep

8 **a** $1^2-0^2=1$ **b** **i** 33
$2^2-1^2=3$ **ii** 177
$3^2-2^2=5$
$4^2-3^2=7$

9 **a**

$$\begin{aligned}1^3 &= 1=1^2\\ 1^3+2^3 &= 9=3^2\\ 1^3+2^3+3^3 &= 36=6^2\\ 1^3+2^3+3^3+4^3 &= 100=10^2\end{aligned}$$

b **i** $1^3+2^3+3^3+4^3+5^3=15^2=225$
ii $1^3+2^3+3^3+......+10^3=55^2=3025$

EXERCISE 1G

1 **a** 5 **b** 11 **c** 11 **d** 7 **e** 99 **f** 21
g 2 **h** 28 **i** 36 **j** 13 **k** 16 **l** 3

2 **a** 30 **b** 18 **c** 1 **d** 50 **e** 26 **f** 18
g 36 **h** 3 **i** 30 **j** 16 **k** 6 **l** 13
m 28 **n** 40 **o** 18

3 **a** 45 **b** 6 **c** 14 **d** 10 **e** 10 **f** 1
g 4 **h** 25 **i** 1 **j** 18

4 **a** 90 **b** 28 **c** 54 **d** 31 **e** 9 **f** 168
g 4 **h** 15

5 **a** 3 **b** 2 **c** 3 **d** 2 **e** 8 **f** 3
g 3 **h** 0 **i** 4

6 **a** 48 **b** 54 **c** 17 **d** 3 **e** 37 **f** 11
g 2 **h** 5 **i** 2 **j** 38 **k** 36 **l** 18

7 **a** $3+15\div3=8$ **b** $10-7+15=18$
c $8\times4-10=22$ **d** $(18+2)\div10=2$
e $(10-3)\div7=1$
f $15\div3+2\times5=15$ or $15+3+2-5=15$

8 **a** $(18-6)\times3+2=38$ **b** $48-6\times(3+4)=6$
c $32\div(8\div2)=8$ **d** $(8+4)\div(2+2)=3$
e $(5+3)\times6-10=38$ **f** $(13+5)\div(5+4)=2$

PUZZLE

1 **a** $a=7,\ c=6$ **b** $d=5,\ e=6$
c $f=1,\ g=3,\ h=8$ **d** $p=2,\ q=1,\ r=9$

2

486	506	256	603	152
× 12	× 14	× 14	× 15	× 30
5832	7084	3584	9045	4560

REVIEW SET 1A

1 four thousand seven hundred and thirty eight **2** 400
3 1823 **4** 48 500 **5** 30 000 **6** 4428 **7** 7 buses
8 **a** 3300 **b** 7000 **9** 377 pencils **10** \$43.20
11 1200 **12** 81 **13** €80 **14** 55 **15** **a** 729 **b** 512

16 **a** 19 **b** 8 **c** 10 **d** 5

17 **a** $4\times(2+3)-5=15$ **b** $20\div(4+1+5)=2$

REVIEW SET 1B

1 0 **2** 3000 **3** 47 000 **4** 3190 **5** 166
6 4020 **7** 97 520 **8** 32 **9** 89 **10** 355
11 219 **12** 3900 **13** 532 **14** 23
15 1 500 000 **16** 18 students **17** £2685 **18** 4

19 **a** 18 **b** 75 **c** 37 **d** 22 **e** $\frac{1}{4}$

20 **a** $9-8\div(1+3)=7$ **b** $(2+8)\div(4-2)=5$

EXTENSION

1 **a** 1_7, 2_7, 3_7, 4_7, 5_7, 6_7, 10_7, 11_7, 12_7, 13_7, 14_7, 15_7, 16_7, 20_7, 21_7, 22_7, 23_7, 24_7, 25_7, 26_7
b 100_7

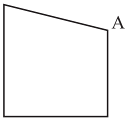
2 **a** 14_7 **b** 35_7 **c** 50_7 **d** 61_7 **e** 82_7
f 120_7 **g** 231_7 **h** 265_7

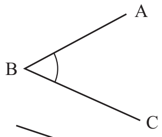
3 **a** 6_{10} **b** 10_{10} **c** 27_{10} **d** 30_{10} **e** 68_{10}
f 84_{10} **g** 105_{10} **h** 236_{10}

4 **a** 164_7 **b** 43_7 **c** 33_7

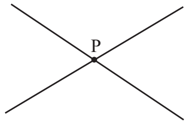
EXERCISE 2A

2 **a** vertex A **b** angle ABC

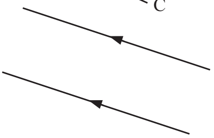




c



d



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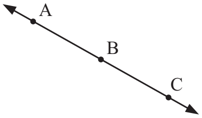
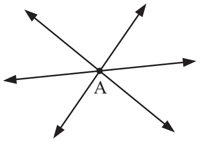


f



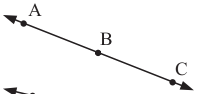
3 **a** (PQ), (QP) **b** (AB), (AC), (BA), (BC), (CA), (CB)

4 **a** B **b** C **c** A

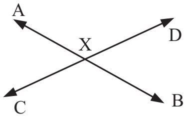
5 **a** [AB], [BC], [CD], [DA] **b** [AC], [BD] **c** X
d 3 **e** collinear **f** concurrent at B

6 **a** 1 **b** 1 **c** infinitely many **d** 0

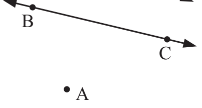
7 **a**



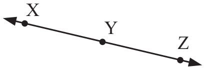
b



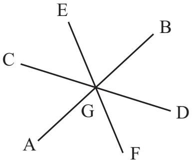
c



d

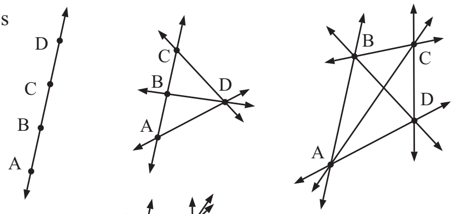


e

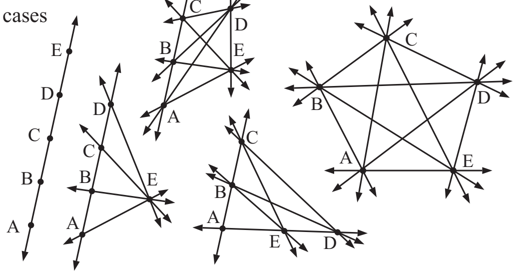


8 **a** any two of (CA), (AE), (EA), (CE), (EC) **b** (AD), (BF)
c **i** collinear **ii** intersect at B **iii** concurrent at D

9 **a** 3 cases



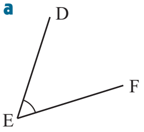
b 5 cases



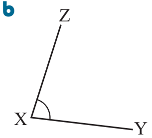
EXERCISE 2B

1 **a** **C** **b** **A** **c** **D** **d** **B**

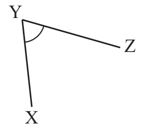
2 **a**



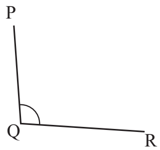
b



c



d



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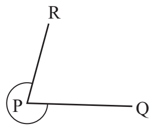
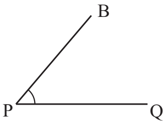
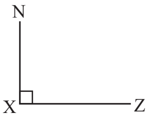


3 **a** 35^o **b** 24^o **c** 59^o **d** 94^o

5 **a**



b



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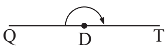
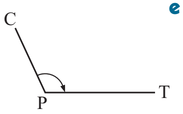
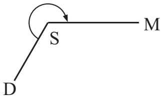


d



e



f

6 **a** **i** 114^o **ii** 66^o **iii** 90^o

b **i** 110^o **ii** 299^o **iii** 340^o

7 28^o **8** 248^o

9 **a** **i** 8 **ii** 8 **b** **i** 8 **ii** 14 **c** **i** 14 **ii** 68

10 **a** 6

b 10

c

No. of vertices	*No. acute angles*
2	1
3	3
4	6
5	10

d **i** 15

ii 45

EXERCISE 2C

1 **a** 90^o, complementary **b** 180^o, supplementary

c 150^o, neither **d** 90^o, complementary

e 170^o, neither **f** 90^o, complementary

2 **a** 60^o **b** 85^o **c** 5^o **3** **a** 80^o **b** 175^o **c** 90^o

4 **a** complementary **b** supplementary **c** neither

d supplementary

5 **a** $(90 - x)^o$ **b** $(180 - y)^o$

6 **a** $a = 65$ **b** $b = 33$ **c** $c = 14$ **d** $d = 60$

e $e = 45$ **f** $f = 26$ **g** $g = 29$ **h** $h = 29$

i $x = 60$ **j** $y = 39$ **k** $t = 44$ **l** $p = 42$

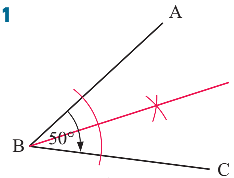
7 **a** $x = 270$ **b** $y = 94$ **c** $z = 333$

8 **a** $p = 130$ **b** $q = 122$ **c** $r = 45$ **d** $s = 42$

e $t = 73$ **f** $u = 16,\ w = 254$

EXERCISE 2D

1



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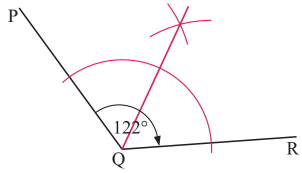
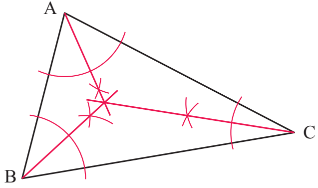
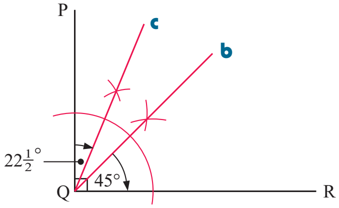


3 **a**



d "The three angle bisectors of a triangle are concurrent (meet at the one point)."

4

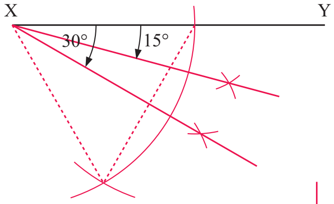


5 **a** XA = XB = AB, as each are equal radii.

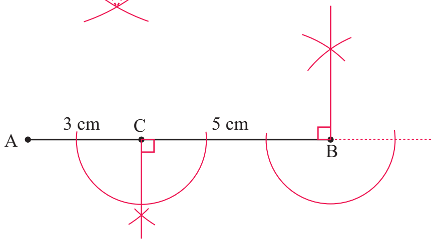
∴ ΔABX is equilateral.

b 60^o

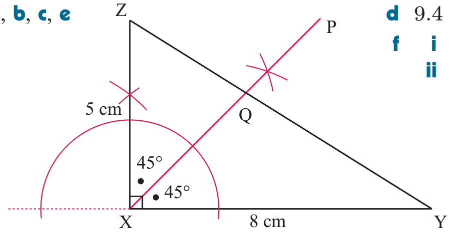
c



6



7 **a**, **b**, **c**, **e**

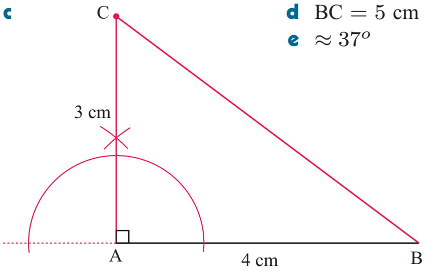


d 9.4 cm

f **i** 5.8 cm

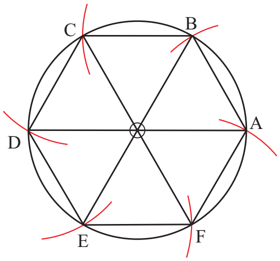
ii 103^o

8 **a**, **b**, **c**



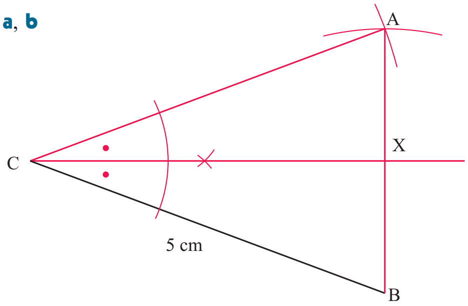
d BC = 5 cm
e ≈ 37^o

9



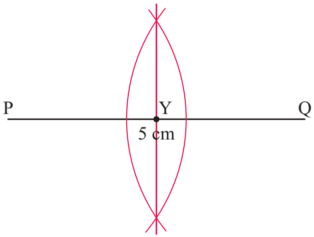
Each of the 6 triangles are identical equilateral triangles.
So, AB = BC = CD = DE = EF = FA.
Also, the angles of the triangles are 60^o and so all interior angles of the hexagon are 120^o.
So, ABCDEF is a regular hexagon.

10 **a**, **b**



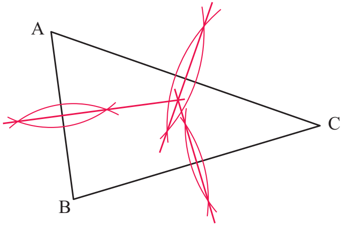
c **i** 2 cm **ii** 2 cm **iii** 47^o **iv** 90^o

11



PY = QY = 2.5 cm

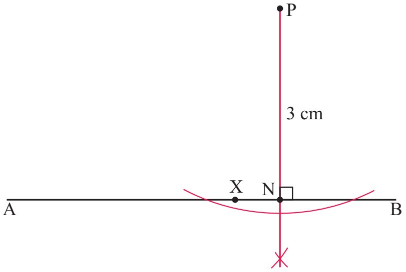
12 **a**



c The 3 perpendicular bisectors seem to meet at the one point.
d "The three perpendicular bisectors of the sides of a triangle are concurrent (meet at the same point)."

13 **c** AP = BP ≈ 4 cm
d Both angles are equal and are ≈ 30^o.

14 **a**



b Let the perpendicular from P meet [AB] at N. If X lies on [AB] not at N, ΔPNX is right angled at N, and PN is always $\leqslant$ PX.
$\therefore$ PN is the shortest distance.

EXERCISE 2E

1 **a** alternate **b** not alternate **c** not alternate
d not alternate

2 **a** p **b** n **c** l **d** m

3 **a** corresponding **b** not corresponding
c corresponding **d** not corresponding

4 **a** l **b** n **c** p **d** m

5 **a** not co-interior **b** not co-interior **c** co-interior
d not co-interior

6 **a** p **b** n **c** l **d** m

7 **a** corresponding **b** alternate **c** co-interior
d corresponding **e** corresponding
f vertically opposite **g** vertically opposite
h co-interior **i** alternate

EXERCISE 2F

1 **a** $a = 126$ {alternate} **b** $a = 49$ {corresponding}
c $a = 99$ {co-interior} **d** $a = 60$ {corresponding}
e $a = 50$ {alternate}
f $a = 72$ {vertically opposite}, $b = 72$ {corresponding}
g $a = 115$ {vertically opposite}, $b = 65$ {co-interior}
h $a = 45$ {angles on a line}, $b = 45$ {corresponding}
i $a = 74$ {co-interior}, $b = 106$ {co-interior}

2 **a** $a = 118$ {co-interior}, $b = 242$ {angles at a point}
b $a = 43$ {alternate}, $b = 43$ {corresponding}
c $a = 50$ {co-interior}, $b = 70$ {co-interior}, $c = 60$ {angles on a line}
d $a = 60$ {corresponding}, $b = 80$ {alternate}
e $a = 40$ {alternate}, $b = 70$ {alternate}, $c = 40$ {alternate}
f $a = 40$ {alternate}, $b = 140$ {angles on a line}, $c = 140$ {co-interior}

3 **a** $x = y$ {corresponding} **b** $x + y = 180$ {co-interior}
c $x + y = 180$ {co-interior} **d** $x = y$ {alternate}

4 $a = 40$ {alternate}, $b = 40$ {vertically opposite}, $c = 40$ {alternate}, $d = 90$ {corresponding}, $e = 90$ {vertically opposite}, $f = 50$ {angles on a line}, $g = 50$ {corresponding}

5 **a** parallel {alternate angles equal}
b parallel {corresponding angles equal}
c not parallel {alternate angles not equal}
d not parallel {co-interior angles not supplementary}

e parallel {corresponding angles equal}
f not parallel {co-interior angles not supplementary}

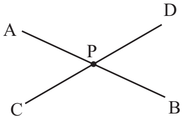
REVIEW SET 2A

1 **a** 49^o **b** 360^o **c** 162^o

2 **a** **i** 6 **ii** 4 **iii** 2 **b** **i** reflex **ii** obtuse **iii** acute

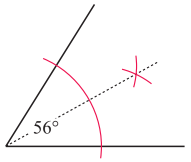
3 **a** $n = 61$ **b** $x = 11$ **c** $y = 120$

4 2 points **5**



6 **a** any two of (SR), (ST), (TS), (RT), (TR)
b any two of (PQ), (PU), (PR), (PS), (QR), (US), (QP), (UP), (RP), (SP), (RQ), (SU)
c **i** collinear **ii** intersect at R

7

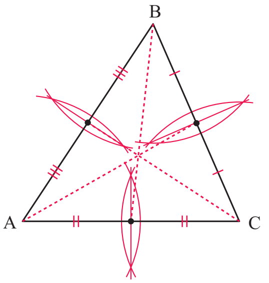


8 **a** g **b** e **c** b **d** d

9 **a** $m = 46$ {vertically opposite}
b $m = 116$ {corresponding}
c $m = 119$ {co-interior}
d $m = 45$ {vertically opposite}

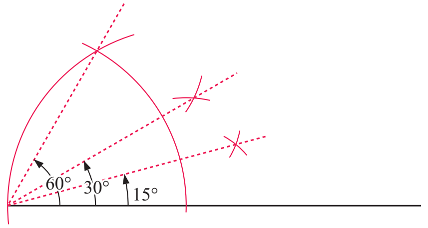
10 **a** not parallel, alternate angles not equal
b not parallel, corresponding angles not equal

11 **a**



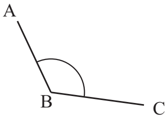
b All 3 medians of a triangle meet at the one point.

12



REVIEW SET 2B

1 **a**



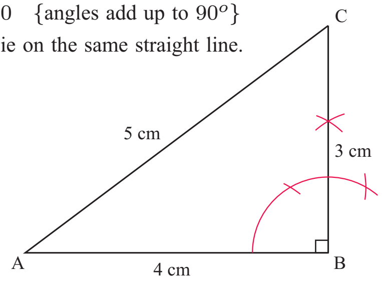
b 72^o
c 338^o

2 **a** 5 **b** 1 **c** 2 **3** **a** 27^o **b** 110^o

4 **a** $y = 57$ {supplementary angles}
b $y = 139$ {angles at a point}
c $y = 30$ {angles add up to 90^o}

5 ...... they lie on the same straight line.

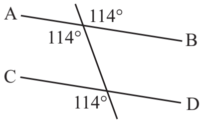
6



7 **a** $x = 70$ {alternate angles}
b $x = 110$ {corresponding angles}
c $x = 41$ {vertically opposite}

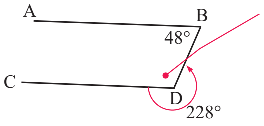
8 **a** $x + y = 180$ {co-interior angles}
b $p = q$ {opposite angles of a parallelogram}

9 **a**



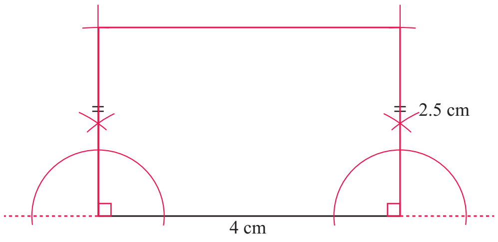
Are parallel {vertically opposite and then corresponding angles are equal}

b



This angle is
$360^o - 228^o = 132^o$
{angles at a point}
Then $48^o + 132^o = 180^o$
$\therefore$ [AB] $\parallel$ [CD]
{co-interior angles add to 180^o}

10



EXERCISE 3A

1 **a** true **b** true **c** false **d** false **e** true **f** true
g false **h** true **i** true

2 **a** yes **b** yes **c** no **d** yes **e** no **f** yes
g yes **h** no **i** yes **j** yes **k** yes **l** yes

3 **a** no **b** no **c** yes **d** no

4 **a** yes **b** yes **c** no **d** yes

5 **a** 2, 3, 4, 5, 10 **b** 2, 4 **c** 2, 3, 4, 5, 10 **d** 3

6 **a** 2, 5, 8 **b** 3, 6, 9 **c** 2, 5, 8 **d** 1, 4, 7

7 **a** divisible by 3 and divisible by 4
b divisible by 3 and divisible by 5
c divisible by 3 and divisible by 8

8 3, 6, 9, 12, 15, 18 **9** 210

EXERCISE 3B.1

1 **a** 1, 3, 9 **b** 1, 2, 3, 4, 6, 12 **c** $12 = 2 \times 6$ **d** 3×4

2 **a** 1, 2, 5, 10 **b** 1, 2, 3, 6, 9, 18
c 1, 2, 3, 5, 6, 10, 15, 30 **d** 1, 5, 7, 35

e 1, 2, 4, 11, 22, 44 **f** 1, 2, 4, 7, 8, 14, 28, 56
g 1, 2, 5, 10, 25, 50
h 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84 **i** 1, 3, 13, 39
j 1, 2, 3, 6, 7, 14, 21, 42 **k** 1, 2, 3, 6, 11, 22, 33, 66
l 1, 3, 5, 15, 25, 75

3 **a** 4 **b** 5 **c** 7 **d** 20 **e** 8 **f** 44 **g** 18 **h** 12
i 4 **j** 7 **k** 11 **l** 12 **m** 5 **n** 4 **o** 12

4 **a** 6 **b** 9 **c** 9 **d** 24 **e** 22 **f** 25 **g** 45 **h** 13

5 **a** 30 **b** 105 **c** 210

6 **a** is even **b** is odd **c** is even **d** is even

7 **a** even **b** odd **c** even **d** odd **e** even **f** odd

8 59

EXERCISE 3B.2

1 **a** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
b No, prime has exactly two factors, 1 and itself. **c** Yes, 2.

2 **a** $5485 = 5 \times 1097$ **b** $8230 = 2 \times 4115$
c $7882 = 2 \times 3941$ **d** $999 = 3 \times 333$

3 **a** $2^3 \times 3^1$ **b** $2^2 \times 7^1$ **c** $3^2 \times 7^1$ **d** $2^3 \times 3^2$
e $2^3 \times 17^1$ **f** $2^2 \times 3^1 \times 7^1$ **g** $2^3 \times 3^3$
h $2^4 \times 3^1 \times 11^1$ **i** $3^4 \times 5^1$ **j** $2^4 \times 7^2$
k $2^1 \times 3^1 \times 23^1$ **l** $2^1 \times 5^3$ **m** $3^3 \times 7^1$
n $2^1 \times 3^1 \times 11^2$ **o** $2^1 \times 3^1 \times 5^1 \times 7^1 \times 11^1$

4 **a** 3 **b** 15 **c** 5 **5** 11

6 **a** **i** 3 **ii** 3 **iii** 3 **iv** 9
b perfect squares of whole numbers
c **ii** Numbers with an even number of factors finish closed whereas those with an odd number finish open. So, lockers with perfect square numbers finish open.
iii They are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961
In all there are 31 lockers open.

EXERCISE 3B.3

1 **a** 3 **b** 8 **c** 6 **d** 14 **e** 6 **f** 8 **g** 12
h 3 **i** 24 **j** 36 **k** 3 **l** 26

2 **a** 25 **b** 11 **c** 21 **d** 13

3 **a** 5 **b** 4 **c** 2 **d** 8

EXERCISE 3C.1

1 **a** 3, 6, 9, 12, 15, 18 **b** 8, 16, 24, 32, 40, 48
c 12, 24, 36, 48, 60, 72 **d** 17, 34, 51, 68, 85, 102
e 25, 50, 75, 100, 125, 150
f 34, 68, 102, 136, 170, 204

2 **a** 30 **b** 40 **c** 150 **d** 4900

3 **a** 1, 2, ③, $\boxed{4}$, 5, ⑥, 7, $\boxed{8}$,
⑨, 10, 11, $\boxed{⑫}$, 13, 14, ⑮, $\boxed{16}$,
17, ⑱, 19, $\boxed{20}$, ㉑, 22, 23, $\boxed{㉔}$,
25, 26, ㉗, $\boxed{28}$, 29, ㉚
c 12, 24

4 **a** 30, 60, 90, 120, 150 **b** 45, 90, 135
c 60, 120 **d** 60, 120

5 **a** 63, 99 **b** 35

EXERCISE 3C.2

1 **a** 6 **b** 12 **c** 40 **d** 60 **e** 24 **f** 24 **g** 45
h 70

2 **a** 12 **b** 180 **c** 36 **d** 84 **3** 60 **4** 36 m **5** 210

6 36 seconds **7** **a** 204 **b** 495

EXERCISE 3D.1

1 **a** -7 **b** 3 **c** 0 **d** -2.8 **e** 3.17 **f** 0.6731

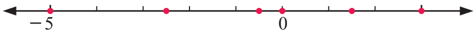
2 **a** 8 **b** 4 **c** 1 **d** 0 **e** -4

3 **a** 2 **b** -4 **c** -4 **d** -7 **e** $-6\frac{1}{2}$

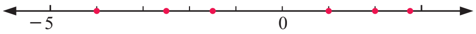
4 **a** 5 **b** 9 **c** 9 **d** 9 **e** 7 **f** 10

5 **a** true **b** false **c** false **d** true

6 **a**



b



7 **a** 3 **b** 2 **c** -3 **d** 1 **e** 0 **f** $-6\frac{1}{2}$
g 3.3 **h** 0.9

8 **a** 8 **b** -4 **c** 10 **d** -4 **e** 18 **f** -4
g 26 **h** -6 **i** 31 **j** -15 **k** 46 **l** -16

9 **a** 8 **b** -14 **c** -5 **d** -21 **e** 11 **f** -25
g 11 **h** -35

10 **a** 8 **b** 2 **c** -2 **d** -8 **e** -1 **f** 1
g -15 **h** 3.5 **i** -1.5 **j** 1.5 **k** -3.5 **l** -4.5

11 **a** -25°C **b** 59 m below sea level **c** 1.5 m below

12 Yes

EXERCISE 3D.2

1 **a** 3 **b** 13 **c** -13 **d** -3 **e** -5 **f** 13
g -13 **h** 5 **i** 5 **j** -5 **k** 29 **l** 29
m -7 **n** -5 **o** 11 **p** -1 **q** -8 **r** -30
s 30 **t** 8

2 **a** 9 **b** 10 **c** -7 **d** 1 **e** -3 **f** -18
g 9 **h** 4 **i** -5 **j** -1 **k** -1 **l** -12
m -321 **n** -143 **o** -114

EXERCISE 3D.3

1 **a** 12 **b** -12 **c** -12 **d** 12 **e** 55 **f** -55
g 55 **h** -55 **i** 54 **j** -54 **k** -54 **l** 54
m 9 **n** -9 **o** 9 **p** -9 **q** 1 **r** -1
s -1 **t** -18

2 **a** -40 **b** 6 **c** -48 **d** 45 **e** -64 **f** -80
g 27 **h** -50

3 **a** 5 **b** -5 **c** -5 **d** 5 **e** 4 **f** -4
g 4 **h** -4 **i** 1 **j** 1 **k** -1 **l** -1

EXERCISE 3D.4

1 **a** 10 **b** 33 **c** -8 **d** 13 **e** -5 **f** 8
g 0 **h** -18 **i** -1 **j** -12 **k** 31 **l** $-\frac{1}{2}$

EXERCISE 3E

1 **a** 5 **b** 6 **c** 9 **d** 12 **e** 17 **f** 21
g 25 **h** 32 **i** 0 **j** 37 **k** 83 **l** 100

2 **a** ≈ 1.73 **b** ≈ 2.65 **c** ≈ 3.16 **d** ≈ 14.14 **e** 42

3 **a** 1 **b** 4 **c** 5 **d** 7 **e** 1 **f** 0 **g** 2
h -1 **i** -3 **j** cannot be found **k** 5 **l** 1
m -1 **n** -2 **o** 2

4 **a** $\sqrt{223} \approx 14.9$, so we divide 223 by 2, 3, 5, 7, 11 and 13.
Each results in a non-whole number.
$\therefore$ 223 is a prime number.
b $527 = 17 \times 31$, $\therefore$ is not a prime number. **c** yes

REVIEW SET 3A

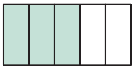
1 □ = 0 or 5 **2** □ = 4 **3** 1, 3, 7, 9, 21, 63 **4** 36, 42, 48
5 odd **6** 41, 43, 47 **7** 60 **8** 24 **9** $2^2 \times 3 \times 5 \times 7$
10 21 **11** 32 or 72 (both answers must be given) **12** 41
13 308 **14** 90 sweets **15** **a** 13 **b** -5
16 No, as it is divisible by 3. **17** **a** -3 **b** 12 **c** 2

REVIEW SET 3B

1 □ = 0, 3, 6 or 9 **2** 2, 4, 8, 16, 32, 64
3 1, 2, 3, 6, 9, 18, 27, 54, 81, 162 **4** 56 **5** 84
6 **a** divisible by 3 **b** divisible by 4 **c** not divisible by 5
7 **a** 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72 **b** 23, 29
c $2^2 \times 31$ **d** 7
8 **a** $3^2 \times 5^3$ **b** 144 **9** 90 **10** 13 **11** 71
12 **a** 2^4 **b** 11^2 **13** 10 weeks **14** 12°C
15 **a** cannot be found **b** 2 **16** No, as it is divisible by 13.
17 **a** 14 **b** -25 **c** -9 **d** 3 **e** $-\frac{1}{3}$ **f** 1

EXERCISE 4A

1 **a**



b

c $\frac{3}{5}$

2 **a** $\frac{8}{12}$ **b** $\frac{9}{12}$ **c** $\frac{10}{12}$ **d** $\frac{4}{12}$ **e** $\frac{4}{12}$
3 **a** $\frac{12}{28}$ **b** $\frac{12}{10}$ **c** $\frac{12}{27}$ **d** $\frac{12}{14}$ **e** $\frac{12}{28}$
4 **a** $\frac{3}{5}$ **b** $\frac{1}{3}$ **c** $\frac{5}{2}$ **d** $\frac{2}{5}$ **e** $\frac{3}{7}$ **f** $\frac{2}{3}$ **g** $\frac{2}{3}$ **h** $\frac{1}{8}$
5 **a** 5 **b** -3 **c** -5 **d** -11 **e** 3 **f** -3
g 3 **h** $-\frac{1}{2}$ **i** $\frac{1}{4}$ **j** $-\frac{1}{3}$ **k** $\frac{1}{2}$ **l** $-\frac{1}{2}$
6 **a** -3 **b** 2 **c** $-\frac{1}{2}$ **d** $\frac{3}{4}$ **e** $-\frac{1}{6}$ **f** 4 **g** 1 **h** 1
7 **a** -2 **b** -2 **c** -10 **d** 2 **e** -6 **f** 1
g -10 **h** -2

8 **a**

b

9 **a** $\frac{3}{5}$ **b** $\frac{5}{7}$ **c** $\frac{2}{11}$ **d** $\frac{3}{10}$
10 **a** $-\frac{3}{4}, -\frac{2}{3}, -\frac{1}{6}, \frac{1}{8}, \frac{3}{11}$ **b** $-\frac{3}{4}, -\frac{6}{11}, \frac{5}{7}, \frac{4}{3}, \frac{7}{5}$
11 **a** $\frac{6}{10}, \frac{4}{9}, \frac{3}{7}, \frac{2}{5}, \frac{5}{13}$ **b** $-\frac{6}{13}, -\frac{1}{2}, -\frac{4}{7}, -\frac{5}{8}, -\frac{7}{11}$

EXERCISE 4B.1

1 **a** 1 **b** $\frac{1}{2}$ **c** $1\frac{1}{5}$ **d** $-\frac{2}{7}$ **e** 2 **f** -1
g $\frac{3}{5}$ **h** $-1\frac{1}{2}$
2 **a** $\frac{9}{10}$ **b** $\frac{7}{20}$ **c** $-\frac{1}{6}$ **d** $1\frac{7}{15}$ **e** $-\frac{1}{14}$ **f** $\frac{1}{4}$
g $-1\frac{1}{2}$ **h** $1\frac{2}{3}$ **i** $-\frac{7}{10}$ **j** $1\frac{4}{9}$ **k** $-1\frac{1}{8}$ **l** $\frac{1}{14}$
3 **a** $-\frac{1}{3}$ **b** $2\frac{1}{4}$ **c** $-1\frac{3}{4}$ **d** $4\frac{11}{12}$ **e** $6\frac{1}{2}$
f $-3\frac{1}{6}$ **g** $\frac{7}{8}$ **h** $2\frac{1}{30}$
4 **a** $\frac{11}{15}$ **b** $\frac{5}{12}$ **c** $-2\frac{1}{3}$ **d** $1\frac{11}{12}$
5 **a** $\frac{7}{15}$ **b** $-2\frac{1}{4}$ **6** **a** $1\frac{23}{30}$ **b** $1\frac{31}{40}$ **c** $\frac{5}{12}$ **d** $\frac{11}{60}$

EXERCISE 4B.2

1 **a** $\frac{3}{10}$ **b** $\frac{3}{16}$ **c** $\frac{9}{25}$ **d** $-\frac{2}{15}$
2 **a** $\frac{1}{4}$ **b** $\frac{3}{10}$ **c** $-\frac{1}{3}$ **d** 16 **e** $-\frac{5}{6}$ **f** $\frac{3}{4}$
g $-\frac{1}{2}$ **h** $\frac{9}{16}$ **i** 93 **j** $\frac{1}{6}$ **k** $\frac{3}{4}$ **l** -26
3 $\frac{2}{5}$
4 **a** $\frac{1}{10}$ **b** $\frac{1}{5}$ **c** $1\frac{1}{6}$ **d** $-2\frac{11}{15}$ **e** $\frac{11}{30}$
f $\frac{5}{9}$ **g** $-\frac{29}{36}$ **h** $3\frac{19}{21}$

EXERCISE 4B.3

1 **a** $\frac{4}{3}$ **b** $\frac{7}{2}$ **c** $\frac{1}{4}$ **d** -2 **e** $-\frac{1}{2}$ **f** $\frac{8}{5}$
g $-\frac{2}{5}$ **h** -1
2 **a** 4 **b** $2\frac{1}{7}$ **c** $-1\frac{1}{2}$ **d** $\frac{4}{15}$ **e** $\frac{3}{20}$ **f** $4\frac{1}{8}$
g -2 **h** $2\frac{2}{5}$
3 **a** $-1\frac{5}{24}$ **b** $4\frac{2}{3}$ **c** $-\frac{17}{40}$ **d** $-\frac{1}{2}$
4 **a** $\frac{1}{2}$ **b** $\frac{1}{12}$ **c** $\frac{23}{36}$ **d** $\frac{4}{9}$

EXERCISE 4B.4

1 **a** 7 **b** 3 **c** $1\frac{1}{2}$ **d** -1 **e** 23 **f** $\frac{1}{2}$
g $-2\frac{5}{6}$ **h** -6 **i** -4 **j** 7 **k** -2 **l** $2\frac{1}{2}$

EXERCISE 4B.5

1 **a** 3 **b** $\frac{7}{9}$ **c** 1 **d** 5 **e** $2\frac{1}{3}$ **f** 7
g 10 **h** $6\frac{2}{3}$ **i** $1\frac{13}{42}$

EXERCISE 4C

1 \$142 **2** £186 **3** €801.75 **4** \$458 **5** €45
6 $\frac{3}{8}$ **7** $\frac{1}{24}$ **8** $\frac{13}{60}$ **9** 500 packets
10 3200 containers **11** €18 000 **12** \$298 556

EXERCISE 4D

1 **a** \$259.50 **b** \$778.50
2 **a** 526 tonnes **b** 2630 tonnes **3** \$814 **4** 130 truffles
5 6312 kg **6** **a** $\frac{4}{5}$ **b** 4300 kg
7 **a** £525 **b** £27 300 **8** **a** $\frac{23}{60}$ **b** \$360 **c** \$72
9 **a** **i** $\frac{7}{10}$ **ii** $\frac{9}{10}$ **b** 850 leaves

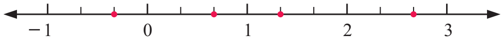
EXERCISE 4E

1 **a** ......, $\sqrt{\frac{1}{4}} = \frac{1}{2}$
b Since $(\frac{2}{3})^2 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, $\sqrt{\frac{4}{9}} = \frac{2}{3}$
c Since $(\frac{3}{7})^2 = \frac{3}{7} \times \frac{3}{7} = \frac{9}{49}$, $\sqrt{\frac{9}{49}} = \frac{3}{7}$
2 **a** $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** $\frac{1}{11}$ **d** $\frac{2}{5}$ **e** $\frac{3}{4}$ **f** $\frac{4}{7}$
g $1\frac{2}{3}$ **h** $2\frac{1}{2}$ **i** $\frac{9}{10}$ **j** $1\frac{3}{7}$ **k** $\frac{3}{11}$ **l** $\frac{6}{13}$
3 **a** $2\frac{1}{2}$ **b** $1\frac{1}{3}$ **c** $1\frac{1}{4}$ **d** $2\frac{1}{3}$ **e** $1\frac{3}{4}$ **f** $3\frac{1}{3}$

REVIEW SET 4A

1 **a** -3 **b** $\frac{1}{3}$ **c** $-\frac{1}{5}$ **d** $\frac{5}{7}$

2

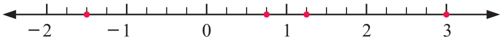


3 $-1\frac{1}{2}, -\frac{3}{4}, \frac{2}{3}, \frac{4}{5}, 1\frac{1}{4}$

4 **a** $\frac{11}{14}$ **b** $-\frac{2}{15}$ **c** $-1\frac{11}{12}$ **d** $-\frac{17}{20}$ **5** $1\frac{5}{12}$
6 **a** 1 **b** $-1\frac{1}{3}$ **c** $-1\frac{1}{3}$ **d** \$360 **7** $-\frac{1}{8}$
8 **a** $1\frac{2}{5}$ **b** $\frac{7}{32}$ **9** $\frac{7}{20}$ **10** 1333 frogs **11** €517.50
12 **a** $\frac{181}{220}$ **b** $\frac{39}{220}$ **13** **a** $\frac{4}{9}$ **b** $6\frac{1}{4}$ **c** $2\frac{1}{2}$

REVIEW SET 4B

1



2 **a** 3 **b** $-\frac{1}{5}$ **c** -5 **d** -4 **3** $\frac{5}{6}$
4 **a** $2\frac{1}{4}$ **b** $\frac{8}{27}$ **c** $1\frac{2}{3}$ **5** $\frac{5}{6}, \frac{1}{2}, -\frac{1}{10}, -\frac{4}{5}, -1\frac{1}{3}$
6 **a** $-\frac{1}{15}$ **b** $\frac{5}{6}$ **c** $-\frac{3}{10}$ **d** $-\frac{1}{15}$
7 **a** $-1\frac{1}{2}$ **b** $-1\frac{1}{5}$ **c** \$63 **d** $-1\frac{1}{2}$
8 $1\frac{1}{12}$ **9** **a** $\frac{1}{9}$ **b** -4 **10** $1\frac{2}{15}$ **11** $\frac{23}{60}$ remains
12 \$41 000 **13** **a** £875 **b** £45 500

EXERCISE 5A

1 **a**

Figure number	*Figure*	*Matches*	*Pattern*	*Explanation*
1		3 +3	1×3	
2		6 +3	2×3	
3		9 +3	3×3	
4		12 +3	4×3	
5		15	5×3	

b **i** 18 **ii** 60 **c** multiplied by 3 **d** $M = 3n$

2 **a**

Figure number	*Figure*	*Matches*	*Pattern*	*Explanation*
1		4 +2	$1 \times 2 + 2$	
2		6 +2	$2 \times 2 + 2$	
3		8 +2	$3 \times 2 + 2$	
4		10 +2	$4 \times 2 + 2$	
5		12	$5 \times 2 + 2$	

b **i** 14 **ii** 32 **c** figure number × 2 plus 2
d $M = 2n + 2$

3 **a**

Figure number	*Figure*	*Matches*	*Pattern*	*Explanation*
1		5 +3	$1 \times 3 + 2$	
2		8 +3	$2 \times 3 + 2$	
3		11 +3	$3 \times 3 + 2$	
4		14 +3	$4 \times 3 + 2$	
5		17	$5 \times 3 + 2$	

b **i** 32 **ii** 47 **c** figure number × 3 plus 2
d $M = 3n + 2$

4 **a**

b 4, 13, 22, 31, 40

c

Figure (n)	1	2	3	4	5	8
Matchsticks (M)	4	13	22	31	40	67

d Number of matchsticks is the figure number multiplied by 9 less 5.

e $M = 9n - 5$ **f** 265

5 **a**

b 5, 12, 19, 26, 33

c

Figure (n)	1	2	3	4	5	8
Matchsticks (M)	5	12	19	26	33	54

d Number of matchsticks is the figure number multiplied by 7 less 2.

e $M = 7n - 2$ **f** 208

EXERCISE 5B.1

1 **a**

Input	*Output*
1	4
2	8
3	12
4	16

b

Input	*Output*
4	7
6	9
12	15
26	29

c

Input	*Output*
0	6
1	9
2	12
5	21

d

Input	*Output*
1	1
2	4
3	9
5	25

e

Input	*Output*
1	3
2	5
3	7
8	17

f

Input	*Output*
3	0
4	2
10	14
15	24

g

Input	*Output*
2	5
6	7
10	9
23	$15\frac{1}{2}$

h

Input	*Output*
2	4
8	6
23	11
98	36

EXERCISE 5B.2

1 **a** $1 \to 5$, $3 \to 7$, $11 \to 15$ **b** $0 \to 2$, $1 \to 4$, $4 \to 10$
c $2 \to 7$, $3 \to 12$, $4 \to 17$ **d** $0 \to 2$, $2 \to 6$, $5 \to 12$
e $1 \to 2$, $2 \to 5$, $4 \to 17$ **f** $1 \to 2$, $2 \to 6$, $4 \to 20$

EXERCISE 5B.3

1 **a** $M = 3n$ **b** $M = 2n + 1$ **c** $M = 2n + 5$
d $M = 4n + 1$ **e** $M = 5n - 2$ **f** $M = 3n + 4$

2 **a** $N = 2a + 2$ **b** $y = x + 3$ **c** $K = 4c + 3$
d $Q = 7d - 3$ **e** $C = 6h + 2$ **f** $M = 8n - 2$

EXERCISE 5C

1 **a** **i** $y = 29$ **ii** $y = 61$ **iii** $y = 89$

b **i** $y = 21$ **ii** $y = 11$ **iii** $y = -7$

c **i** $y = 5$ **ii** $y = 11$ **iii** $y = 18\frac{1}{2}$

d **i** $y = 13$ **ii** $y = 5$ **iii** $y = 18$

2

a	0	2	5	-1	-4
N	7	13	22	4	-5

3

n	-5	-2	0	2	9
K	31	19	11	3	-25

4 **a** \$17 **b** \$11 **c** \$20

5 **a** $V = 5000$ litres and is the starting volume of water in the tank.

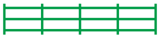
b **i** 4900 litres **ii** 3800 litres **iii** 800 litres

6 **a** $1 = \frac{1\times 2}{2}$ ✓, $1 + 2 = 3 = \frac{2\times 3}{2}$ ✓,
$1 + 2 + 3 = 6 = \frac{3\times 4}{2}$ ✓,
$1 + 2 + 3 + 4 = 10$ and $\frac{4\times 5}{2} = 10$ ✓

b **i** 1275 **ii** 20 100 **iii** 500 500

EXERCISE 5D

1 **a**



4 section

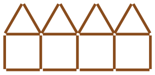
5 section

b

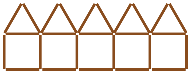
Section number (n)	1	2	3	4	5
Number of lengths (S)	5	9	13	17	21

c $S = 4n + 1$ **d** 177 lengths

2 **a**



4 houses



5 houses

b

Houses (h)	1	2	3	4	5
Toothpicks (T)	6	11	16	21	26

c $T = 5h + 1$ **d** 126 toothpicks

3 **a**

Garden beds (b)	1	2	3	4	5
Sleepers (S)	4	7	10	13	16

b $S = 3b + 1$ **c** **i** 55 sleepers **ii** 112 sleepers

4 **a**

Figure number (n)	1	2	3	4	5
No. of matches (M)	2	4	6	8	10

b $M = 2n$

5 **a**

Figure number (n)	1	2	3	4	5
No. of matches (M)	3	7	11	15	19

b $M = 4n - 1$

EXERCISE 5E

1 **a** After 1 week, she has $2000 + 120 \times 1$ pounds.
After 2 weeks, she has $2000 + 120 \times 2$ pounds.
After 3 weeks, she has $2000 + 120 \times 3$ pounds.

$\vdots$ etc.

So, after n weeks, she has $2000 + 120 \times n$ pounds.
$\therefore$ $M = 2000 + 120n$ pounds

b **i** £2360 **ii** £5120 **iii** £11 360

2 **a** The level rises $b \times 1.5$ cm **b** $D = 2 + 1.5b$ cm

c **i** 9.5 cm **ii** 29 cm

3 **a** Water used is $0.6 \times b$ litres. **b** $W = 50 - 0.6b$ litres

c **i** 41 litres **ii** 27.8 litres

4 **a** $\$25t$ **b** $C = 60 + 25t$ dollars **c** **i** \$135 **ii** \$185

5 **a** **i** 7000 trees **ii** 9000 trees **iii** 11 000 trees

b $T = 3000 + 2000n$ trees **c** 23 000 trees

EXERCISE 5F

1 **a** 16, 19, 22; The next number is equal to the previous number plus 3.

b 31, 35, 39; The next number is equal to the previous number plus 4.

c 37, 44, 51; The next number is equal to the previous number plus 7.

d 30, 36, 42; The next number is equal to the previous number plus 6.

e 49, 58, 67; The next number is equal to the previous number plus 9.

f 59, 72, 85; The next number is equal to the previous number plus 13.

g -2, -3, -4; The next number is equal to the previous number minus 1.

h -1, -4, -7; The next number is equal to the previous number minus 3.

i 1, 5, 9; The next number is equal to the previous number plus 4.

2 **a** 28, 26, 24; The next number is equal to the previous number minus 2.

b 17, 14, 11; The next number is equal to the previous number minus 3.

c 33, 27, 21; The next number is equal to the previous number minus 6.

d 88, 85, 82; The next number is equal to the previous number minus 3.

e 218, 210, 202; The next number is equal to the previous number minus 8.

f 45, 41, 37; The next number is equal to the previous number minus 4.

g 32, 64, 128; The next number is equal to the previous number multiplied by 2.

h 162, 486, 1458; The next number is equal to the previous number multiplied by 3.

i 512, 2048, 8192; The next number is equal to the previous number multiplied by 4.

j 2, 1, $\frac{1}{2}$; The next number is equal to the previous number divided by 2.

k 5, $2\frac{1}{2}$, $1\frac{1}{4}$; The next number is equal to the previous number divided by 2.

l 3, 1, $\frac{1}{3}$; The next number is equal to the previous number divided by 3.

3 **a** 13, 19, 25 **b** 12, 21, 30 **c** $5\frac{1}{2}$, 7, $8\frac{1}{2}$

d 45, 34, 23 **e** 125, 100, 75 **f** 11, 25, 53

g 26, 256, 2556 **h** 49, 25, 13

4 **a** $\square = 15$ **b** $\square = 24$ **c** $\square = 45$ **d** $\square = 12$

e $\square = 15$ **f** $\square = 81$ **g** $\square = 21$ **h** $\square = 120$

i $\square = 25$ **j** $\square = 23$ **k** $\square = 24$

5 **a** 23, 30, 38; To the nth number add n to get the next number.

b 21, 34, 55; The next number is the sum of the previous two numbers.

REVIEW SET 5A

1 **a**

No. of diamonds (n)	1	2	3	4	5
No. of dots (D)	0	2	4	6	8

b $D = 2n - 2$ **c** 62 dots

2 **a** 7 **b** 20 **3**

x	1	3	6	10
y	2	10	22	38

4

Input	Output
2	11
5	20
9	32

5 **a** $Q = 6t - 5$
b $G = 7m - 3$

6 **a**

Figure number (n)	1	2	3	4	5
No. of matches (M)	4	16	28	40	52

b $M = 12n - 8$ **c** 232

7 **a** $5w$ dollars **b** $A = 263 - 5w$ dollars
c **i** \$203 **ii** \$3

8 **a** 54, 47; The next number is equal to the previous number minus 7.
b 48, 96; The next number is equal to the previous number multiplied by 2.
c $\frac{1}{4}$, $\frac{1}{8}$; The next number is equal to the previous number divided by 2.

REVIEW SET 5B

1 **a**

Figure number (n)	1	2	3	4	5
Matchsticks (M)	7	12	17	22	27

b $M = 5n + 2$ **c** 252

2

Input	Output
5	7
8	13
14	25

3

x	1	4	7	15
y	8	17	26	50

4 **a** $D = 5r - 2$ **b** $N = 4s - 1$

5 **a**

Figure number (n)	1	2	3	4	5
No. of matches (M)	2	5	8	11	14

b $M = 3n - 1$ **c** 89 matchsticks

6 **a** €750 **b** €1650 **c** €2850

7 **a** **i** -1, -4; The next number is the previous one, minus 3.
ii 16, 22; To the nth number we add n to get the next number.
b $\square = 20$ $\{2 = 1 \times 2,\ 6 = 2 \times 3,\ 12 = 3 \times 4,$ etc.$\}$

8 **a** \$$5p$ **b** $C = 80 + 5p$ dollars **c** **i** \$140 **ii** \$215

EXERCISE 6A.1

1 **a** $3 + \frac{6}{10}$ **b** $8 + \frac{7}{100}$
c $\frac{1}{10} + \frac{2}{100} + \frac{3}{1000}$ **d** $2 + \frac{6}{100} + \frac{1}{1000}$
e $3 + \frac{7}{1000} + \frac{1}{10\,000}$ **f** $\frac{5}{10\,000} + \frac{4}{100\,000}$
g $3 + \frac{5}{100} + \frac{8}{1000}$ **h** $\frac{6}{100} + \frac{3}{1000} + \frac{2}{10\,000}$
i $50 + 3 + \frac{7}{10} + \frac{7}{1000}$ **j** $\frac{6}{1000} + \frac{7}{100\,000}$

2 **a** 0.2 **b** 0.13 **c** 0.241 **d** 0.83 **e** 0.13
f 0.037 **g** 0.037 **h** 0.659 **i** 0.0037 **j** 0.091
k 0.96 **l** 0.7517

3 **a** 500 **b** $\frac{5}{10}$ **c** 50 **d** $\frac{5}{1000}$ **e** $\frac{5}{100}$
f 50 000 **g** $\frac{5}{100}$ **h** 500

4 **a** 8.8 **b** 2.57 **c** 13.18 **d** 1.461 **e** 7.041
f 3.007 **g** 5.0006 **h** 5.039 **i** 6.48 **j** 5.666
k 68.1 **l** 70.61

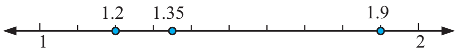
EXERCISE 6A.2

1 **a** $\frac{3}{10}$ **b** $\frac{9}{10}$ **c** $1\frac{1}{5}$ **d** $2\frac{1}{2}$ **e** $1\frac{7}{10}$ **f** $3\frac{1}{5}$
g $\frac{3}{20}$ **h** $\frac{4}{25}$ **i** $\frac{1}{50}$ **j** $\frac{7}{100}$ **k** $\frac{1}{25}$ **l** $\frac{1}{8}$

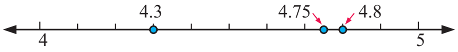
2 **a** $\frac{27}{100}$ **b** $\frac{21}{25}$ **c** $\frac{1}{250}$ **d** $\frac{3}{200}$ **e** $\frac{1}{2500}$ **f** $\frac{11}{40}$
g $\frac{33}{40}$ **h** $\frac{1}{400}$ **i** $\frac{5}{8}$ **j** $\frac{1}{20\,000}$ **k** $4\frac{2}{25}$ **l** $\frac{3}{40}$

EXERCISE 6B

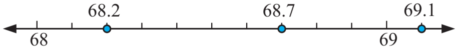
1 **a**



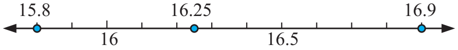
b



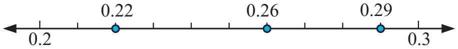
c



d



e



f

2 **a** A = 2.3, B = 2.6 **b** A = 0.58, B = 0.525
c A = 0.203, B = 0.2075 **d** A = 2.526, B = 2.5285
e A = 10.912, B = 10.915 **f** A = 6.7535, B = 6.7539

3 **a** $3.63 > 3.6$ **b** $7.07 < 7.7$ **c** $0.008\,76 < 0.0786$
d $0.229 < 0.292$ **e** $0.47 < 0.5$ **f** $21.101 > 21.011$
g $0.746 > 0.467$ **h** $0.076 < \frac{67}{100}$ **i** $0.306 < 0.603$
j $\frac{150}{1000} = 0.15$ **k** $7.5 = 7.500$ **l** $0.7 = \frac{70}{100}$

4 **a** 2.036, 2.3, 2.36 **b** 9.04, 9.3, 9.34, 9.43
c 0.052, 0.495, 0.5 **d** 18.6, 18.7, 18.71, 19.1
e 7.99, 8.055, 8.1 **f** 7.092, 7.209, 7.29, 7.902
g 3.009, 3.09, 3.1, 3.2 **h** 0.09, 0.099, 0.9, 0.99

5 9.09 sec, 9.89 sec, 9.9 sec, 9.99 sec

6 0.7212, 0.7211, 0.7201, 0.7122, 0.7102

EXERCISE 6C

1 **a** 0.9 **b** 1.03 **c** 1.53 **d** 3.1 **e** 0.686
f 19.632 **g** 0.548 **h** 13.962 **i** 0.6317 **j** 1
k 21.781 **l** 3.145

2 **a** 0.7 **b** 0.9 **c** 0.7 **d** 2.28 **e** 2.55
f 0.01 **g** 0.0001 **h** 0.7 **i** 1.6 **j** 0.0739
k 0.61 **l** 0.122

3 **a** 180.632 **b** 1327.91 **c** 379.8741 **d** 120.992

4 **a** 11.222 **b** 13.93 **c** 8.397 **d** 58.626

5 3.73 m **6** 432.4 kg **7** \$1.55 **8** £927.58 **9** €962.46

10 **a** 0.4, 0.5, 0.6 **b** 0.8, 1.0, 1.2 **c** 0.09, 0.11, 0.13
d 0.08, 0.09, 0.1 **e** 4.9, 4.8, 4.7 **f** 0.44, 0.55, 0.66
g 6.8, 6.6, 6.4 **h** 2.0, 1.6, 1.2 **i** 0.45, 0.4, 0.35

EXERCISE 6D.1

1 **a** **i** 87 **ii** 870 **iii** 8700 **iv** 870 000
b **i** 0.73 **ii** 73 **iii** 730 **iv** 73 000

2 **a** 380 **b** 900 **c** 32 **d** 8 **e** 71 **f** 280
g 60 **h** 83 **i** 18 900 **j** 53 **k** 58.3 **l** 18 700

EXERCISE 6D.2

1 **a** **i** 0.09 **ii** 0.009 **iii** 0.000 09
b **i** 0.706 **ii** 0.007 06 **iii** 0.000 000 706

2 **a** 0.7 **b** 8.9 **c** 46.3 **d** 4.63 **e** 0.463
f 0.0463 **g** 0.08 **h** 0.0008 **i** 0.0073 **j** 0.007
k 0.0007 **l** 0.000 083 **m** 0.000 002 3
n 0.000 000 28 **o** 0.000 000 051

EXERCISE 6E

1 **a** 0.06 **b** 0.035 **c** 0.008 **d** 0.16
e 0.0036 **f** 0.000 12 **g** 0.16 **h** 48
i 42 **j** 2400 **k** 1600 **l** 0.105

2 **a** 1853.1 **b** 18.531 **c** 185.31 **d** 18.531
e 0.185 31 **f** 1.8531 **g** 1.8531 **h** 185.31
i 0.018 531

3 **a** 1.8 **b** 2 **c** 0.21 **d** 21 **e** 14
f 0.48 **g** 0.04 **h** 0.02 **i** 0.04 **j** 6.25
k 0.07 **l** 0.0009 **m** 0.008 **n** 0.027 **o** 0.01
p 1.06 **q** 0.064 **r** 0.49

4 **a** €414 **b** $39.73 **5** $180.45 **6** 67.5 kg

7 **a** 10.75 tonnes **b** 6 truck loads

EXERCISE 6F

1 **a** 4.2 **b** 5.2 **c** 5.1 **d** 0.03 **e** 0.014
f 0.23 **g** 8.3 **h** 0.76 **i** 0.056 **j** 0.0135
k 0.49 **l** 0.056

2 **a** 3 **b** 7 **c** 30 **d** 0.6 **e** 5 **f** 5
g 40 **h** 900 **i** 7 **j** 63 **k** 40 **l** 560

3 **a** 0.09 **b** 0.6 **c** 12 **d** 0.02 **e** 0.2
f 20 **g** 0.24 **h** 0.0024 **i** 24
j 0.000 024 **k** 0.0024 **l** 0.24 **m** 24
n 4.75 **o** 0.375 **p** 0.14 **q** 0.014 **r** 0.7

4 **a** 27 lengths **b** €235.40
c **i** 3.973 kg **ii** 0.993 25 kg **d** 20 packets
e 300 lengths **f** 23 tins

5 **a** 72 **b** 2.5 **c** 79 **d** 800 **e** 0.1 **f** 7.5

EXERCISE 6G.1

1 **a** 0.9 **b** 0.8 **c** 0.25 **d** 0.6 **e** 0.125
f 0.075 **g** 0.35 **h** 1.2 **i** 3.25 **j** 8.62

2 **a** $\frac{5}{5}$ **b** $\frac{2}{2}$ **c** $\frac{125}{125}$ **d** $\frac{25}{25}$ **e** $\frac{5}{5}$

3 **a** 0.75 **b** 1.4 **c** 0.34 **d** 0.465 **e** 0.104
f 0.162 **g** 1.25 **h** 3.44 **i** 1.152 **j** 2.35

EXERCISE 6G.2

1 **a** $0.\overline{6}$ **b** $0.\overline{5}$ **c** $0.\overline{428\,571}$ **d** $0.\overline{63}$ **e** $0.8\overline{3}$

2 **a** $0.1\overline{3}$ **b** $0.6\overline{42\,857\,1}$ **c** $0.\overline{076\,923}$ **d** $0.5\overline{1}$
e $0.4\overline{259}$

EXERCISE 6H

1 **a** **i** 0.8 **ii** 0.77 **iii** 0.769
b **i** 0.1 **ii** 0.07 **iii** 0.0715

2 **a** 9 **b** 15.6 **c** 0.64 **d** 0.465 **e** 0.7778
f 0.888 89

3 **a** 5.3 **b** 9.1 **c** 0.6 **d** 0.52 **e** 0.35
f 0.18 **g** 0.002 **h** 3.833 **i** 1.077

4 53.7 **5** £136.7 million **6** 32.59 points **7** $78.20

8 **a** 4.65 **b** 15.139 **c** 8.66 **d** 99.0 **e** 16.00
f 21.0 **g** 1.7850 **h** 17 **i** 0.0 **j** 2.2941
k 4.863 **l** 0.135 **m** 590.801 **n** 0.0281 **o** 0.309

EXERCISE 6I

1 **a** ≈ 1.99 times **b** ≈ 2.33 times **c** ≈ 1.30 times

2 **a** ≈ 3.27 times **b** ≈ 1.69 times **c** ≈ 5.97 times

3 **a** ≈ 2.64 times **b** ≈ 3.83 times **c** ≈ 4.77 times

4 1707.081 ha

REVIEW SET 6A

1 **a** $\frac{3}{10}$ **b** $\frac{6}{1000}$ **c** 8460 **d** 7.43 **e** 7020
f $\frac{2}{1000}$ **g** 0.0002 **h** $2\frac{16}{25}$

2 $23\frac{113}{250}$ **3** **a** 3.151 **b** 0.1204

4 **a** 23.55 **b** 0.47 **c** 0.55 **5** $3.25 **6** $112\frac{1}{2}$ laps

7 200 cm **8** **a** ≈ 9.33 **b** ≈ 16.0

REVIEW SET 6B

1 **a** 0.383 **b** $\frac{5}{100}$ **c** $4\frac{3}{250}$ **d** 0.6232
e 0.0042 **f** $0.\overline{54}$

2 **a** 2.915 **b** 6.72

3 **a** $2.01 < 2.101$ **b** $0.966 > 0.696$

4 **a** 0.440 **b** 3.28 **5** **a** 7.5 **b** 0.5

6 **a** 384.443 **b** 60 **c** 0.48

7 **a** 1.124 **b** €8.25 **c** 10.17 tonnes

EXERCISE 7A

1 **a** 15% **b** 34% **c** 57% **d** 93%

2 **a** 40% **b** 200% **c** 6% **d** 68% **e** 45%
f 80% **g** 73% **h** 31% **i** 60% **j** 30%
k 160% **l** 213% **m** 50% **n** 100% **o** 75%
p 350%

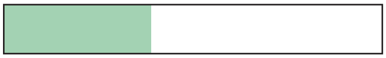
3 **a** **B** **b** **D** **c** **A** **d** **C**

4 **a**

b

c

d



5 **a** Homer **b** Lisa **c** Marge **d** Bart

EXERCISE 7B

1 **a** 60% **b** 70% **c** 240% **d** 250%
e 402.5% **f** $33\frac{1}{3}\%$ **g** 83% **h** 600%
i 0.4% **j** 6.7% **k** $22\frac{2}{9}\%$ **l** 137.5%

2 **a** 30% **b** 400% **c** 14% **d** 44% **e** 85%
f 40% **g** 67% **h** 85% **i** 100% **j** 70%
k 20% **l** 130% **m** 317% **n** 50% **o** 75%
p 3.1% **q** 0.14% **r** 10.5% **s** 5.6% **t** 31.25%

3 **a** $\frac{1}{10}$ **b** $\frac{1}{4}$ **c** $\frac{3}{5}$ **d** $\frac{17}{20}$ **e** $\frac{1}{20}$ **f** $\frac{3}{20}$
g $\frac{21}{50}$ **h** 1 **i** 5 **j** $\frac{1}{100}$ **k** $1\frac{7}{20}$ **l** $\frac{3}{25}$

4 **a** $\frac{3}{40}$ **b** $\frac{133}{400}$ **c** $\frac{1}{8}$ **d** $\frac{1}{3}$ **e** $\frac{1}{400}$ **f** $\frac{1}{200}$
g $\frac{1}{16}$ **h** $\frac{37}{500}$

5 **a** 0.4 **b** 0.23 **c** 0.5 **d** 0.17 **e** 0.083
f 1.5 **g** 2 **h** 0.368 **i** 50 **j** 0.0001
k 1.179 **l** 0.867

6

Percentage	*Fraction*	*Decimal*
100%	1	1
75%	$\frac{3}{4}$	0.75
50%	$\frac{1}{2}$	0.5
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
10%	$\frac{1}{10}$	0.1
5%	$\frac{1}{20}$	0.05
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0.\overline{3}$
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0.\overline{6}$
$12\frac{1}{2}\%$	$\frac{1}{8}$	0.125

7 **a** $\frac{3}{4}$ **b** 75% **c** 25% **8** **a** $\frac{2}{5}$ **b** 40% **c** 60%

EXERCISE 7C

1 **a** 20% **b** 25% **c** 75% **d** 42.86% **e** 30%
f 500% **g** $33\frac{1}{3}\%$ **h** 4.17% **i** 6.25% **j** 20%
k 200% **l** $166\frac{2}{3}\%$ **m** 0.25% **n** 140% **o** 37.5%

2 **a** 80% **b** 57.5% **c** 66.4% **d** 17.5% **e** 77.6%

3 **a** 40% **b** 11% **c** 0.4% **d** 0.425%
e 35% **f** $66\frac{2}{3}\%$

4 **a** 120% **b** £90 000 **c** 20%

EXERCISE 7D

1 **a** $7.20 **b** €1512 **c** 90 cm **d** £750
e 220 kg **f** 129.6 min **g** 54 min
h 456 mm **i** 4.75 tonnes **j** 5.4 tonnes

2 18 **3** €214 620

EXERCISE 7E

1 **a** 1200 mL **b** 800 g **c** $1800 **d** 40 kg
e €900 **f** 80 seconds

2 **a** 180 **b** 576 **c** 2580 **d** 40 **3** 400 students
4 **a** 125 kg **b** 105 kg **5** 120 **6** **a** 90 kg **b** 79.2 kg

EXERCISE 7F

1 **a** €9200 **b** 91.2 kg **c** $17 360 **d** $17 500
e 80.34 kg **f** £66.30

2 **a** 920 m **b** 33 tonnes **c** £4700 **d** €3780
e 7650 hectares **f** 40 880 tonnes

3 $99 **4** $0.9 \times 0.9 = 0.81 = 81\%$ **5** 72.8% increase overall

EXERCISE 7G

1 **a** 8 cm increase **b** £17 decrease **c** 3 kg decrease
d 6 m increase **e** $7 increase **f** 3 tonnes increase

2 **a** 25% increase **b** 40% decrease **c** 60% increase
d 60% decrease **e** 46.67% decrease **f** 75% increase

3 19.64% increase **4** 21.88%

EXERCISE 7H.1

1

	Cost Price	*Selling Price*	*Profit or loss*	*How much profit or loss*
a	$45	$60	profit	$15
b	£125	£95	loss	£30
c	$255	$199	loss	$56
d	€2225	€2555	profit	€330

2

	Cost Price	*Selling Price*	*Profit or loss*
a	$60	$85	$25 profit
b	€230	€195	€35 loss
c	£275	£180	£95 loss
d	$162	$297	$135 profit

3 **a** **i** $20 loss **ii** 40% loss
b **i** £1250 profit **ii** 25% profit
c **i** €115 profit **ii** 57.5% profit
d **i** $136 profit **ii** 20% profit
e **i** €97.50 profit **ii** 30% profit

4 25% loss **5** 20% profit **6** $4500, 20% **7** €1530, 45%
8 £3.60 loss, 2.5% loss **9** €12 915 profit, ≈ 140.4%
10 **a** £600 **b** $262.50 **c** £4860 **d** €5200
11 $660.80

EXERCISE 7H.2

1 **a** €1144 **b** €408

2

	Marked Price	*Discount*	*Selling Price*	*Discount as % of marked price*
a	€125	€25	€100	20%
b	£240	£62.40	£177.60	26%
c	€1.85	37 cents	€1.48	20%
d	$3.00	55 cents	$2.45	18.3%
e	$142	$15	$127	10.6%
f	¥6000	¥600	¥5400	10%

EXERCISE 7I

1 **a** $600 **b** £400 **c** €2200 **d** $37 500
2 **a** €3120 **b** $9440 **c** £9000 **d** $24 150
3 $645.83

REVIEW SET 7A

1 **a** 60% **b** 50% **c** 1% **d** 200%

2 **a** 1.07 **b** $\frac{11}{25}$ **c** $\frac{16}{100} = 16\%$
d 35% **e** 3600 **f** €800 **g** $37\frac{1}{2}\%$

3 **a** £336 **b** 345 tonnes **4** **a** 18% **b** 152 students
5 $768 **6** **a** $48 **b** $26\frac{2}{3}\%$ **7** €1890

REVIEW SET 7B

1 **a** 8.5% **b** 17.5% **c** 40% **d** €3.90 **e** 0.025
f 200 **g** 40% **h** 210

2 $423 500 **3** 21% **4** 80 kg **5** 15%
6 £357 **7** 1750 ha **8** $8160

EXERCISE 8A.1

1 **a** $2 \times 3 + 5$ nails **b** $2 \times 4 + 5$ nails **c** $2b + 5$ nails
d $2p + 5$ nails

2 **a** $4 \times 2 + 3$ nails **b** $4 \times 5 + 3$ nails **c** $4a + 3$ nails
d $4n + 3$ nails

3 **a** $bx + 3$ apples **b** $by + 4$ apples **c** $bt + 7$ apples
d $bm + n$ apples

4 **a** **i** $(n+1)+(n+1)$ nails **ii** $2n+2$ nails

iii $(n+2)+n$ nails

b $(n+1)+(n+1)=2n+2=(n+2)+n$

5 **a** $4c+2$ nails, $(2c+1)+(2c+1)$ nails, $2c+2+2c$ nails

b $2c+3$ nails, $c+3+c$ nails, $(c+1)+(c+2)$ nails

c $3c+4$ nails, $3c+2+2$ nails, $2c+c+4$ nails

6 **a** true **b** true **c** true **d** false

7 **a** $4n+7$ nails **b** **i** 19 nails **ii** 47 nails **iii** 107 nails

8 **a** 13 **b** 13 **c** 4 **d** 12 **e** 13 **f** 0 **g** 21

h 16 **i** 67 **j** 14

9 **a** $3c$ **b** $7c$ **c** $2c+2$ **d** $3c+4$ **e** $2c+8$

f $2c+3$ **g** $3c+6$ **h** $4c+4$

10 **a** c **b** $4c$ **c** 0 **d** $3c+3$ **e** $3c+4$

f $c+2$ **g** $3c+1$ **h** $c+1$ **i** $4c+2$

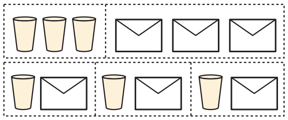
11 **a** **E** **b** **A** **c** **D** **d** **B** **e** **F** **f** **C**

EXERCISE 8A.2

1 **a** $c+1$ **b** $c+e$ **c** $e+3$ **d** $2c+e+3$

e $3e+2$ **f** $e+c+3$ **g** $2c+e+2$

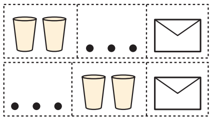
2 **a**



$3c+3e$

$=3(c+e)$

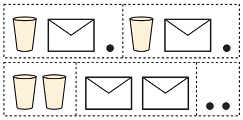
b



$2c+3+e$

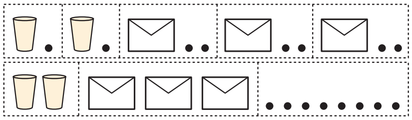
$=3+2c+e$

c



$2(c+e+1)=2c+2e+2$

d



$2(c+1)+3(e+2)=2c+3e+8$

EXERCISE 8B

1 **a** false, an expression **b** true **c** true

d false, an expression **e** true **f** false, coefficient is -5

g false, constant term is -6

2 **a** 4 **b** 7 **c** 2 **d** 3 **e** 1 **f** -2 **g** 1

h -1 **i** -3

3 **a** 1 **b** 2 **c** 3 **d** 3 **e** 3 **f** 3 **g** 2 **h** 2 **i** 2

4 **a** 4 **b** -5 **c** 6 **d** $4x$, x

5 **a** $3x$ and $6x$, 8 and 4 **b** x and $5x$, $2y$ and $-y$

c $3x$ and x, y and $2y$ **d** 2 and 6, t and $4t$

e no like terms **f** ab and $2ab$

EXERCISE 8C

1 **a** $2x$ **b** $3c$ **c** $2a+2b$ **d** $3a+b$ **e** $3+2x+y$

f $2a+3b$ **g** $3g+2$ **h** $3-2a$ **i** $4y-4$ **j** $6-3b$

k $4+2t+3s$ **l** $4m$

2 **a** $4a$ **b** $5x$ **c** cannot be simplified

d cannot be simplified **e** $9a$ **f** $5d$

g cannot be simplified **h** $5m$ **i** n

j cannot be simplified **k** 0 **l** $2x+2$ **m** $2x-2$

n a **o** cannot be simplified **p** $10d$

q cannot be simplified **r** $3p$ **s** $4a$ **t** $13x$

3 **a** $-3x$ **b** $-2a$ **c** cannot be simplified **d** $-n$

e 0 **f** $-3a$ **g** $-3a$ **h** $-7x$ **i** $5x-5$

j cannot be simplified **k** $-5x$ **l** $5-5x$

m cannot be simplified **n** $-20x$ **o** $-4b$ **p** $-3m$

q $3a$ **r** $-3a$ **s** $4x$ **t** $5x-5$

4 **a** $4a+6b$ **b** $5x+5p$ **c** $8g+5h$ **d** $8x+6y$

e 0 **f** $p+3q+5$ **g** $4x+2z$ **h** $5y+4$

i $8x+4y$ **j** cannot be simplified

k cannot be simplified **l** $11x-12y$

5 **a** sometimes, when $x=y$ **b** always **c** never

d sometimes, when $x=y$ **e** sometimes, when $y=1$

f sometimes, when $x=1$

6 **a** -5, 7 **b** $4-3p$, $4+p$

EXERCISE 8D

1 **a** $5d$ **b** $5d$ **c** $3a$ **d** $10a$ **e** $6m$ **f** $6ab$

g $21c$ **h** $6cd$ **i** $7pq$ **j** abc **k** bdh **l** $ab+3$

2 **a** $ac+b$ **b** $2a+3b$ **c** $ab-c$ **d** $a-bc$

e $b-2c$ **f** $ab+cd$ **g** $6-3bc$ **h** $5(a+2)$

i $3(k+2)$ **j** $7(a+4)$ **k** $4(b-a)$ **l** $ab(c+2)$

3 **a** $a\times a$ **b** $b\times b\times b\times b$ **c** $2\times n\times n$

d $3\times m\times m\times m$ **e** $11\times m\times m\times n$

f $7\times a\times b\times b\times b$ **g** $2\times a\times 2\times a$

h $3\times b\times 3\times b\times 3\times b$ **i** $a\times a\times a+2\times b\times b$

j $2\times d\times d-4\times n\times n\times n$

4 **a** $2p^2$ **b** t^3 **c** $5c^3$ **d** $2ab^3$ **e** p^2+q

f $2r^2s^2$ **g** p^2q^2r **h** $9a^2$ **i** $25x^3$ **j** $a+a^2$

k x^3+2 **l** a^2-2a **m** $2a^3+b$ **n** $6a-2b^2$

o $2ab+c^3$ **p** st^2-3t **q** $3b+4n^2$ **r** $3r^3+4r$

s $6a$ **t** $4a+a^2$ **u** $9a^2$

5 **a** $2x^2$ **b** $2x^2$ **c** $10x^2$ **d** $10x^2$ **e** $7x$

f $12x^2$ **g** $12x^2$ **h** x **i** $2x^2$ **j** $2x^2$ **k** $-10x$

l $-20x$ **m** $6x^2$ **n** $20x^2$ **o** $-15x^2$ **p** $-12x^3$

EXERCISE 8E

1 **a** 20 **b** 17 **c** 22 **d** 3 **e** -3 **f** 9 **g** 26

h 7 **i** 6 **j** 7 **k** 6 **l** 2

2 **a** 5 **b** 6 **c** 25 **d** 10 **e** 3 **f** -3 **g** 30

h 18 **i** 30 **j** 48 **k** 21 **l** 21

3 **a** 14 **b** 10 **c** 18 **d** 18 **e** 16 **f** 32

g 64 **h** 8 **i** 6 **j** 36 **k** 8 **l** -6

m -10 **n** 40 **o** 20 **p** 100

4 **a** 9 **b** -12 **c** 12 **d** -1 **e** -15 **f** 4

g -14 **h** -1 **i** -5 **j** 11 **k** 1 **l** -41

5 **a** 0 **b** 2 **c** 10 **d** -21 **e** 6 **f** 21

g 49 **h** -27

REVIEW SET 8A

1 **a** $2\times 2+3=7$ nails **b** $2\times 5+3=13$ nails

c $2n+3$ nails

2 **a** $2c$ **b** $7a+5$ **c** $5p-4$ **d** $10y+6$

3 3 terms **4** 4

5 **a** $3x^2$ and x^2, $-x$ and $3x$ **b** $2a$ and $-a$, 4 and 1

c $2c$ and $3c$ **d** $2c$ and $-4c$, cd and dc

6 **a** $11x-10y$ **b** $5+5r-2m$
c cannot be simplified **d** $5e-3f$

7 **a** $40x$ **b** $7(a-c)$

8 **a** $3a^2b^3$ **b** $2x+x^2+x^3$ **c** $6x^3$

9 **a** 8 **b** 7 **10** 6

REVIEW SET 8B

1 **a** $5y+2$ oranges **b** $5t+r$ oranges **2** 3 terms **3** -5

4 **a** $5x$ and $-2x$ **b** a^2 and $2a^2$, 4 and -1
c there are no like terms

5 **a** $10x-10$ **b** $5a+1$ **c** $2d-4c$ **d** $4q+qt-3t$

6 **a** $5p^2-p^3$ **b** $4x^5$ **c** $-12x^2$ **d** $12x^3$

7 **a** 21 **b** 121 **8** **a** -12 **b** -7 **c** 23 **d** -37

EXERCISE 9A.1

1 **a** mm or cm **b** m **c** km **d** cm **e** mm **f** m

2 **a** **B** **b** **C** **c** **D** **3** **a** **C** **b** **A** **c** **D** **d** **B**

4 **a** 25 mm **b** 26 mm **c** 8 mm **d** 59 mm
e 5 mm **f** 18 mm

EXERCISE 9A.2

1 **a** 800 cm **b** 1640 cm **c** 4 cm **d** 600 cm **e** 20 cm
f 500 000 cm **g** 40 000 cm **h** 2 000 000 cm

2 **a** 5000 m **b** 2 000 000 m **c** 6 m **d** 36.5 m

3 **a** 6000 mm **b** 70 mm **c** 2500 mm **d** 748 000 mm

4 **a** 10 km **b** 0.45 km **c** 4 km **d** 0.2 km

5 **a** 70 mm **b** 5 m **c** 4 km **d** 5 cm
e 600 cm **f** 8000 m **g** 64 mm **h** 3.4 m
i 5.2 km **j** 2.5 cm **k** 380 cm **l** 1.5 km

6 **a** 2131.2 m **b** 5854.5 m **c** 870.4 cm **d** 643.2 cm

7 **a** 16 **b** 20 **c** 40 000 **d** 2000 **e** 5000

EXERCISE 9B

1 **a** 12 units **b** 12 units **c** 22 units **d** 12 units
e 18 units **f** 18 units

2 **a** 10.1 cm **b** 11.3 cm **c** 11.2 cm

3 **a** 15 cm **b** 18 m **c** 21 m **d** 12 m **e** 8 m
f 120 cm **g** 56 m **h** 42 m **i** 57 m

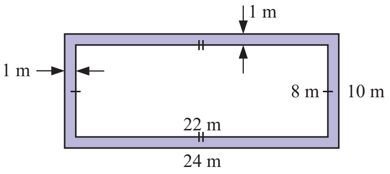
4 **a** 12 m **b** 46 cm **c** 10.2 m

5 **a** 7000 m **b** 120 mm **c** 630 cm

6 **a** 30 m **b** 24 cm **c** 38 cm

7 **a** 300 cm **b** 340 cm **8** **a** 146.08 m **b** 69.38 m

9 **a**



b 68 m

10 £4035.60 **11** 40.8 km **12** 40 min 48 sec

13 **a** 16 posts, 32 m **b** 60 m **c** 150 pickets, 180 m
d $618

EXERCISE 9C

1 **a** 15 square units **b** 12 square units
c 30 square units **d** 36 square units

2 **a** 12 000 cm^2 **b** 80 ha **c** 9700 mm^2 **d** 50 000 m^2
e 1.5 ha **f** 4.76 m^2 **g** 16 km^2 **h** 790 mm^2
i 53 ha **j** 0.56 ha **k** 23 700 cm^2 **l** 3800 mm^2

3 63 000 mm^2 **4** **a** Badan **b** Dahari

EXERCISE 9D.1

1 **a** 32 cm^2 **b** 180 m^2 **c** 80 cm^2 **d** 144 m^2

4 **a** 640 ha **b** $51 200

5 **a** 4500 m^2 (or 0.45 ha) **b** 1 hour 15 min

6 **a** 500 tiles **b** €3425 **7** £3105 **8** $56 960

EXERCISE 9D.2

1 **a** 3 cm^2 **b** 6 cm^2 **c** 20 cm^2 **d** 20 km^2
e 14 m^2 **f** 4 cm^2 **g** 640 m^2 **h** 12 cm^2

2 **a** 24 cm^2 **b** 48 m^2 **c** 120 cm^2

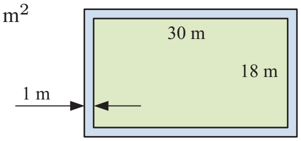
3 **a** 70 cm^2 **b** 105 cm^2 **c** 84 m^2 **4** 64 cm^2

5 3500 cm^2 **6** 600 cm^2 **7** **a** 6 m^2 **b** $x=2.4$

EXERCISE 9E

1 **a** 90 cm^2 **b** 34 m^2 **c** 33 m^2 **d** 30 km^2
e 24 cm^2 **f** 50 m^2

2 **a** 41 cm^2 **3** 100 m^2
b 45 m^2



4 **a** **i** $P=2x+10$ cm **ii** $A=5x$ cm^2
b **i** $P=4x+6$ m **ii** $A=x(x+3)$ m^2
c **i** $P=12x$ cm **ii** $A=6x^2$ cm^2

5 **a** $A=10x^2$ m^2 **b** $A=\dfrac{11x^2}{2}$ m^2
c $A=4x^2+15x$ m^2

REVIEW SET 9A

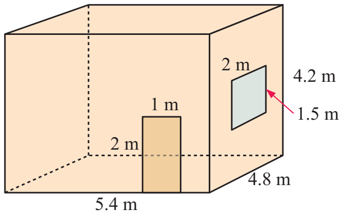
1 **a** 7430 m **b** 1630 cm **c** 1.5 km **d** 4.69 m
e 9.438 cm **f** 2.5 m

2 **a** 19 cm **b** 20 m **c** 23 cm

3 **a** 3.5 m^2 **b** 30 cm^2 **c** 48 m^2

4 40 **5** $275 **6** 12.96 m^2

7 **a**



b €443.74

8 **a** **i** $P=6x$ m **ii** $A=2x^2$ m^2
b **i** $P=30x$ cm **ii** $A=30x^2$ cm^2

REVIEW SET 9B

1 **a** 3.23 cm^2 **b** 2.3462 ha **c** 84 200 cm^2
d 0.028 cm^2 **e** 2530 mm^2 **f** 292 ha

2 **a** 8.04 m **b** 12 cm **c** 28 cm

3 **a** 72 m^2 **b** 6 m^2 **c** 18 m^2 **d** 10 cm^2
e 84 cm^2 **f** 8 m^2

4 5.55 m **5** 43.73 m^2 **6** 280 bricks

7 **a** **i** $P=4x+6$ m **ii** $A=6x$ m^2
b **i** $P=8x$ m **ii** $A=3x^2$ m^2

8 **a** 72 m **b** 6 m **c** 36 m^2 **d** 13 m^2

EXERCISE 10A

1 **a** 10 **b** $5y$ **c** 18 **d** $7b$ **e** 12 **f** $8a$

2 **a** $3a+6$ **b** $2x+10$ **c** $5a+20$ **d** $14x+21$
e $6y+3$ **f** $16c+28$ **g** $30+3y$ **h** $10+5x$
i $4+2b$ **j** $4m+4n$ **k** $8a+4b$ **l** $6x+9y$

3 **a** a^2-4a **b** $2a^2-6a$ **c** a^2-6a **d** $4y^2-10y$
e $6p^2-18p$ **f** r^2-2r **g** $5z-z^2$ **h** k^2-k
i $y-y^2$ **j** $15x^2-10x$ **k** $14p^2-28p$ **l** q^2-q

4 **a** $3x+6$ **b** $4x+4y$ **c** $6+3y$ **d** $ac+bc$
e $dm+dn$ **f** $7k+49$ **g** k^2+7k **h** p^2+4p

5 **a** $kl+3k$ **b** $kl-k$ **c** $kl+5k$ **d** $xy-2x$
e $ab-2b$ **f** $xy+6y$ **g** $kl+7l$ **h** $pz-p$
i $10xy-15x$ **j** $2a^2+2ac$ **k** $4k^2-8kl$ **l** $6x^2-8xy$

6 **a** $3z+6$ **b** $9z-6$ **c** $20z-30y$ **d** $7x+21z+7$
e $12-18a-30b$ **f** $20z-8x+12y$
g $6ax-8ay+14a$ **h** $5x-2x^2+3xy$
i $6p+2px-4pq$ **j** $8x-20y-8$
k $6m+12n+48$ **l** $7x^2+21xy+28x$
m $5x^2+15xy+35xz$ **n** $8ax-24bx+8cx$
o $10x^2+50x+1$ **p** $9xy-9yz+9py$
q $6a^2+30ab+12ac$ **r** $3x^3+9x^2+27x$

EXERCISE 10B

1 **a** $3x+8$ **b** $5a+37$ **c** $5n+9$ **d** $5n+6$
e $7x-17$ **f** $11y+2$ **g** $9x+42$ **h** $14y+20$
i $15x+50$ **j** $15y+45$ **k** x^2-2x-8

2 **a** $3m^2+3m$ **b** $2x$ **c** a^2+6a **d** $5x^2+10x-2$
e $8a^2+11a$ **f** $6p+16q$ **g** $3x^2+5xy$
h $4x^2+12x+12$

EXERCISE 10C

1 **a** 10 **b** 10 **c** 6 **d** 6 **e** 3
f 3 **g** $-8m$ **h** $-8m$

2 **a** $-2x-10$ **b** $-6x-3$ **c** $-12+3x$ **d** $-6a-6b$
e $-x-6$ **f** $-x+3$ **g** $-5-x$ **h** $x-8$
i $-5x-5$ **j** $-12-4x$ **k** $-3b+2$ **l** $-10+2c$

3 **a** $x+4$ **b** $6x-34$ **c** $x-10$ **d** $y-18$
e $3y+16$ **f** $3b-15$

4 **a** $2x$ **b** $3x^2-8x$ **c** $-3x-8$ **d** $x-13$
e a^2-4a **f** $-a-23$

EXERCISE 10D

1 **a** $am+bm+an+bn$ **b** $cp+cq+dp+dq$
c $ax+ay+bx+by$ **d** $ac-bc+ad-bd$
e $cr-dr+cs-ds$ **f** $ax-ay+2x-2y$
g $am+bm-an-bn$ **h** $cx+dx-3c-3d$
i $pr+ps-4r-4s$

2 **a** x^2+5x+6 **b** $x^2+8x+15$ **c** $x^2+14x+48$
d $2x^2+5x+2$ **e** $3x^2+20x+12$ **f** $4x^2+9x+2$
g x^2+4x-5 **h** $x^2+3x-28$ **i** $x^2-3x-18$
j $x^2-xy+3x-3y$ **k** x^2-x-6 **l** $x^2-4x-21$
m $x^2-7x+12$ **n** $x^2-13x+40$ **o** $x^2-10x+24$
p a^2+4a+4 **q** $b^2+10b+25$ **r** $c^2+14c+49$
s x^2-2x+1 **t** $x^2-8x+16$ **u** $y^2-2dy+d^2$

EXERCISE 10E

1 **a** $P=4x+16$ cm **b** $P=4x+12$ cm
c $P=4x+4$ cm

2 **a** $P=3x+6$ cm **b** $P=3x+10$ cm
c $P=4x+12$ cm **d** $P=5x+30$ cm
e $P=6x-18$ cm **f** $P=7x-3$ cm

3 **a** $A=x^2+2x\text{ cm}^2$ **b** $A=x^2+7x+6\text{ m}^2$
c $A=\frac{1}{2}x^2+2x\text{ cm}^2$ **d** $A=x^2+10x\text{ m}^2$
e $A=4x^2+6x\text{ cm}^2$ **f** $A=\frac{3}{2}x^2+x\text{ m}^2$

4 $P=6x+14$ cm, $A=2x^2+11x+12\text{ cm}^2$

EXERCISE 10F

1 **a** x **b** $4b$ **c** $2xy$ **d** x **e** $2y$ **f** a
g $2y$ **h** $3x$ **i** $2y$

2 **a** 4 **b** 3 **c** 2 **d** $3b$ **e** $4y$ **f** $5a$
g $2x$ **h** $3y$ **i** $3a$ **j** $x-1$ **k** $x+2$ **l** $2(x+3)$

3 **a** $5(a+2)$ **b** $2(3a+4)$ **c** $6(a+2b)$ **d** $4(1+2x)$
e $11(a+2b)$ **f** $8(2x+1)$ **g** $4(a+2)$ **h** $5(2+3y)$
i $5(5x+4)$ **j** $x(1+a)$ **k** $x(3+m)$ **l** $a(c+n)$

4 **a** $2(a-5)$ **b** $4(y-5)$ **c** $3(b-4)$ **d** $6(x-4)$
e $2(3x-7)$ **f** $7(2y-1)$ **g** $5(a-3)$ **h** $5(2-3b)$
i $5(4b-5)$ **j** $8(2b-3)$ **k** $x(1-y)$ **l** $a(b-c)$

5 **a** $x(x+3)$ **b** $2x(x+4)$ **c** $3x(x-4)$ **d** $x(6-x)$
e $4x(2-x)$ **f** $3x(5-2x)$ **g** $2x^2(x+2)$ **h** $x^2(2x+5)$

6 **a** $2x(x^2+x+2)$ **b** $x^2(x^2+2x+3)$
c $x(6x^2-3x+5)$ **d** $ax(x+2+a)$
e $3my(y+1+2m)$ **f** $x^2a(4+6a+x^2a^2)$

7 **a** $(x+a)(2+p)$ **b** $(x-2)(n+p)$ **c** $(y+5)(r+4)$
d $(x+4)(3-x)$ **e** $(7-x)(a-b)$ **f** $(x+11)(4+y)$
g $(x+2)(x+1)$ **h** $(x+2)(x-1)$ **i** $(x+3)(x+2)$
j $(x-1)(x+2)$ **k** $(x+5)(x+3)$ **l** $(x-4)(x-2)$

REVIEW SET 10A

1 $a(b+c+d)=ab+ac+ad$

2 **a** $xy+xz$ **b** $6x-15$ **c** x^2-3x **d** $dx+5d$

3 **a** $3x^2-18x+12$ **b** $-2x^2+2x-2$

4 **a** $8x+13$ **b** $6-x$ **c** $x-18$
d $x^2+8x+30$ **e** $x-x^2+1$ **f** $4y^2-4y$

5 **a** $P=3x+6$ m **b** $P=8x-6$ cm **c** $P=7x+4$ cm

6 **a** $A=3x^2+3x\text{ cm}^2$ **b** $A=\frac{3}{2}x+6\text{ m}^2$
c $A=\frac{3}{2}x^2+5x+4\text{ mm}^2$

7 **a** $x^2+7x+12$ **b** $2x^2-5x-3$ **c** $2x^2-15x+7$

8 **a** $3(x+4)$ **b** $x(x-3)$ **c** $b(a+c-2)$
d $(x-2)(a+3)$ **e** $(x+3)(x+2)$

REVIEW SET 10B

1 **a** $6-3y$ **b** $12t+8$ **c** $-a^2-2a$ **d** $nx+6n$

2 **a** $3x+2$ **b** $8-x$

3 **a** $2x^2+2xy-6x$ **b** $2x^2-6x$

4 **a** $5x+9$ **b** $4y+3x+26$ **c** $3x-8$
d $3x^2+x$ **e** $3x^2+14x+5$ **f** $3n^2$

5 **a** $P=3x+18$ m **b** $P=6x-2$ cm
c $P=10x+20$ cm

6 **a** $A = \frac{1}{2}x^2 + x$ cm^2 **b** $A = x^2 + 2x + 1$ mm^2
c $A = \frac{1}{2}x^2 + \frac{1}{2}x - 3$ m^2
7 **a** $x^2 + 11x + 18$ **b** $x^2 + x - 6$ **c** $x^2 - 11x + 28$
8 **a** $4(x + 6y)$ **b** $2x(x - 4)$ **c** $3a(1 + 2b + 3a)$
d $(x - 6)(3 + d)$ **e** $(x + 4)(2x + 3)$

EXERCISE 11A

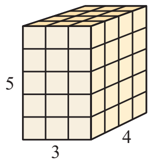
1 **a** 36 units^3 **b** 20 units^3 **c** 96 units^3
2 **a** cm^3 **b** mm^3 **c** cm^3 **d** m^3 **e** cm^3
f cm^3 **g** mm^3 **h** m^3 **i** cm^3
3 **a** 145 mm^3 **b** 3000 cm^3 **c** 14.971 cm^3
d 0.0485 m^3 **e** 2 700 000 cm^3 **f** 0.118 m^3
g 34 300 mm^3 **h** 0.001 694 cm^3 **i** 6 250 000 mm^3

EXERCISE 11B.1

1 **a** 18 units^3 **b** 36 units^3 **c** 45 units^3
2 **a** 60 mm^3 **b** 24 m^3 **c** 27 cm^3
3 29 600 mm^3 or 29.6 cm^3
4 **B** 3000 cm^3 **5** 20.83 m^3 **6** 180 m^3 **7** 64 m^3

EXERCISE 11B.2

1 **a** 18 m^3 **b** 252 m^3 **c** 600 cm^3 **d** 36 mm^3
e 90 m^3 **f** 510 cm^3
2 8.2 cm **3** **a** 36 m^3 **b** 240 cm^3 **c** 0.24 m^3
4 320 m^3 **5** $360
6 €921.60
7 No, greatest number is 60 cartons.



EXERCISE 11C.1

1 **B** **2** **C** **3** **D** **4** **C** **5** **E**
6 **a** 5000 mL **b** 8600 mL **c** 0.4 L **d** 5.83 kL
e 1 000 000 000 mL **f** 3250 L
7 58 bottles

EXERCISE 11C.2

1 **a** 2000 cm^3 **b** 2000 mL **c** 2 litres
2 1.12 litres **3** 32 kL **4** 50 litres
5 **a** 1500 cm^2 **b** 300 000 cm^3 **c** **i** 300 L **ii** 0.3 kL
6 **a** 40 m^2 **b** 0.32 ML **7** 30 kL **8** 2100 ML

EXERCISE 11D.1

1 **a** t **b** kg **c** mg **d** g **e** mg **f** kg
2 **a** 6000 g **b** 0.047 g **c** 3.75 g **d** 7 300 000 g
e 450 g **f** 24 500 g **g** 320 000 g **h** 3.642 g
3 **a** 5000 kg **b** 6 kg **c** 0.5 kg **d** 0.3 kg
e 0.001 847 kg **f** 0.386 kg **g** 4500 kg **h** 5.642 kg
4 896 grams **5** 2000 pegs **6** 6400 tiles
7 2800 kg or 2.8 tonne **8** 130 grams

EXERCISE 11D.2

1 3 kg **2** 20 g **3** 2.75 kg
4 **a** 45 L **b** 45 kg **c** 46 kg
5 **a** 80 g **b** 0.9979 m^2 **c** 4.9896 g

EXERCISE 11E.1

1 **a** 780 min **b** 26 min **c** 2880 min **d** 377 min
e 4658 min
2 **a** 2922 days **b** 3 days **c** 36 days **d** 10 days
3 **a** 10 800 s **b** 2820 s **c** 18 420 s **d** 3 024 000 s
4 **a** 1000 s **b** 6 h **c** 2 weeks **d** 8000 min
5 ≈ 25 days **6** 7.8 min or 7 min 48 s
7 **a** 6 h 23 min **b** 7 h 11 min **c** 5 h 58 min
d 4 h 36 min **e** 5 h 29 min **f** 26 h 20 min
8 2 h 33 min **9** 8 h 45 min
10 **a** 22 min **b** 22 min **c** 16 min
11 **a** 6 **b** 9:15 am **c** 5:30 pm
d **i** 2 h 45 min **ii** 5 h 10 min **e** 9 h 30 min
f Bus E **g** Bus A
12 **a** 6:16 pm **b** 6:15 am **c** 4:50 pm **d** 1:30 pm
e 3:15 am **f** 7:35 pm
13 5:10 pm **14** 7:17 am

EXERCISE 11E.2

1 **a** 3:00 pm **b** 8:00 pm **c** 9:00 pm **d** 8:00 am
2 **a** 6:00 pm Tuesday **b** 3:00 pm Tuesday
c 9:00 am Wednesday **d** 1:00 am Wednesday
3 **a** 11:00 pm Friday **b** 1:00 am Saturday
c 3:00 pm Friday **d** 5:00 pm Friday
4 Abu Dhabi **5** Ottowa or New York **6** 7:00 pm

REVIEW SET 11A

1 **a** 314 min **b** 2600 kg **c** 5.683 kg **d** 1 000 000 cm^3
e 40 000 000 mg **f** 1 day 19 hours 54 min
g 4500 mL **h** 0.026 15 ML
2 **a** 48 units^3 **b** 0.56 m^3 or 560 000 cm^3 **c** 195 cm^3
3 **a** 480 000 cm^3 **b** 480 L **4** 576 L **5** 0.72 kL
6 29.28 kg **7** 4:33 pm **8** 40 000 cm^3 or 0.04 m^3

REVIEW SET 11B

1 **a** 1600 kg **b** 3300 min **c** 5300 cm^3 **d** 0.5126 kg
e 420 mL **f** 2.5 L **g** 25 000 g **h** 0.046 kL
2 4 h 22 min **3** **a** 4500 cm^3 **b** 4.5 L
4 **a** 120 units^3 **b** 28.8 m^3 **c** 7500 cm^3 **5** £2268.00
6 173 containers **7** 2 m **8** **a** 9 pm **b** 9 am

EXERCISE 12A

1 **a** 8 : 5 **b** 7 : 13 **c** 5 : 2 **d** 2 : 7 **e** 13 : 1 **f** 8 : 5
2 **a** 5 : 4 **b** 2 : 3 **c** 1 : 7 **d** 3 : 8
3 **a** 65 : 1000 **b** 87 : 100 **c** 5 : 24 **d** 60 : 240
e 200 : 80 **f** 200 : 1000
4 **a** 11 : 9 **b** 3 : 1 **c** 5 : 1 **d** 2 : 5 **e** 15 : 4
f 2 : 5 **g** 50 : 2000 **h** 5 : 10 000 000

EXERCISE 12B

1 **a** **i** 3 : 1 **ii** 3 : 4 **iii** $\frac{3}{4}$
b **i** 5 : 1 **ii** 5 : 6 **iii** $\frac{5}{6}$
c **i** 3 : 5 **ii** 3 : 8 **iii** $\frac{3}{8}$
d **i** 3 : 6 **ii** 3 : 9 **iii** $\frac{3}{9}$ or $\frac{1}{3}$
e **i** 4 : 2 **ii** 4 : 6 **iii** $\frac{4}{6}$ or $\frac{2}{3}$

f **i** 9 : 7 **ii** 9 : 16 **iii** $\frac{9}{16}$

2 **a** **i** 50 : 30 **ii** 50 : 100 **b** $\frac{50}{100}$ or $\frac{1}{2}$

3 **a** 64 students **b** **i** 8 : 18 **ii** 14 : 64 **c** $\frac{14}{64}$ or $\frac{7}{32}$

EXERCISE 12C

1 **a** 1 : 2 **b** 2 : 1 **c** 3 : 7 **d** 1 : 3 **e** 2 : 5
f 3 : 5 **g** 3 : 5 **h** 3 : 7 **i** 4 : 1

2 **a** 1 : 4 **b** 5 : 1 **c** 2 : 1 **d** 2 : 1 **e** 4 : 1
f 1 : 6 **g** 6 : 1 **h** 7 : 16 **i** 5 : 3 **j** 3 : 2

3 **a** 4 : 5 **b** 13 : 18 **c** 1 : 9 **d** 1 : 3 **e** 2 : 1
f 2 : 1 **g** 1 : 5 **h** 1 : 6 **i** 3 : 1

4 **a** 1 : 2 **b** 2 : 1 **c** 3 : 1 **d** 1 : 4 **e** 3 : 2 **f** 4 : 5

5 **a** 1 : 3 **b** 1 : 2 **c** 1 : 1 **d** 1 : 1 **e** 2 : 3 **f** 1 : 2

6 **a** 1 : 5 **b** 5 : 1 **c** 1 : 2 **d** 1 : 2 **e** 1 : 3
f 1 : 7 **g** 4 : 5 **h** 2 : 1 **i** 4 : 9 **j** 17 : 5
k 8 : 5 **l** 4 : 15 **m** 8 : 3 **n** 7 : 3 **o** 1 : 13
p 1 : 4 **q** 12 : 1 **r** 4 : 1

7 **a** equal **b** equal **c** not equal **d** equal **e** equal
f not equal **g** not equal **h** equal **i** equal

EXERCISE 12D

1 **a** 8 **b** 24 **c** 6 **d** 25 **e** 9 **f** 6 **g** 11
h 50 **i** 12 **j** 2 **k** 40 **l** 9

2 **a** 45 doctors **b** 26 engineers **3** **a** 54^o **b** 21^o

4 45 teachers **5** 8 : 9 **6** 105 km per hour

7 **a** 480 sheep **b** 8 : 11 **c** 11 : 3 **8** $120

9 €43 200 in stocks, €28 800 in shares

EXERCISE 12E

1 **a** **i** 1 : 4 **ii** $\frac{1}{5}$ **b** **i** 3 : 1 **ii** $\frac{3}{4}$
c **i** 1 : 1 **ii** $\frac{1}{2}$ **d** **i** 2 : 1 **ii** $\frac{2}{3}$
e **i** 1 : 8 **ii** $\frac{1}{9}$ **f** **i** 2 : 1 **ii** $\frac{2}{3}$

2 **a** 1 : 2 **b** 1 : 1 **c** 2 : 1 **d** 2 : 3

3 **a** F **b** L **c** E **d** E **e** I **f** G **g** G **h** D

4 **a** 60 mL **b** 400 mL **5** 428.6 mL

6 2.5 L oil, 17.5 L petrol

7 **a** $\frac{2}{5}$ **b** 10 marbles **c** 15 marbles

8 **a** $4 : $16 **b** €35 : €14 **9** £240 **10** ¥60 000

11 $360 000, $270 000, $180 000 **12** 1 tonne

13 **a** 300 g **b** 100 g

14 **a** Joe $\frac{3}{10}$, Bob $\frac{7}{10}$ **b** Joe £10.50, Bob £24.50
c £28 **d** £60

EXERCISE 12F.1

1 **a** 1 : 1000 scale factor 1000 **b** 1 : 5 000 000, 5 000 000
c 1 : 2000, 2000 **d** 1 : 25 000 000, 25 000 000
e 1 : 5000, 5000 **f** 1 : 20 000 000, 20 000 000

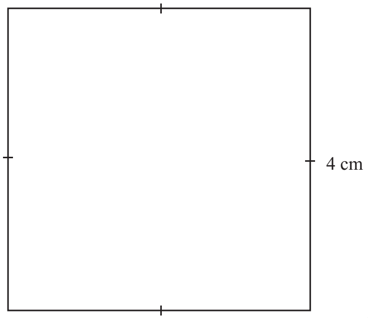
2 **a** 2 m **b** 30 m **c** 5 m **d** 200 m **e** 1 km **f** 50 km

EXERCISE 12F.2

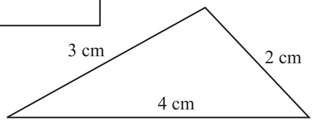
1 **a** 40 cm **b** 9.2 cm **c** 6.8 cm **d** 14.4 cm

2 **a** 2.5 m **b** 120 m **c** 46 m **d** 7.4 m

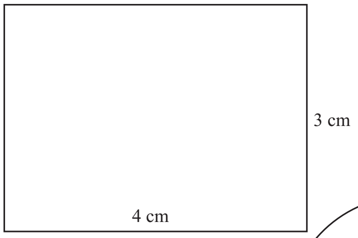
3 **a**



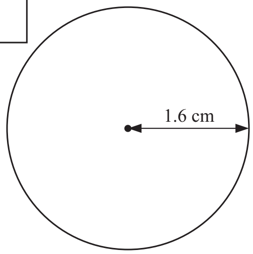
b



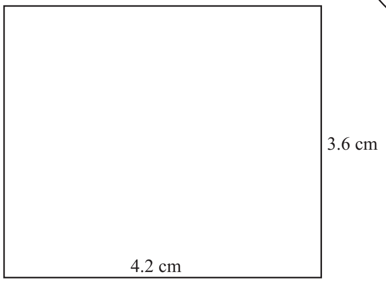
c



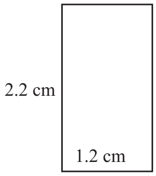
d



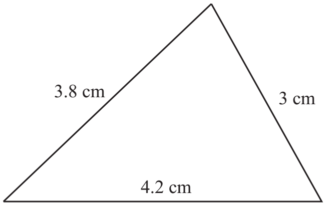
4 **a** *Scale:*
1 cm
represents
10 m



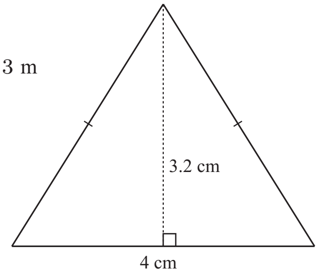
b *Scale:*
1 cm represents 1 m



c *Scale:*
1 cm represents 20 m



d *Scale:* 1 cm represents 3 m



5 6.4 m × 3.8 m, **D**

6 **a** 50 m, 30 m **b** 2.5 m **c** 3.2 mm

7 **a** 4.88 m **b** 64 cm **c** 1.44 m **d** 1.12 m

8 **a** ≈ 1900 km (38 mm) **b** ≈ 2600 km (52 mm)
c ≈ 1600 km (32 mm)

9 **a** 1 : 1000 **b** 54 m **c** 6 m

10 **a** 11 mm **b** 5.9 mm **c** 7.8 mm

11 **a** 10.8 m by 6.6 m **b** 3.24 m by 1.62 m **c** $968.55

12 **a** 0.034 mm **b** 0.025 mm

EXERCISE 12G

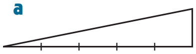
1 **D** **2**

Line [PQ]	*rise*	*run*	*rise : run*
A	1	3	1 : 3
B	2	3	1 : 1.5
C	3	3	1 : 1
D	4	3	1 : 0.75

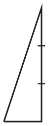
3 The run decreases for a given rise.

4 **a** 1 : 0.8 **b** 1 : 2.5 **c** 1 : 0.5 **d** 1 : 1

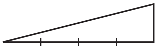
5 **a**



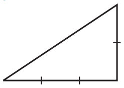
b



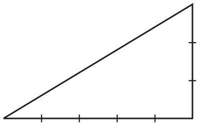
c



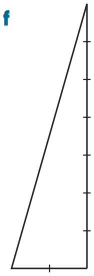
d



e



f



6 **a** PQ 3 : 2, RS 3 : 2 **b** same slope **c** parallel

REVIEW SET 12A

1 **a** 2 : 5 **b** 2 : 3 **c** ☐ = 40 **d** 3 : 2
e 4 : 3 **f** 1 : 2

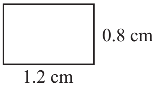
2 £120 : £280 **3** 3 : 4 **4** 96 students

5 **a** 1 : 200 **b** 80 cm **c** 3.4 m

6 1600 apricot, 2000 peach **7** 3 : 5

8 **a** $80, $120 **b** €1600, €2400, €3200

9



10 12 secretaries

11 **a** 2 : 3
b 1 : 2

REVIEW SET 12B

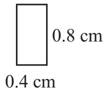
1 **a** 4 : 3 **b** 3 : 8 **c** 5 : 6 **d** 13 : 25
e 4 : 9 **f** 7 : 12 **g** 6 : 5

2 **a** ☐ = 12 **b** ☐ = 15 **3** **a** 5 : 7 **b** 5 : 12

4 35 kg, 40 kg **5** 8 matches

6 2100 hectares, 3150 hectares, 1750 hectares

7



8 **a** D **b** C **c** E

9 4.8 m

10 **a** 1 : 1 **b** 2 : 5

11 45 people

EXERCISE 13A

1 **a** $x=3$ **b** $x=3$ **c** $x=14$ **d** $x=10$
e $x=-1$ **f** $x=4$ **g** $x=-3$ **h** $x=13$
i $x=27$ **j** $x=-81$ **k** $x=-3$ **l** $x=3$

2 **a** $x=1$ **b** $x=-1$ **c** $x=-2$ **d** $x=3$
e $p=-8$ **f** $a=5$

3 **a** **A** **b** **C** **c** **A** **d** **C** **e** **A** **f** **A** **g** **B**
h **C** **i** **D** **j** **C** **k** **D** **l** **C**

4 **a** q is any integer **b** s is any integer **c** $a=0$ or $a=1$
d none **e** $*$ is any integer **f** k is any integer
g none **h** k is any integer **i** $k=0$
j $y=4$ **k** $t=0$ **l** n is any integer

5 **a** identity **b** identity **c** not an identity
d identity **e** not an identity **f** identity

EXERCISE 13B

1 **a** $x=7$ **b** $x=7$ **c** $4x+4=x$ **d** $7x=x+6$

2 **a** $x=-1$ **b** $3x=-6$ **c** $2x+2=x$ **d** $4x-1=x$

3 **a** $x=26$ **b** $x-1=24$ **c** $2x=30$ **d** $2x-1=-4$

4 **a** $x=-11$ **b** $x=-3$ **c** $2+x=-5$ **d** $2x-1=7$

EXERCISE 13C

1 **a** $\div 6$ **b** $+4$ **c** $\times 2$ **d** -10 **e** $+6$ **f** $\times 8$
g $\div\frac{1}{2}$ or $\times 2$ **h** -11

2 **a** x **b** x **c** x **d** x **e** x **f** x **g** x
h x **i** $4x$

3 **a** $x=6$ **b** $x=11$ **c** $x=5$ **d** $x=8$
e $x=7$ **f** $x=-4$ **g** $x=-6$ **h** $x=6$
i $x=-3$ **j** $x=-4$ **k** $x=32$ **l** $x=36$
m $x=-8$ **n** $x=22$ **o** $x=7$ **p** $x=-14$
q $x=5$ **r** $x=6$ **s** $x=-7$ **t** $x=-14$
u $x=7$ **v** $x=-84$ **w** $x=7$ **x** $x=-9$

4 **a** $a=-3$ **b** $b=\frac{2}{7}$ **c** $c=-5$ **d** $d=-4$
e $e=2$ **f** $f=2$ **g** $g=1$ **h** $h=-48$

EXERCISE 13D

1 **a**
$$\boxed{x}\xrightarrow{\times 4}\boxed{4x}\xrightarrow{+1}\boxed{4x+1}$$
$$\boxed{4x+1}\xrightarrow{-1}\boxed{4x}\xrightarrow{\div 4}\boxed{x}$$

b
$$\boxed{x}\xrightarrow{\times 3}\boxed{3x}\xrightarrow{+8}\boxed{3x+8}$$
$$\boxed{3x+8}\xrightarrow{-8}\boxed{3x}\xrightarrow{\div 3}\boxed{x}$$

c
$$\boxed{x}\xrightarrow{\times 4}\boxed{4x}\xrightarrow{-2}\boxed{4x-2}$$
$$\boxed{4x-2}\xrightarrow{+2}\boxed{4x}\xrightarrow{\div 4}\boxed{x}$$

d
$$\boxed{x}\xrightarrow{\times 7}\boxed{7x}\xrightarrow{-9}\boxed{7x-9}$$
$$\boxed{7x-9}\xrightarrow{+9}\boxed{7x}\xrightarrow{\div 7}\boxed{x}$$

e
$$\boxed{x}\xrightarrow{\div 2}\boxed{\frac{x}{2}}\xrightarrow{+6}\boxed{\frac{x}{2}+6}$$
$$\boxed{\frac{x}{2}+6}\xrightarrow{-6}\boxed{\frac{x}{2}}\xrightarrow{\times 2}\boxed{x}$$

f
$$\boxed{x}\xrightarrow{+6}\boxed{x+6}\xrightarrow{\div 2}\boxed{\frac{x+6}{2}}$$
$$\boxed{\frac{x+6}{2}}\xrightarrow{\times 2}\boxed{x+6}\xrightarrow{-6}\boxed{x}$$

g $\boxed{x} \xrightarrow{\div 5} \boxed{\frac{x}{5}} \xrightarrow{-4} \boxed{\frac{x}{5}-4}$

$\boxed{\frac{x}{5}-4} \xrightarrow{+4} \boxed{\frac{x}{5}} \xrightarrow{\times 5} \boxed{x}$

h $\boxed{x} \xrightarrow{-4} \boxed{x-4} \xrightarrow{\div 5} \boxed{\frac{x-4}{5}}$

$\boxed{\frac{x-4}{5}} \xrightarrow{\times 5} \boxed{x-4} \xrightarrow{+4} \boxed{x}$

i $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-5} \boxed{3x-5}$

$\boxed{3x-5} \xrightarrow{+5} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

j $\boxed{x} \xrightarrow{-5} \boxed{x-5} \xrightarrow{\times 3} \boxed{3(x-5)}$

$\boxed{3(x-5)} \xrightarrow{\div 3} \boxed{x-5} \xrightarrow{+5} \boxed{x}$

k $\boxed{x} \xrightarrow{+8} \boxed{x+8} \xrightarrow{\div 3} \boxed{\frac{x+8}{3}}$

$\boxed{\frac{x+8}{3}} \xrightarrow{\times 3} \boxed{x+8} \xrightarrow{-8} \boxed{x}$

l $\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{+8} \boxed{\frac{x}{3}+8}$

$\boxed{\frac{x}{3}+8} \xrightarrow{-8} \boxed{\frac{x}{3}} \xrightarrow{\times 3} \boxed{x}$

m $\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{+1} \boxed{4x+1}$

$\boxed{4x+1} \xrightarrow{-1} \boxed{4x} \xrightarrow{\div 4} \boxed{x}$

n $\boxed{x} \xrightarrow{+1} \boxed{x+1} \xrightarrow{\times 4} \boxed{4(x+1)}$

$\boxed{4(x+1)} \xrightarrow{\div 4} \boxed{x+1} \xrightarrow{-1} \boxed{x}$

o $\boxed{x} \xrightarrow{\div -7} \boxed{\frac{x}{-7}} \xrightarrow{+9} \boxed{\frac{x}{-7}+9}$

$\boxed{\frac{x}{-7}+9} \xrightarrow{-9} \boxed{\frac{x}{-7}} \xrightarrow{\times -7} \boxed{x}$

p $\boxed{x} \xrightarrow{+9} \boxed{x+9} \xrightarrow{\div -7} \boxed{\frac{x+9}{-7}}$

$\boxed{\frac{x+9}{-7}} \xrightarrow{\times -7} \boxed{x+9} \xrightarrow{-9} \boxed{x}$

2 **a** $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x+1} \xrightarrow{\div 3} \boxed{\frac{2x+1}{3}}$

$\boxed{\frac{2x+1}{3}} \xrightarrow{\times 3} \boxed{2x+1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$

b $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{\div 3} \boxed{\frac{2x}{3}} \xrightarrow{+1} \boxed{\frac{2x}{3}+1}$

$\boxed{\frac{2x}{3}+1} \xrightarrow{-1} \boxed{\frac{2x}{3}} \xrightarrow{\times 3} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$

c $\boxed{x} \xrightarrow{+1} \boxed{x+1} \xrightarrow{\times 2} \boxed{2(x+1)} \xrightarrow{\div 3} \boxed{\frac{2(x+1)}{3}}$

$\boxed{\frac{2(x+1)}{3}} \xrightarrow{\times 3} \boxed{2(x+1)} \xrightarrow{\div 2} \boxed{x+1} \xrightarrow{-1} \boxed{x}$

d $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-2} \boxed{3x-2} \xrightarrow{\div 4} \boxed{\frac{3x-2}{4}}$

$\boxed{\frac{3x-2}{4}} \xrightarrow{\times 4} \boxed{3x-2} \xrightarrow{+2} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

e $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{\div 4} \boxed{\frac{3x}{4}} \xrightarrow{-2} \boxed{\frac{3x}{4}-2}$

$\boxed{\frac{3x}{4}-2} \xrightarrow{+2} \boxed{\frac{3x}{4}} \xrightarrow{\times 4} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

f $\boxed{x} \xrightarrow{-2} \boxed{x-2} \xrightarrow{\times 3} \boxed{3(x-2)} \xrightarrow{\div 4} \boxed{\frac{3(x-2)}{4}}$

$\boxed{\frac{3(x-2)}{4}} \xrightarrow{\times 4} \boxed{3(x-2)} \xrightarrow{\div 3} \boxed{x-2} \xrightarrow{+2} \boxed{x}$

g $\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{+7} \boxed{4x+7} \xrightarrow{\div 5} \boxed{\frac{4x+7}{5}}$

$\boxed{\frac{4x+7}{5}} \xrightarrow{\times 5} \boxed{4x+7} \xrightarrow{-7} \boxed{4x} \xrightarrow{\div 4} \boxed{x}$

h $\boxed{x} \xrightarrow{+7} \boxed{x+7} \xrightarrow{\times 4} \boxed{4(x+7)} \xrightarrow{\div 5} \boxed{\frac{4(x+7)}{5}}$

$\boxed{\frac{4(x+7)}{5}} \xrightarrow{\times 5} \boxed{4(x+7)} \xrightarrow{\div 4} \boxed{x+7} \xrightarrow{-7} \boxed{x}$

i $\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{\div 5} \boxed{\frac{4x}{5}} \xrightarrow{+7} \boxed{\frac{4x}{5}+7}$

$\boxed{\frac{4x}{5}+7} \xrightarrow{-7} \boxed{\frac{4x}{5}} \xrightarrow{\times 5} \boxed{4x} \xrightarrow{\div 4} \boxed{x}$

j $\boxed{x} \xrightarrow{\times -3} \boxed{-3x} \xrightarrow{+1} \boxed{1-3x} \xrightarrow{\div 2} \boxed{\frac{1-3x}{2}}$

$\boxed{\frac{1-3x}{2}} \xrightarrow{\times 2} \boxed{1-3x} \xrightarrow{-1} \boxed{-3x} \xrightarrow{\div -3} \boxed{x}$

k $\boxed{x} \xrightarrow{\times -3} \boxed{-3x} \xrightarrow{\div 2} \boxed{\frac{-3x}{2}} \xrightarrow{+1} \boxed{1-\frac{3x}{2}}$

$\boxed{1-\frac{3x}{2}} \xrightarrow{-1} \boxed{\frac{-3x}{2}} \xrightarrow{\times 2} \boxed{-3x} \xrightarrow{\div -3} \boxed{x}$

l $\boxed{x} \xrightarrow{\times -1} \boxed{-x} \xrightarrow{+1} \boxed{1-x} \xrightarrow{\times 3} \boxed{3(1-x)} \xrightarrow{\div 2} \boxed{\frac{3(1-x)}{2}}$

$\boxed{\frac{3(1-x)}{2}} \xrightarrow{\times 2} \boxed{3(1-x)} \xrightarrow{\div 3} \boxed{1-x} \xrightarrow{-1} \boxed{-x} \xrightarrow{\div -1} \boxed{x}$

m $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+1} \boxed{2x+1} \xrightarrow{\div 3} \boxed{\frac{2x+1}{3}} \xrightarrow{+4} \boxed{\frac{2x+1}{3}+4}$

$\boxed{\frac{2x+1}{3}+4} \xrightarrow{-4} \boxed{\frac{2x+1}{3}} \xrightarrow{\times 3} \boxed{2x+1} \xrightarrow{-1} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$

n $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-2} \boxed{3x-2} \xrightarrow{\div 5} \boxed{\frac{3x-2}{5}} \xrightarrow{-4} \boxed{\frac{3x-2}{5}-4}$

$\boxed{\frac{3x-2}{5}-4} \xrightarrow{+4} \boxed{\frac{3x-2}{5}} \xrightarrow{\times 5} \boxed{3x-2} \xrightarrow{+2} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

o $\boxed{x} \xrightarrow{\times -1} \boxed{-x} \xrightarrow{+2} \boxed{2-x} \xrightarrow{\div 4} \boxed{\frac{2-x}{4}} \xrightarrow{+6} \boxed{\frac{2-x}{4}+6}$

$\boxed{\frac{2-x}{4}+6} \xrightarrow{-6} \boxed{\frac{2-x}{4}} \xrightarrow{\times 4} \boxed{2-x} \xrightarrow{-2} \boxed{-x} \xrightarrow{\div -1} \boxed{x}$

EXERCISE 13E

1 **a** $x=7$ **b** $x=4$ **c** $x=5$ **d** $x=3$

e $x=\frac{5}{6}$ **f** $x=-\frac{4}{7}$ **g** $x=4$ **h** $x=10\frac{1}{2}$

i $x=2$ **j** $x=-\frac{5}{6}$ **k** $x=0$ **l** $x=-4\frac{1}{4}$

2 **a** $x=14$ **b** $x=18$ **c** $x=9$ **d** $x=-4$

e $x=-15$ **f** $x=78$ **g** $x=-64$ **h** $x=81$

i $x=-110$

3 **a** $x=3$ **b** $x=-2$ **c** $x=-1$ **d** $x=-\frac{5}{7}$

e $x=5$ **f** $x=2\frac{3}{5}$

4 **a** $x=6$ **b** $x=18\frac{1}{2}$ **c** $x=-1\frac{1}{4}$ **d** $x=-6$

e $x=-17$ **f** $x=2\frac{1}{2}$

5 **a** $x=7$ **b** $x=-\frac{1}{4}$ **c** $x=6$ **d** $x=4\frac{1}{3}$

e $x=-5$ **f** $x=0$

6 **a** $a=4$ **b** $x=-12$ **c** $y=21$ **d** $x=3$

e $m=12$ **f** $x=11$ **g** $a=20$ **h** $x=2$

i $m=8$

EXERCISE 13F

1 **a** $x=2$ **b** $x=-14$ **c** $x=10$ **d** $y=7\frac{1}{5}$

e $x=-\frac{2}{5}$ **f** $a=3\frac{3}{5}$ **g** $x=2$ **h** $x=4\frac{2}{7}$

i $x=6$ **j** $x=8$

2 **a** $-x$ **b** $-x$ **c** $-2x$ **d** $-7x$ **e** $-5x$ **f** $-10x$

3 **a** $x=5$ **b** $x=4$ **c** $x=0$ **d** $x=-1$

e $x=-6$ **f** $x=-1$

4 **a** $+x$ **b** $+3x$ **c** $+4x$ **d** $+x$ **e** $+8x$ **f** $+6x$

5 **a** $x=2$ **b** $x=\frac{1}{2}$ **c** $x=-\frac{1}{9}$ **d** $x=6$

e $x=-1$ **f** $x=-1\frac{1}{3}$

6 **a** $x=5$ **b** $t=3$ **c** $y=-1$ **d** $a=-1\frac{1}{4}$

e $b=0$ **f** $a=-3$

7 **a** simplifies to $0=0$ **b** true for all values of a

8 **a** simplifies to $6=5$ which is false **b** no solutions

9 **a** $x=-2$ **b** $x=1$ **c** $x=4$ **d** $p=\frac{3}{8}$

e $x=1\frac{1}{5}$ **f** $x=1\frac{1}{10}$ **g** $x=-1\frac{2}{3}$ **h** $x=-1\frac{1}{3}$

i $x=-5$ **j** $x=-13$ **k** $x=-2\frac{1}{7}$ **l** $x=3\frac{1}{3}$

m $x=-2$ **n** $x=1\frac{4}{7}$

10 **a** $x=4$ **b** $p=1$ **c** $x=-1$ **d** $x=3$

e $x=-1\frac{4}{5}$ **f** $x=\frac{2}{9}$ **g** $x=-1$ **h** $x=-3$

i $x=1\frac{2}{9}$ **j** $x=-7\frac{2}{3}$

REVIEW SET 13A

1 **a** $\div 5$ **b** $3x=12$ **c** $x=-4$ **2** $x=2$

3 **a** $\boxed{x}\xrightarrow{+8}\boxed{x+8}\xrightarrow{\div 5}\boxed{\frac{x+8}{5}}$

b $\boxed{4x+3}\xrightarrow{-3}\boxed{4x}\xrightarrow{\div 4}\boxed{x}$

4 **a** $\boxed{x}\xrightarrow{+4}\boxed{x+4}\xrightarrow{\div 6}\boxed{\frac{x+4}{6}}$

b $\boxed{x}\xrightarrow{\times 4}\boxed{4x}\xrightarrow{-5}\boxed{4x-5}$

5 **a** $\boxed{\frac{x}{5}+8}\xrightarrow{-8}\boxed{\frac{x}{5}}\xrightarrow{\times 5}\boxed{x}$

b $\boxed{3(x-9)}\xrightarrow{\div 3}\boxed{x-9}\xrightarrow{+9}\boxed{x}$

6 **a** $x=1\frac{3}{4}$ **b** $x=-12$ **c** $x=36$

7 **a** $x=-5$ **b** $x=-1$ **c** $x=1$ **d** $x=1\frac{5}{6}$

e $x=-\frac{1}{2}$

REVIEW SET 13B

1 **a** $a=30$ **b** $4x=-2$ **c** $\times 4$ **d** $x=-21$

2 $x=-13$

3 **a** $\boxed{x}\xrightarrow{\times 2}\boxed{2x}\xrightarrow{+3}\boxed{2x+3}\xrightarrow{\div 4}\boxed{\frac{2x+3}{4}}$

b $\boxed{\frac{x}{5}-6}\xrightarrow{+6}\boxed{\frac{x}{5}}\xrightarrow{\times 5}\boxed{x}$

4 **a** $\boxed{x}\xrightarrow{\times 3}\boxed{3x}\xrightarrow{-7}\boxed{3x-7}\xrightarrow{\times 2}\boxed{2(3x-7)}$

b $\boxed{x}\xrightarrow{\times 2}\boxed{2x}\xrightarrow{+3}\boxed{2x+3}\xrightarrow{\div 6}\boxed{\frac{2x+3}{6}}$

5 **a** $\boxed{\frac{5x-3}{4}}\xrightarrow{\times 4}\boxed{5x-3}\xrightarrow{+3}\boxed{5x}\xrightarrow{\div 5}\boxed{x}$

b $\boxed{6(2x+1)}\xrightarrow{\div 6}\boxed{2x+1}\xrightarrow{-1}\boxed{2x}\xrightarrow{\div 2}\boxed{x}$

6 **a** $x=9$ **b** $x=1\frac{1}{2}$ **c** $x=4$

7 **a** $x=4$ **b** $x=\frac{1}{5}$ **c** $x=7$ **d** $x=1\frac{5}{7}$ **e** $x=2\frac{3}{8}$

EXERCISE 14A

1 **a** 27 mm, 24 mm, 19 mm, scalene

b 26 mm, 21 mm, 21 mm, isosceles

c 22 mm, 22 mm, 19 mm, isosceles

d 19 mm, 22 mm, 16 mm, scalene

2 **a** 58^o, 58^o, 64^o, acute **b** 122^o, 27^o, 31^o, obtuse

c 78^o, 46^o, 56^o, acute **d** 88^o, 51^o, 41^o, acute

3 **a** equilateral acute **b** right angled isosceles

c right angled scalene **d** isosceles acute

EXERCISE 14B

1 Reason: angle sum of triangle (all parts)

a $x=48$ **b** $x=42$ **c** $x=60$

2 Reason: exterior angle of a triangle (all parts)

a $a=55$ **b** $a=125$ **c** $a=146$ **d** $a=69$

e $a=55$ **f** $a=130$

3 Reason: **a**, **b**, **c**, **g**, **h**, **i**: angle sum of a triangle

d, **e**, **f**: exterior angle of a triangle

a $x=40$ **b** $t=45$ **c** $a=55$ **d** $a=49$

e $a=40$ **f** $k=55$ **g** $n=60$ **h** $b=45$

i $r=30$

4 **a** The sum of two obtuse angles is greater than 180^o.

b The sum of one obtuse angle and one right angle is greater than 180^o.

5 **a** $x=40$ **b** $x=35$ **c** $x=40$ **d** $x=36$

e $x=11$ **f** $x=20$

6 $\mathrm{Q\widehat{A}C}=c^o$ {equal alternate angles}

$\mathrm{P\widehat{A}B}=b^o$ {equal alternate angles}

But $\mathrm{P\widehat{A}B}+\mathrm{B\widehat{A}C}+\mathrm{Q\widehat{A}C}=180^o$ {angles on a line}

So, $b+a+c=180^o$

7 **a** $a=40$, $b=80$, $c=55$ **b** $a=30$, $b=120$

c $a=45$, $b=105$

EXERCISE 14C

1 **a** $x = 46$ **b** $x = 66$ **c** $x = 4$ **d** $x = 40$
e $x = 70,\ y = 4$ **f** $p = 90$ **g** $n = 102$
h $p = 44$ **i** $t = 120$ **j** $x = y = 65$
k $y = 35,\ x = 145$ **l** $x = 31,\ y = 56$

2 **a** $x = 43$, ΔPQR is isosceles with PR = QR
b $x = 45$, ΔKLM is right angled isosceles with KL = ML
c $x = 36$, $M\widehat{N}L = 72^o$, ΔLMN is isosceles with LM = MN

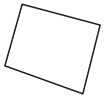
EXERCISE 14D

1 **a** A polygon. **b** Not a polygon as side is curved.
c A polygon. **d** Not a polygon as sides cross over.
e Not a polygon as is not closed. **f** A polygon.
g Not a polygon as sides are curved.
h Not a polygon as a side is curved.
i Not a polygon as sides cross over.
j Not a polygon as is not closed (or sides cross over).
k A polygon. **l** Not a polygon as sides cross over.

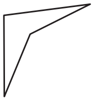
2 **a** triangle **b** quadrilateral **c** pentagon
d hexagon **e** octagon **f** decagon

3 **a** a convex quadrilateral **b** a convex pentagon
c a triangle **d** a convex hexagon
e a convex octagon **f** a non-convex heptagon (7-gon)
g a non-convex 11-gon **h** a non-convex nonagon (9-gon)

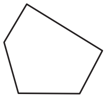
4 **a**



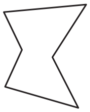
b



c



d



5 **a** All angles are not equal. **b** All angles are not equal.
c All sides are not equal in length.
d All sides are not equal in length.

EXERCISE 14E

1 **a** $x = 63,\ y = 27$ **b** **i** 113^o **ii** 67^o **iii** 113^o
c **i** 30^o **ii** 41^o **iii** 109^o **iv** 109^o
d **i** $x = 90$ **ii** $y = 45$ **iii** $z = 45$

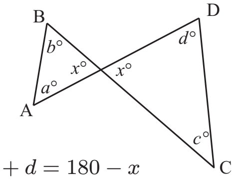
2 **a** trapezium, alternate angles equal, one pair parallel sides
b rectangle, four equal angles of 90^o
c rhombus, parallelogram with diagonals meeting at 90^o
d parallelogram, alternate angles equal
$\therefore$ both pairs opposite sides parallel
e parallelogram, opposite sides equal
f square, diagonals equal length and bisect each other at 90^o

3 **a** parallelogram $a = b = 70$ **b** rectangle $a = b = 20$
c trapezium $a = 70,\ b = 110$
d trapezium $a = 40,\ b = 90$
e square $a = 90,\ b = 45$ **f** rhombus $a = 40,\ b = 50$

EXERCISE 14F

1 **a** $x = 72$ {angles of a quadrilateral}
b $a = 70$ {angles on a line}
$b = 80$ {angles of a quadrilateral}
c $a = 96$ {angles of a quadrilateral}
d $a = 80$ {angles of a quadrilateral}
$b = 100$ {angles on a line}
e $a = 90$ {angles of a quadrilateral}
f $a = 75$ {angles of a quadrilateral}
g $a = 80$ {angles on a line}
$b = 70$ {angles of a quadrilateral}
$c = 110$ {angles on a line}
h $a = 63.5$ {angles of a quadrilateral}
i $a = 108$ {angles on a line}
$b = 88$ {angles on a line}
$c = 90$ {angles of a quadrilateral}
$d = 90$ {angles on a line}

2 **a** It crosses over.
b $a + b + x = 180$ and
$c + d + x = 180$
{vertically opposite angles x^o
and angle sum of a triangle}
$\therefore\ a + b = 180 - x$ and $c + d = 180 - x$
$\therefore\ a + b = c + d$



EXERCISE 14G

1 **a** 540^o **b** 900^o **c** 720^o **d** 1800^o **e** 2340^o

2 **a** $x = 108$ {angle sum of a pentagon}
b $x = 150$ {angle sum of a hexagon}
c $x = 60$ {angle sum of a pentagon}
d $x = 120$ {angle sum of a hexagon}
e $x = 125$ {angle sum of a heptagon}
f $x = 135$ {angle sum of an octagon}

3 135^o **4**

Regular polygon	*No. of sides*	*Sum of angles*	*Size of each angle*
square	4	360^o	90^o
pentagon	5	540^o	108^o
hexagon	6	720^o	120^o
octagon	8	1080^o	135^o
decagon	10	1440^o	144^o

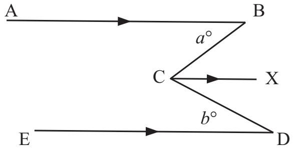
5 $\theta = \dfrac{(n-2) \times 180^o}{n}$ **6** 15 sides **7** impossible

8 **a** The angles at the centre of the pentagon are angles at a point and they are all equal to x^o $\therefore\ 5x = 360$.
b $x = 72,\ y = 54$
c The angles all have size $2y$, i.e., 108^o.

EXERCISE 14H

1 **a** **i** a^o **ii** $2a^o$ **iii** $(90 - a)^o$
b $A\widehat{B}C = A\widehat{B}D + D\widehat{B}C$
$= (90 - a)^o + a^o$
$= 90^o$

2



$B\widehat{C}X = A\widehat{B}C = a^o$
{equal alternate angles}
$X\widehat{C}D = C\widehat{D}E = b^o$
{equal alternate angles}
$\therefore\ B\widehat{C}D = (a+b)^o$

3 [AC] bisects $D\widehat{A}B$
$\therefore\ D\widehat{A}C = B\widehat{A}C = a^o$, say
Likewise $A\widehat{B}D = C\widehat{B}D = b^o$, say
$\therefore\ D\widehat{A}B = 2a$ and $C\widehat{B}A = 2b$
But $D\widehat{A}B$ and $C\widehat{B}A$ are co-interior with (AD) $\parallel$ (BC)

$\therefore \quad 2a + 2b = 180^o$

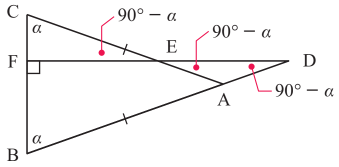
$\therefore \quad a + b = 90^o$

$\therefore \quad A\widehat{X}B = 180^o - (a + b)$

i.e., $\quad A\widehat{X}B = 90^o$

$\therefore$ [BD] is perpendicular to [AC].

4



Let $\quad C\widehat{B}A = \alpha \quad \therefore \quad B\widehat{C}A = \alpha \quad$ {base angles of isosceles Δ}

In ΔCFE, $\quad C\widehat{E}F = 90 - \alpha \quad$ {angles of a triangle}

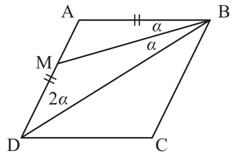
$\therefore \quad D\widehat{E}A = 90 - \alpha \quad$ {vertically opposite}

In ΔBFD, $\quad F\widehat{D}B = 90 - \alpha \quad$ {angles of a triangle}

$\therefore \quad A\widehat{E}D = A\widehat{D}E = 90 - \alpha$

$\therefore \quad \Delta$ADE is isosceles $\quad$ {two equal angles}

5



As [BM] bisects $A\widehat{B}D$,

$A\widehat{B}M = D\widehat{B}M = \alpha \quad$ say

But ΔABD is isosceles

as $\quad$ AD = AB

$\therefore \quad A\widehat{D}B = 2\alpha \quad$ {equal base angles}

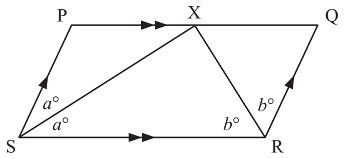
$\therefore \quad A\widehat{M}B = M\widehat{D}B + M\widehat{B}D \quad$ {exterior angle of a triangle}

$= 2\alpha + \alpha$

$= 3\alpha$

$= 3 \times A\widehat{B}M$

6



Let $\quad P\widehat{S}X = X\widehat{S}R = a^o \quad$ and

$S\widehat{R}X = Q\widehat{R}X = b^o$

$\therefore \quad P\widehat{S}R = 2a^o \quad$ and $\quad S\widehat{R}Q = 2b^o$

$\therefore \quad 2a^o + 2b^o = 180^o \quad$ {co-interior angles add to 180^o}

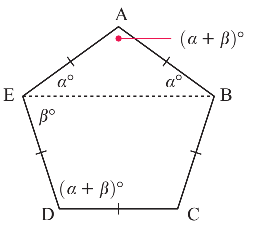
$\therefore \quad a + b = 90^o$

$\therefore \quad S\widehat{X}R = 180^o - (a + b)$

$= 180^o - 90^o$

$= 90^o$

7



ΔABE is isosceles

as $\quad$ AE = AB

$\therefore \quad A\widehat{B}E = \alpha^o \quad$ also

{equal base angles}

As all angles are equal

in the regular pentagon

$$\alpha + \beta = \frac{3 \times 180}{5}$$

$\therefore \quad \alpha + \beta = 108$

But $\quad \alpha + \alpha + \alpha + \beta = 180 \quad$ {in ΔABE}

$\therefore \quad 2\alpha + 108 = 180$

$\therefore \quad 2\alpha = 72$

$\therefore \quad \alpha = 36 \quad$ and $\quad \beta = 72$

Now $\quad B\widehat{E}D + E\widehat{D}C = 72^o + 108^o = 180^o$

$\therefore \quad$ [EB] is parallel to [DC] $\quad$ {co-interior angles add to 180^o}

REVIEW SET 14A

1 **a** $2x^o = 140^o$ {angles of a triangle} So, $x = 70$
b $5t^o = 180^o$ {angles of a triangle} So, $t = 36$
c $b = 70$ {angles of a triangle}

2 **a** $x = 122$ {angles on a line, exterior angle}
b $x = 36$ {exterior angle}
c $x = 100$ {angles of a quadrilateral, angles on a line}
d Other angle is 43^o {isosceles triangle}
So, $x = 94$ {angles of a triangle}
e $x = 125$ {opposite angles of a parallelogram}
f $x = 34$ {exterior angle of a triangle}

3 **a** $a = 85$ {angles of a Δ}, $b = 59$ {exterior angle of a Δ}
b $x = 40$ {angles on a line}, $y = 40$ {isosceles Δ}
$z = 100$ {angles of a Δ}
$t = 30$ {vertically opposite angles and angles of a Δ}
c $x = 18$ {isosceles Δ and angles of a Δ}
$y = 63$ {angles on a line}

4 **a** $a + b = 180$ {opposite angles of parallelogram}
b $a + b = 150$ {angle sum of triangle}
c $x + y = 60$ {co-interior angles}

5 **a** rhombus {parallelogram, diagonal bisects angle}
b trapezium {pair of opposite sides parallel, since alternate angles equal}
c rectangle {one angle 90^o, diagonals bisect each other}

6 **a** yes **b** yes **7** **a** $a = 34$ **b** $b = 112$ **c** $c = 56$

8 a parallelogram; $x = 65$, $y = 115$

9 **a** $a = 120$ **b** $a = 115$ **10** 150^o **11** 20 sides

REVIEW SET 14B

1 **a** $a = 70$ {angle sum of triangle}
b $a = 40$ {isosceles Δ} **c** $a = 130$ {exterior angle}
d $a = 130$ {isosceles triangle, exterior angle}
e $a = 95$ {angle sum of quadrilateral}
f $a = 115$ {angle sum of quadrilateral}

2 **a** $x = 102$ {exterior angle}
b $x = 65$ {vertically opposite, angle sum of triangle}
c $x = 80$ {angle sum of quadrilateral}

3 **a** $t = 75$ {angles on a line}, $x = 25$ {isosceles Δ}
$y = 30$ {angles of a Δ}
b $a = 30$ {angles of a Δ}, $b = 65$ {angles of a Δ}
c $a = 50$ {angles on a line}
$c = 100$ {isosceles Δ and angles of a Δ}
$d = 30$ {vertically opposite and angles of a Δ}

4 **a** $c = a + b$ {exterior angle}
b $a + b = 205$ {angle sum of quadrilateral}
c $a + b = 90$ {isosceles triangle}

5 **a** parallelogram {opposite angles equal}
b right angled isosceles triangle {angles of a Δ}
c rhombus {parallelogram, diagonals bisect angles}

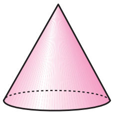
6 $x = 90$, $y = 28$ **7** **a** $x = 108$ **b** $x = 130$

8 **a** yes **b** yes **9** 140^o

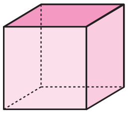
10 Solving for n in $\dfrac{(n-2)180}{n} = 155$ gives $n = 14.4$
which is impossible as n must be a whole number.

EXERCISE 15A

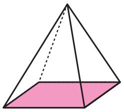
1 **a**



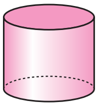
b



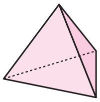
c



d



e



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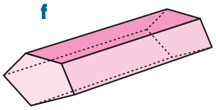

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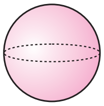

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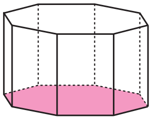

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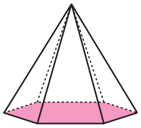


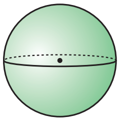
2 **a** a cone **b** a cylinder **c** a rectangular prism
d a sphere **e** a cone **f** a tetrahedron **g** a cylinder

3 **a** P, Q, R, S, T, U, V, W
b PQRS, TUVW, PSVW, QRUT, PQTW, SRUV
c [PQ], [QR], [RS], [SP], [TU], [UV], [VW], [WT], [PW], [QT], [RU], [SV]
d [PW], [QT], [RU] **e** [PQ], [PS], [PW] **f** TUVW

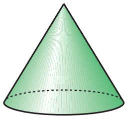
4 **a** rectangular **b** triangular

5 Examples are:

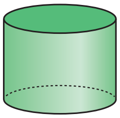
a



b



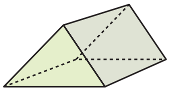
c



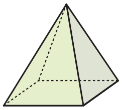
6 **a** 9 faces, 14 vertices, 21 edges
b 30 faces, 56 vertices, 84 edges

EXERCISE 15B

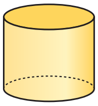
1 **a** triangular prism



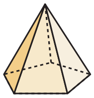
b square-based pyramid



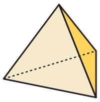
c cylinder



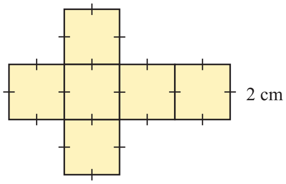
d pentagonal pyramid



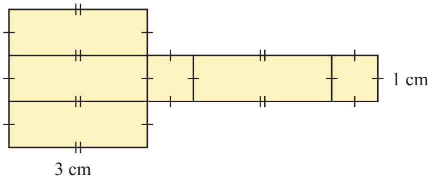
e tetrahedron



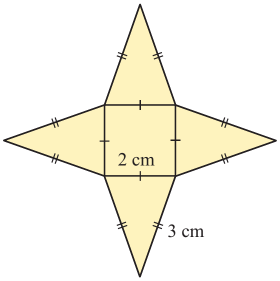
2 **a**



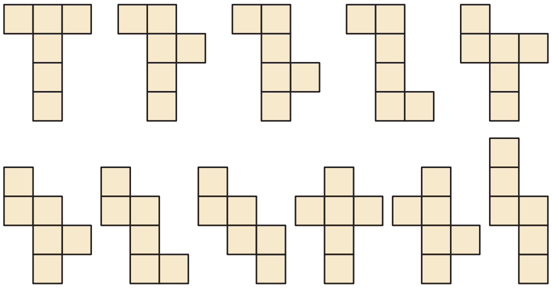
b



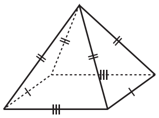
c



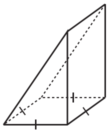
3 There are 11 nets for making a cube:



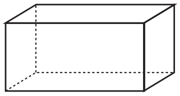
4 **a** rectangular-based pyramid



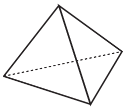
b triangular-based prism



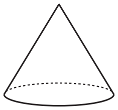
c rectangular prism



d triangular-based pyramid



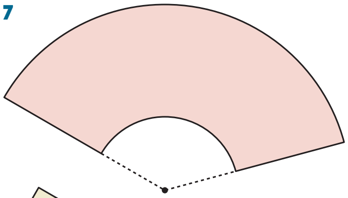
e cone



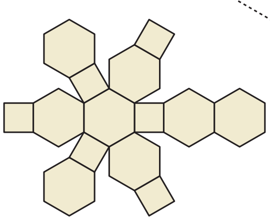
5 **a** No
b Yes
c The second one.

6 **D**

7

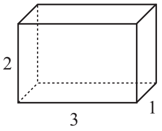


8 **a** 8
b 6
c

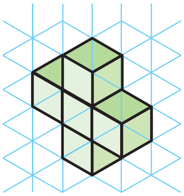


EXERCISE 15C

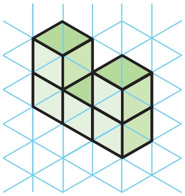
1



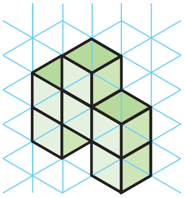
2 **a**



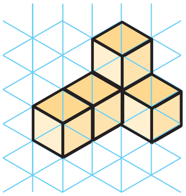
b



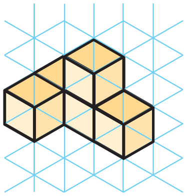
c



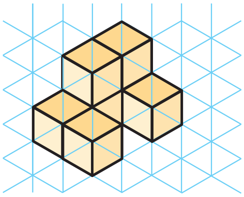
3 **a**



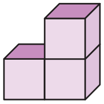
b



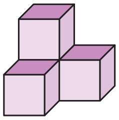
c



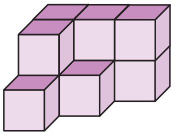
4 **a**



b

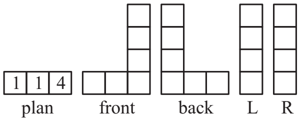


c

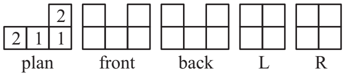


EXERCISE 15D

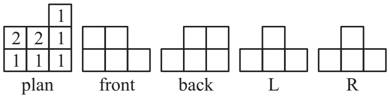
1 **a**



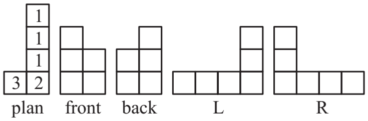
b



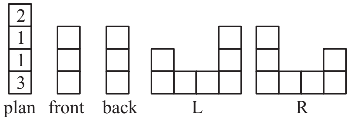
c



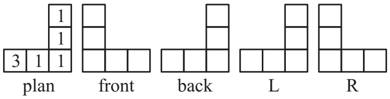
d



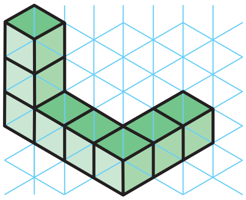
e



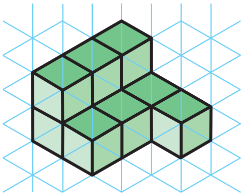
f



2 **a**



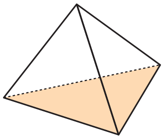
b



REVIEW SET 15A

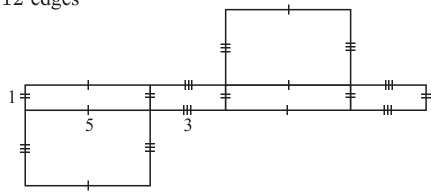
1 **a** [AD], [EF]
b [AE], [BE], [FE]
c DCF

2



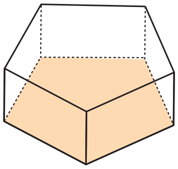
3 12 edges

4



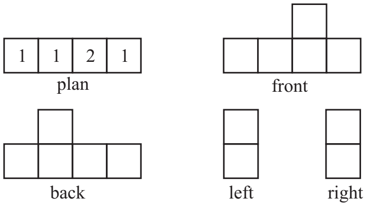
5 8 vertices, 6 faces, 12 edges

6 pentagonal prism



7 **a** cylinder **b** hexagonal prism

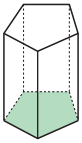
8



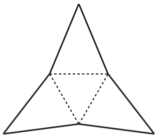
REVIEW SET 15B

1 **a** [AE], [DH], [CG] **b** [EH], [DH], [GH] **c** BFGC

2



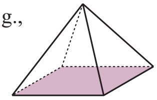
3

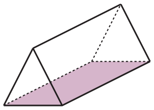


4 **a** triangular prism **b** pentagonal pyramid

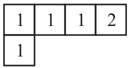
5 6 faces, 10 edges, 6 vertices

6 e.g.,

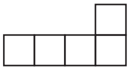




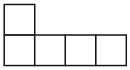
7



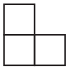
plan



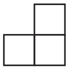
front



back

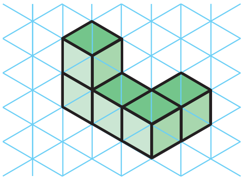


left



right

8



EXERCISE 16A

1 **a** $n - 5 = 16$ **b** $4n + 6 = 18$ **c** $\frac{n}{5} = 7$
d $3(n + 5) + 4 = 40$ **e** $\frac{n + 6}{2} = 9$
f $4(n + 10) + 2 = 50$ **g** $3(3n + 5) = 51$
h $\frac{n + 12}{2} = 7$ **i** $\frac{n}{2} + 7 = 23$ **j** $7n - 6 = 64$

2 **a** I think of a number, add 5 to it and the result is 2.
b I think of a number, subtract 9 from it and the result is 3.
c I think of a number, double it, subtract 5 from the result and the answer is 7.
d I think of a number, multiply it by 3, add 6 to the result and the answer is 21.
e I think of a number, add 1 to it, multiply the result by 2 and the answer is 16.
f I think of a number, subtract 4 from it, multiply the result by 3 and the answer is 9.
g I think of a number, multiply it by 2, add 3 to it, multiply this result by 3 and the answer is 15.
h I think of a number, multiply it by 4 then subtract 1, multiply this result by 9 and the answer is 27.
i I think of a number, add 3 to it then multiply the result by 2, add 4 to this result and the answer is 24.
j I think of a number, multiply it by 2, subtract 4, multiply the result by 3, then subtract 5, and the answer is 25.
k I think of a number, multiply it by 3 and subtract 9, multiply this result by 4, subtract 8 from this result and the answer is 40.
l I think of a number, multiply it by 2 and add 1, multiply this result by 5 then subtract 4 and the answer is 41.
m I think of a number, add 1 to it, multiply the result by 3 then divide by 7 and the answer is 6.
n I think of a number, multiply it by 2, subtract 5, then multiply this result by 4, then divide by 3 and the answer is 20.
o I think of a number, multiply it by -1, add 2, then multiply the result by 3 and finally divide all of it by 7 and the answer is 6.

EXERCISE 16B

1 **a** -3 **b** 52 **c** 15 **d** 47 **e** $4\frac{1}{2}$ **f** 6
2 **a** $-\frac{1}{2}$ **b** $8\frac{1}{2}$ **c** 17 **d** 13 **e** 12

EXERCISE 16C

1 **a** $x = 3$ **b** $x = 7$ **c** $x = 1$
2 **a** $x = 37$ **b** $x = 42\frac{1}{2}$ **c** $x = 40$ **d** $x = 70$
e $x = 40$ **f** $x = 82\frac{1}{2}$

3 $x = 5$ **4** 18 cm **5** $4\frac{1}{2}$ cm **6** $x = 3$

EXERCISE 16D

1 24 cents **2** TV costs \$477, DVD \$377 **3** €17.50
4 RM 2.10 **5** 61 of them **6** 31 of them

EXERCISE 16E

1 60, 71 **2** 17 **3** $x = 5$ **4** 19 cm **5** 4.3 cm

6 **a** 200 m **b** 500 m **7** 80 cm
8 **a** $x = 8$ **b** 6 units **9** $x = 3$, perimeter = 68 cm
10 **a** £52 **b** £92 **11** **a** \$300 **b** \$450
12 **a** $x = 11$
b Yes, since $2 \times 11 - 4 = 18$ and $3 \times 11 - 15 = 18$.
13 8 **14** 15 **15** 6 g **16** **a** €9.50 **b** €5.20 **17** 5
18 $-8\frac{1}{2}$ **19** $-7\frac{1}{2}$

EXERCISE 16F

1 **a** $5x = 50 - 2y$ which is < 50 as $2y > 0$ i.e., $5x < 50$ and so $x < 10$

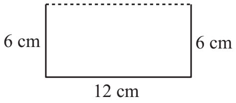
b

x	1	2	3	4	5	6	7	8	9	10
y	$22\frac{1}{2}$	20	$17\frac{1}{2}$	15	$12\frac{1}{2}$	10	$7\frac{1}{2}$	5	$2\frac{1}{2}$	0

c $x = 2$, $y = 20$ or $x = 4$, $y = 15$ or $x = 6$, $y = 10$ or $x = 8$, $y = 5$

2 $x = 5$, $y = 8$ is the only solution
3 17 is the smallest. (The next smallest is 32.)
4 3, 6, 7, 11, 13, 14, 15, 18, 19, 20, 21, 22, 23, 29, 30, 32, 34, 35, 38, 39

5



6 The smallest perimeter occurs when the length is approximately 8 cm. If working to one decimal place it is about 7.7 cm
7 4732 when $a = 7$ and $b = 26$

EXERCISE 16G

1 18 chocolates **2** 3 **3** 16 horses
4 Xia 134 cm, Xuen 130 cm **5** 91
6 X had 300 mL, Y had 100 mL **7** 72 km

REVIEW SET 16A

1 **a** 8 **b** 6 **2** **a** $x = 4$ **b** $x = 6$ **c** $x = 32$
3 17 cm **4** books cost €15, CDs cost €30 **5** 76 cm **6** 25
7 6 of them **8** $x = 6$, $y = 2$ **9** now 14 **10** $x = 4$

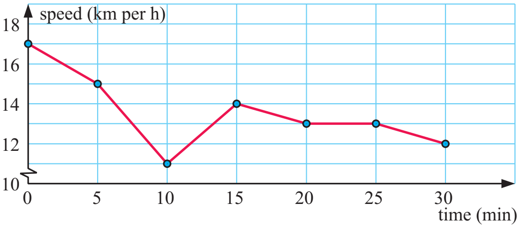
REVIEW SET 16B

1 **a** 7 **b** 16 **2** £2.50
3 **a** $x = 25$ **b** $x = 5$ **c** $x = 12$ **4** 13 cm **5** 14
6 15 marbles **7** 6 ten cent coins
8 6144 when $x = 6$, $y = 32$ **9** \$90 **10** $x = 7$

EXERCISE 17A

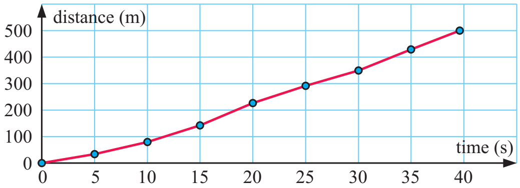
1 **a** independent: *time*; dependent: *distance*
b independent: *time*; dependent: *temperature*
c independent: *distance*; dependent: *speed*
2 **a** **A** **b** **C** **c** **B**
3 **a** dependent: *speed*; independent: *time*

b/c



4 **a** dependent: *distance*; independent: *time*

b/c



d The graph is increasing.

EXERCISE 17B

1 **a** 20°C **b** 2 minutes **c** 1 minute
d 1 minute and $6\frac{1}{2}$ minutes **e** after 3 minutes

2 **a** 11 am **b** 8 pm
c increase in number of shoppers **d** 27

3 **a** 39°C **b** 16 minutes **c** 25°C **d** **i** 18°C **ii** 8°C

4 **a** 225 **b** 30 **c** 19th **d** 7th **e** 45th

5 **a** **i** January **ii** July
b a general increase in temperature **c** March, November

6 **a** 2 am to 6 am and from 2 pm to 6 pm
b 4 m at 10 am and 10 pm
c approx. 1 am, 7 am, 1 pm, 7 pm **d** 8 am

EXERCISE 17C

1 **a** **i** $312 **ii** $576 **b** **i** 167 pounds **ii** 233 pounds

2 **a** **i** 28 miles **ii** 17 miles **b** **i** 77 km **ii** 48 km

3 **a** 100°C **b** **i** 104°F **ii** 167°F **iii** 32°F **iv** −4°C
c 29°C or 30°C

EXERCISE 17D

1 **a** 6 hours **b** 400 km **c** 325 km **d** 4 hours
e **i** 75 km **ii** 100 km **iii** 0 km **iv** 225 km
f The family stopped for a break.

2 **a** 6 am **b** 2 hours 10 minutes **c** **i** 6.5 km **ii** 4.8 km
d **i** 6.2 km **ii** 6.5 km **e** 18 km **f** 5 minutes

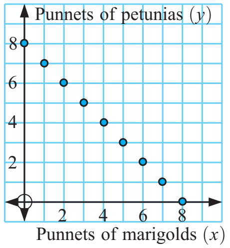
3 **a** the Pellegrinis **b** 2 hours **c** 350 km

EXERCISE 17E

1 **a**

Punnets of marigolds (x)	0	1	2	3	4	5	6	7	8
Punnets of petunias (y)	8	7	6	5	4	3	2	1	0

b

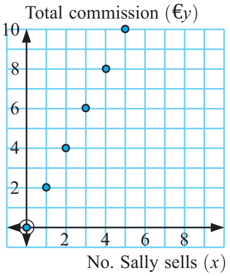


c Yes.
d $x + y = 8$
e No, as punnets are sold in whole numbers only.

2 **a**

No. Sally sells (x)	0	1	2	3	4	5
Total commission (€y)	0	2	4	6	8	10

b

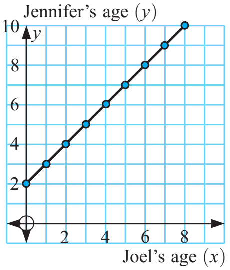


c Yes.
d $y = 2x$
e No, as Sally only sells whole shells.
f No, not in the negative direction. Yes, in the positive direction.

3 **a**

Joel's age (x)	0	1	2	3	4	5	6	7	8
Jennifer's age (y)	2	3	4	5	6	7	8	9	10

b



c Yes.
d Yes, data is continuous.
e **i** $7\frac{1}{2}$ years
ii 3 months

f Find the point on the graph where $x = 28$, i.e., (28, 30). Jennifer is 30 years old.
g $y = x + 2$

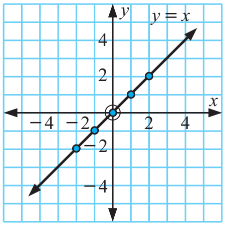
EXERCISE 17F

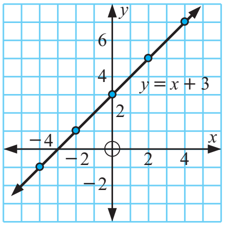
1 **a**

x	−2	−1	0	1	2
y	−2	−1	0	1	2

b

x	−4	−2	0	2	4
y	−1	1	3	5	7



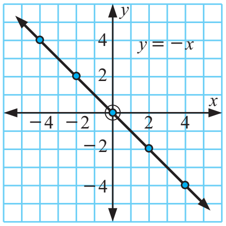


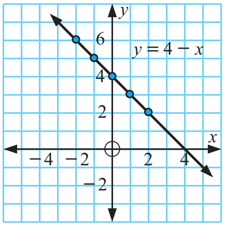
c

x	−4	−2	0	2	4
y	4	2	0	−2	−4

d

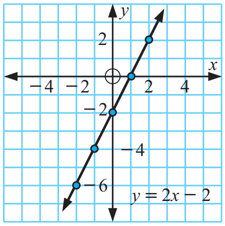
x	−2	−1	0	1	2
y	6	5	4	3	2





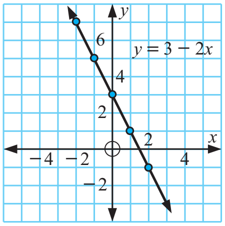
e

x	-2	-1	0	1	2
y	-6	-4	-2	0	2



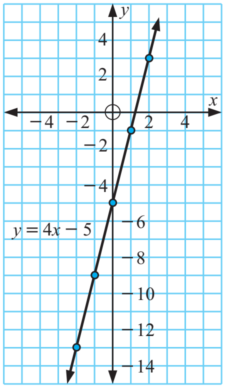
f

x	-2	-1	0	1	2
y	7	5	3	1	-1



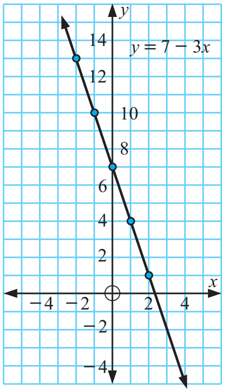
g

x	-2	-1	0	1	2
y	-13	-9	-5	-1	3



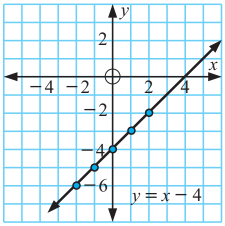
h

x	-2	-1	0	1	2
y	13	10	7	4	1



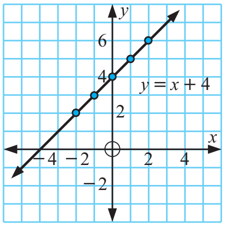
2 **a**

x	-2	-1	0	1	2
y	-6	-5	-4	-3	-2



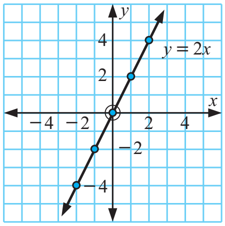
b

x	-2	-1	0	1	2
y	2	3	4	5	6



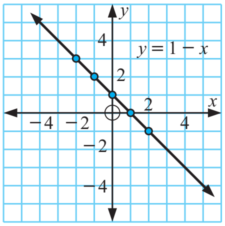
c

x	-2	-1	0	1	2
y	-4	-2	0	2	4



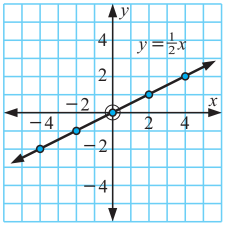
d

x	-2	-1	0	1	2
y	3	2	1	0	-1



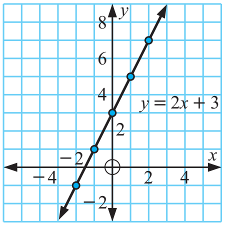
e

x	-4	-2	0	2	4
y	-2	-1	0	1	2



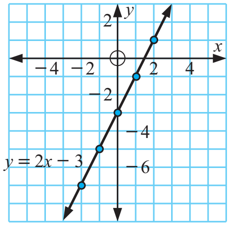
f

x	-2	-1	0	1	2
y	-1	1	3	5	7



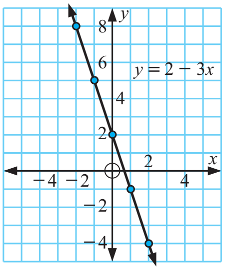
g

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1



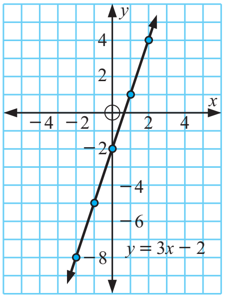
h

x	-2	-1	0	1	2
y	8	5	2	-1	-4



i

x	-2	-1	0	1	2
y	-8	-5	-2	1	4



REVIEW SET 17A

1 **a** **i** 9 pm **ii** 5 pm **b** 32 or 33 **c** 6 pm and 10:30 pm

2 **a** **i** $16\frac{1}{2}$ m per s **ii** 7 m per s

b **i** 54 km per h **ii** 14 km per h

3 **a** 2 hours, between noon and 2 pm

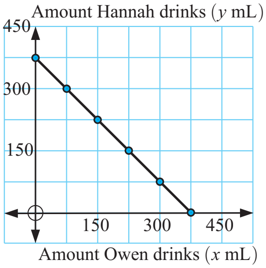
b **i** 60 km **ii** 90 km **c** 30 minutes

d 10:20 am and 5:30 pm

4 **a**

x	0	75	150	225	300	375
y	375	300	225	150	75	0

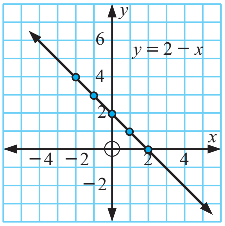
b $x + y = 375$ **c**



d Yes, as all points between the 6 given ones are possible.

5

x	-2	-1	0	1	2
y	4	3	2	1	0



REVIEW SET 17B

1 **a** after 3 years **b** 200 salmon after 9 years

c **i** between 0 and 3 years and between 9 and 10 years

ii between 3 and 9 years

d after 1 year and after 5 years

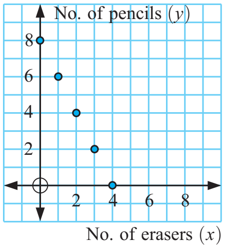
2 **a** **i** $26\frac{1}{2}$ feet **ii** $6\frac{1}{2}$ feet **b** **i** 1.8 m **ii** 6 m
c 1.5 m ≈ 5 feet, so the 5 feet 6 inches tall lady is taller.

3 **a** 24 km **b** Hissam **c** after 80 min **d** 16 km
e Hissam

4 **a**

No. of erasers (x)	0	1	2	3	4
No. of pencils (y)	8	6	4	2	0

b



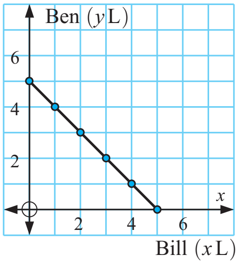
c Points are collinear.
d No, discrete data.

5 **a**

Amount poured in by Bill (x L)	0	1	2	3	4	5
Amount poured in by Ben (y L)	5	4	3	2	1	0

b No, only whole number values. **c** $x+y=5$

d

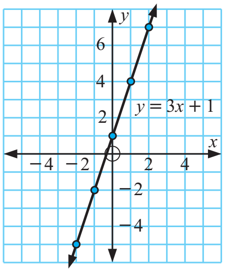


e Yes, data is continuous.

6 **a**

x	-2	-1	0	1	2
y	-5	-2	1	4	7

b

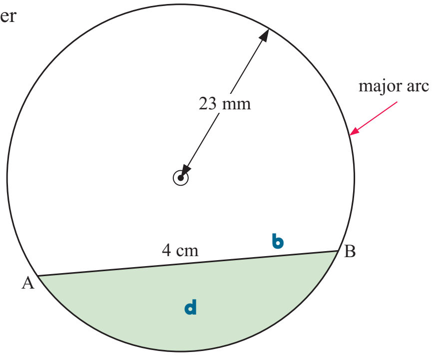


EXERCISE 18A

1 **a** **F** **b** **D** **c** **A** **d** **H** **e** **K** **f** **G**
g **C** **h** **E** **i** **J** **j** **I** **k** **B**

2 a diameter

3 **a**



4 Fold the circular disc exactly in half forming a crease line. Open it out and then repeat, forming a different crease line. Where the two crease lines meet we have the centre of the circular disc.

5 **a** OA = OB {equal radii}
∴ ΔAOB is isosceles
b [OM] is perpendicular to [AB]

EXERCISE 18B.1

1 **a** 31.4 cm **b** 50.2 m **c** 126 km **d** 9.42 m
e 3.45 km **f** 94.2 cm **g** 20.7 m **h** 21.7 m
i 23.4 km

2 **a** 26.7 cm **b** 35.5 m **c** 15.4 km **d** 5.28 km
e 40.1 m **f** 73.6 km

3 9.42 m **4** **a** 25.1 m **b** 26 lengths **c** €650.00

5 **a** 220 cm **b** 220 km

6 **a** 51.4 cm **b** 35.7 cm **c** 46.8 m **d** 77.1 cm
e 47.0 mm **f** 15.7 m

EXERCISE 18B.2

1 **a** 3.18 m **b** 15.9 cm **c** 1.59 km

2 **a** 12.7 cm **b** 4.07 m **c** 0.611 km

3 159 mm **4** 2.01 m

EXERCISE 18C

1 **a** 201 cm^2 **b** 31 400 m^2 **c** 707 km^2

2 452 cm^2 **3** 499 m^2 **4** 201 m^2

5 **a** 25.1 m^2 **b** 36.3 cm^2 **c** 3930 m^2 **6** 9820 m^2

7 **a** 220 cm **b** 1.69 m^2 **8** 7.26 m^2 **9** 1.99 m^2

10 **a** 160 m **b** 1260 m^2 **c** 1600 m^2 **d** 343 m^2
e 126 m

11 **a** 58.9 m^2 **b** 47.1 m

12 **a** **i** 2 : 1 **ii** 4 : 1 **b** **i** 1 : 2 **ii** 1 : 4
c ...... circumference is *doubled* and its area is *four* times greater.

13 **a** **i** 100 cm^2 **ii** 78.5 cm^2 **b** 78.5%

14 **a** **i** 314 cm^2 **ii** 50 cm^2 **iii** 200 cm^2 **b** 63.7%

15 **a** $l = 20$
b When joined up the arc AB of the first figure becomes the base of the second figure.
c $r = 10$ **d** 314 cm^2 **e** 628 cm^2 **f** 942 cm^2

EXERCISE 18D

1 **a** 226 cm^3 **b** 754 mm^3 **c** 33.9 m^3 **d** 942 cm^3
e 15.7 m^3 **f** 4.00 m^3

2 4320 cm^3 **3** 13.7 m^3 **4** 9240 cm^3 **5** 8140 cm^3

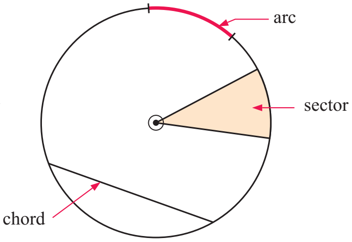
6 **a** 0.332 m^2 **b** 0.664 m^3 **c** 3980 m^3 **7** 6.09 mm

8 24 kL **9** 14.1 kL **10** $h \approx 4.42$ **11** 785 cm^3

REVIEW SET 18A

1

a An **arc** is a part of a circle. It joins any two different points on the circle.



b A **sector** of a circle is a part of the interior of a circle between two radii.

c A **chord** of a circle is a line segment joining any two different points of the circle.

2 **a** 18.8 m **b** 50.3 cm **c** 11.9 m

3 251 cm **4** 9.55 cm **5** 79.6 m

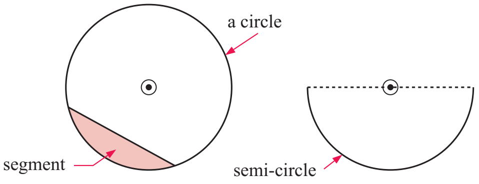
6 **a** 12.6 m^2 **b** 177 cm^2 **c** 78.5 m^2

7 **a** 1.01 m^2 **b** 0.0503 m^3 **8** **a** 254 cm^3 **b** 137 m^3

9 628 mm^3 **10** 14.1 litres

REVIEW SET 18B

1



a A **circle** is a set of points which are all a constant distance from a fixed point.

b A **semi-circle** is an arc which is a half of a circle.

c A **segment** of a circle is a part of the interior of a circle between a chord and the circle.

2 **a** 37.7 cm **b** 11.9 cm **c** 36.4 m

3 **a** 14.3 cm **b** 18.9 m **c** 29.1 m

4 **a** 50.3 cm **b** 201 cm^2 **5** 44.6 cm

6 **a** 28.6 cm **b** 14.3 cm

7 **a** 154 cm^2 **b** 254 m^2 **c** 40.9 cm^2

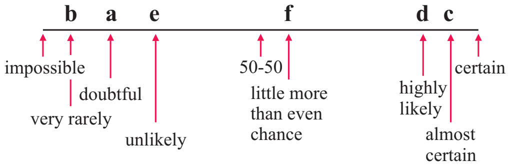
8 **a** 4.52 m^2 **b** 13.7 m^2 **9** **a** 88.0 cm^3 **b** 126 m^3

10 402 mL

EXERCISE 19A

1 **a** unlikely **b** certain **c** highly unlikely
d highly unlikely **e** unlikely **f** highly likely
g highly unlikely **h** certain **i** certain

2



3 **a** highly likely **b** no **c** false

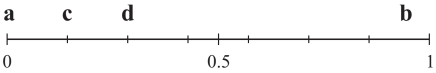
4 **a** No, there are fewer blue discs than white discs.
b white **c** true

5 **a** possible **b** possible **c** possible **d** possible
e impossible **f** impossible
g **i** possible **ii** impossible

EXERCISE 19B

1 **a** $\frac{1}{2}$ **b** **i** 1 **ii** 0

2



3 **a** equally likely **b** not equally likely
c equally likely **d** not equally likely

EXERCISE 19C

1 **a** 25 to 34, 35 to 44, 45 to 54 **b** 0 to 4, 65 and older
c **i** 0.509 **ii** 0.491 **d** no

2 **a** 0.201 **b** 0.399 **c** 0.382 **d** 0.536

3 **a** 0.172 **b** 0.491 **c** 0.152 **d** 0.575

4 **a** 637 people **b**

No. of children	*Frequency*	*Relative frequency*
0	217	0.341
1	218	0.342
2	124	0.195
3	52	0.082
4	17	0.027
5	5	0.008
6 or more	4	0.006
	637	1.001

c **i** 0.195
ii 0.878
iii 0.123

d 231

5 **a**

No. of flaws	*Frequency*	*Relative frequency*
0	27	0.54
1	15	0.30
2	4	0.08
3	2	0.04
4	1	0.02
5	1	0.02
	50	1.00

b **i** 0.54
ii 0.30
iii 0
iv 0.84
v 0.04

EXERCISE 19D

1 **a** {C, D} **b** {A, B, C}
c {Ja, Fe, Ma, Ap, My, Ju, Jl, Au, Se, Oc, No, De}
d {HH, HT, TH, TT} **e** {R, Y}
f {a, b, c, d, e, f, g, ..., w, x, y, z}

2 **a** {ABC, ACB, BAC, BCA, CAB, CBA}
b {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
c {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}
d {11, 12, 13, ..., 64, 65, 66}
e {WXYZ, WXZY, WYXZ, WYZX, WZXY, WZYX, XWYZ, XWZY, XZWY, XZYW, XYWZ, XYZW, YWXZ, YWZX, YXWZ, YXZW, YZWX, YZXW, ZWXY, ZWYX, ZXWY, ZXYW, ZYWX, ZYXW}

3 Label corners A, B, C, D
$\therefore$ sample space = {AB, BC, CD, DA}

EXERCISE 19E

1 **a** **i** 2 green, 3 yellow **ii** $\frac{2}{5}$ **iii** $\frac{3}{5}$
b **i** 4 green, 3 yellow **ii** $\frac{4}{7}$ **iii** $\frac{3}{7}$
c **i** 2 green, 6 yellow **ii** $\frac{1}{4}$ **iii** $\frac{3}{4}$

2 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** 1

3 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{5}{8}$ **e** 0

4 **a** $\frac{4}{9}$ **b** $\frac{1}{3}$ **c** $\frac{2}{9}$ **d** 0 **e** $\frac{5}{9}$ **f** $\frac{2}{3}$
g $\frac{7}{9}$ **h** $\frac{1}{3}$ **i** 1

5 **a** $\frac{1}{4}$ **b** $\frac{1}{52}$ **c** $\frac{1}{4}$ **d** $\frac{1}{26}$ **e** $\frac{1}{26}$ **f** $\frac{2}{13}$
g $\frac{1}{13}$ **h** $\frac{3}{13}$

6 **a** no **b** **i** pink **ii** orange **c** $\frac{1}{4}$

7 {HH, HT, TH, TT} **a** $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{3}{4}$

8 {BB, BG, GB, GG} **a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{2}$

9 {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}

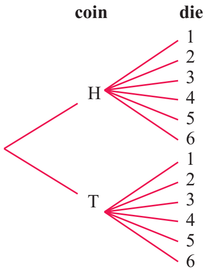
a $\frac{1}{8}$ **b** $\frac{1}{8}$ **c** $\frac{1}{8}$ **d** $\frac{3}{8}$ **e** $\frac{1}{2}$ **f** $\frac{7}{8}$

10 {ABC, ACB, BAC, BCA, CAB, CBA}

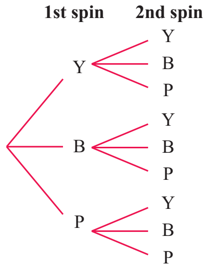
a $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$

EXERCISE 19F

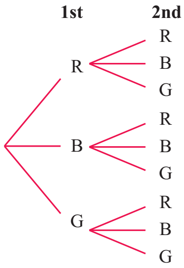
1 **a**



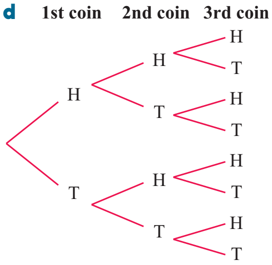
b



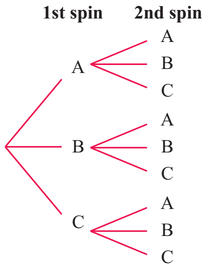
c



d

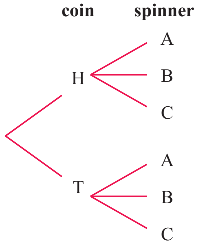


2 **a**



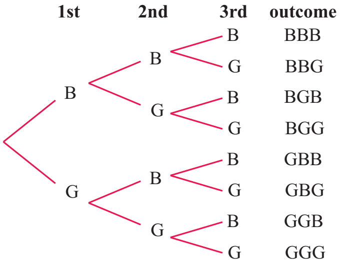
b **i** $\frac{1}{9}$ **ii** $\frac{1}{9}$ **iii** $\frac{5}{9}$ **iv** $\frac{2}{9}$

3 **a**



b **i** $\frac{1}{6}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{3}$

4 **a**



b 8 **c** **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$ **iv** $\frac{1}{8}$ **v** $\frac{7}{8}$

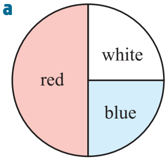
EXERCISE 19G

1 Coin with A on one side, B on the other.
Circular spinner with 180^oA, 180^oB.

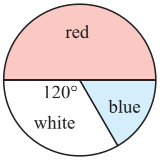
2 Regular pentagonal spinner, 3 sections blue, 2 sections red.
Bag containing 3 blue, 2 red marbles.

3 Die with one face A, 2 faces B, 3 faces C.
Bag containing 3 discs, one labelled A, 2 labelled B, 3 labelled C.

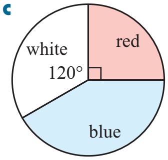
4 **a**



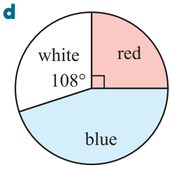
b



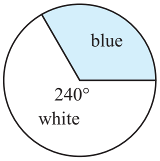
c



d



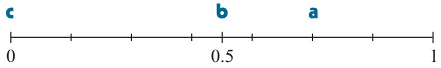
e



5 **a** 10 **b** 15 **c** 1 **d** 7 **e** 0 **f** 20

REVIEW SET 19A

1



2 **a**

No. of sweets	*Frequency*	*Relative frequency*
38	3	0.12
39	6	0.24
40	7	0.28
41	5	0.20
42	4	0.16
	25	1.00

b **i** 0.28 **ii** 0.36 **iii** 0.64

3 {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}, $\frac{3}{8}$

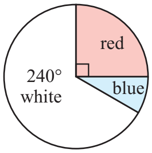
4 **a** $\frac{5}{12}$ **b** $\frac{1}{3}$ **c** $\frac{3}{4}$ **d** $\frac{2}{3}$ **e** $\frac{1}{4}$

5 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{5}{8}$

6 **a** $\frac{1}{4}$ **b** $\frac{1}{26}$ **c** $\frac{3}{13}$ **d** $\frac{2}{13}$ **7** $\frac{23}{50}$

8 A spinner with four 90^o sectors;
1 painted blue and 3 painted yellow.

9



REVIEW SET 19B

1 **a** equally likely **b** equally likely
c not equally likely **d** not equally likely

2 **a**

No. of cups of coffee	*Frequency*	*Relative frequency*
0	11	0.275
1	4	0.100
2	7	0.175
3	7	0.175
4	5	0.125
5	4	0.100
6	1	0.025
7	0	0.000
8	1	0.025
Total	40	1.000

b **i** 0.275 **ii** 0.125 **iii** 0.025

3 **a** $\frac{1}{30}$ **b** $\frac{1}{5}$ **c** $\frac{1}{2}$

4 {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{3}{8}$ **d** $\frac{1}{8}$

5 **a**

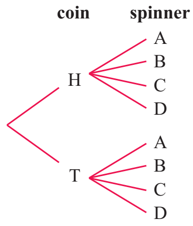
Result	*Frequency*	*Rel. Freq.*
edge	156	0.312
flat	344	0.688
Total	500	1.000

b 0.688

c 312 000 times

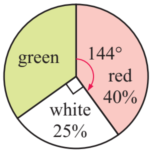
6 **a** 5 **b** 11 **c** 20 **d** 0

7 **a**



b **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$

8



9 A die with 3 faces with A on them, 2 faces with B and 1 with C.

EXERCISE 20A

1 **a** census **b** census **c** sample **d** sample **e** sample **f** census **g** sample

2 **a** age structure of a country's population, marks in a class test, the uniform preferences of students at a high school

b favourite television program, favourite football team, favourite model of motor car

3 **a** The survey would be biased towards people who were home on a Saturday night.

b The survey would be biased towards people who catch the bus.

c The survey would be biased towards people who go to the supermarket and who have access to a car.

d The survey would not include people who do not go to the beach.

e People who do not live in your street would not be represented.

4 **a** Only the views of Year 7 students would be considered. The other year levels are not represented.

b Motorists stopped in peak hour are more likely to be critical of traffic problems.

c Real estate agents are likely to say that house prices are higher than they actually are.

d Politicians are likely to say that the economy is in a better or worse shape than it actually is, depending on which party they belong to.

e Only people with access to a phone are able to vote, and only people with strong opinions about the issue will phone in.

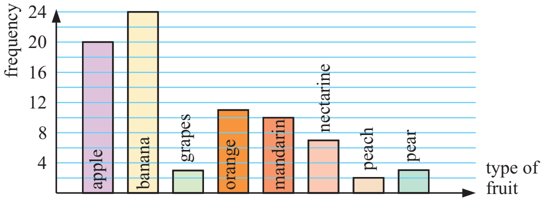
f People who do not have strong opinions about the issue or who are very busy will not take the time to fill out the questionnaire and post it back.

g This claim is unlikely to represent the views of all dog breeders. It is more likely to represent the views of a small (and possibly biased) sample of dog breeders.

EXERCISE 20B

1 **a** horizontal bar chart **b** Science **c** 6 **d** 16% **e** 18%

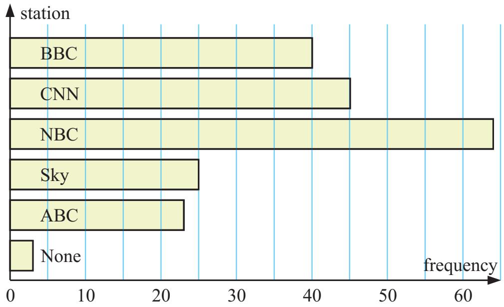
2 **a**



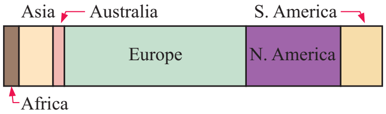
b banana **c** No, this is only one school in a particular area.

3 **a** 200 **b** NBC **c** 32% **d** 22.5%

e

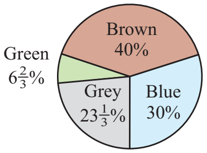


4 **a**



b 52%

5 **a**



b **i** $6\frac{2}{3}\%$ **ii** $53\frac{1}{3}\%$

6 **a** A 45°, B 60°, C 75°, D 60°, E 120°

b A €9 million, B €12 million, C €15 million, D €12 million, E €24 million

7 **a** 2052 people **b** 410 people

8 **a** shop A, June; shop B, January

b shop A, $6000; shop B, $5000

c Increase in length of bars from February to June for shop A.

d Shop A, $28 000; shop B, $27 000

EXERCISE 20C.1

1 **a** **Watermelon data**

4	7 8 5
5	6 5 3 6 0 8
6	9 4 9 1 4 5 9
7	7 6 2 3 8 7 3 0
8	0 2 7 2 0
9	1 Unit = 0.1 kg

b **Watermelon data**

4	5 7 8
5	0 3 5 6 6 8
6	1 4 4 5 9 9 9
7	0 2 3 3 6 7 7 8
8	0 0 2 2 7
9	1 Unit = 0.1 kg

c 30 went to market **d** **i** 4.5 kg **ii** 9.1 kg

2 **a** 0.19 seconds **b** 0.52 seconds **c** 20 **d** 0.39 seconds

3 **a** **Weights of newborn lambs**

2	6 8 9
3	0 1 1 3 3 3 5 6 6 8 9
4	1 2 2 4 7 7 8 8 9 9
5	0 0 1 2 4 5 Unit = 0.1 kg

b Unit = 0.1 kg

c 20%

d 53.3%

4 **a** **Distance (m) of golf shot**

12	7 9
13	2 4 7 8 8 9
14	0 1 1 3 4 4 4 5 5 6 7 8 9 9
15	2 2 2 3 5 6 8 9 9
16	3 5 5 7 7
17	0 3 6

Unit = 1 m

b 127 m
c 56.4%
d 56.4%

5 **a** **Long jump distances (m)**

61	9
62	1 3 7
63	0 2 5 5 8 8 9
64	0 1 1 4 6 7 8 9 9 9
65	2 5
66	1

Unit = 0.01 m

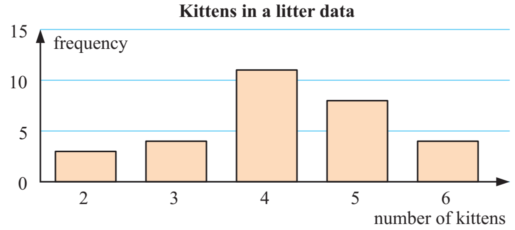
b 50%

EXERCISE 20C.2

1 **a**

Kittens	*Tally*	*Frequency*
2	|||	3
3	||||	4
4	卌 卌 |	11
5	卌 |||	8
6	||||	4
	Total	30

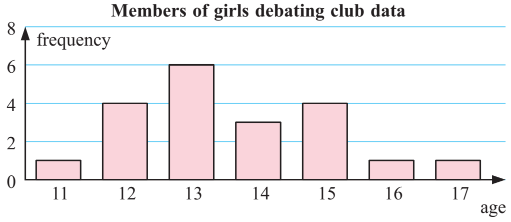
b



Kittens in a litter data

c 23 **d** 23.3%

2 **a**



Members of girls debating club data

b **i** 15% **ii** 90% **iii** 45% **iv** 35%

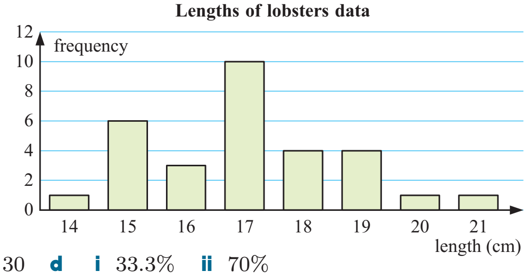
3 **a** 23 games **b** 5 games

c **i** 21.7% **ii** 73.9% **iii** 8.70%

4 **a**

Length (cm)	*Tally*	*Frequency*
14	|	1
15	卌 |	6
16	|||	3
17	卌 卌	10
18	||||	4
19	||||	4
20	|	1
21	|	1
	Total	30

b



Lengths of lobsters data

c 30 **d** **i** 33.3% **ii** 70%

EXERCISE 20D

1 **a** 4 **b** 4 **2** **a** 4 **b** 8

3 **a** **i** 4 **ii** 3 **iii** 0
b the mean as it includes the influence of all scores

4 **a** 1.57 **b** 1.5 **c** modes are 1 and 2

5 **a** ≈ 3.88 **b** 4 **c** 4

6 **a** mean $\approx$ 79.3 clips per box
b No, as the sample of 50 is extremely low and an inaccurate estimate of the overall mean may have been obtained.

7 **a** mode = size 8 **b** median = size $8\frac{1}{2}$

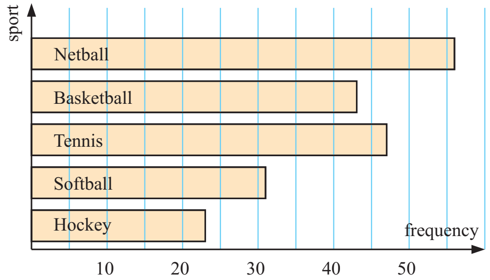
8 **a** mean for group X = 6.5
mean for group Y $\approx$ 7.64
b No, the mean is the average test score per student.
c Group Y with the higher mean.

REVIEW SET 20A

1 **a** census **b** sample **c** sample **d** sample **e** sample

2 **a** The sample is far too small to represent the whole school population. However each year group is mentioned. The results could be biased.
b The principal is only canvassing the views of year 12s and is ignoring all other year groups. The results could be biased.

3 **a** 200 girls **b** 28% **c** hockey
d

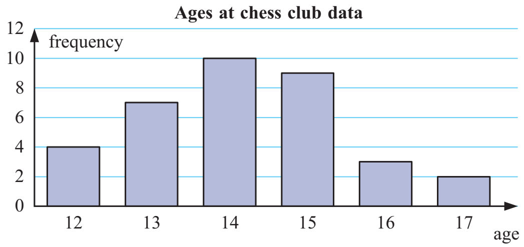


e No, different sports are offered in different schools.

4 **a** vertical column graph **b** train **c** 5 **d** 18% **e** 38%

5 Franki's average $\approx$ 3.21 goals/game
Hugo's average = 3.4 goals/game
$\therefore$ Hugo wins the trophy

6 **a** **i** ≈ 14.2 **ii** 14 **iii** 14
b



Ages at chess club data

REVIEW SET 20B

1 **a** census **b** sample

2 **a** If the survey is about movies there is possibly no bias. If about any other matter it may be biased as non-movie goers are not represented.

b Likely to be biased as non-readers' views are not canvassed.

c Likely to be biased in favour of families with children.

3 **a** AB 15^o, O 160^o, A 125^o, B 60^o **b** 34.7%

c 168 people

4 **a**

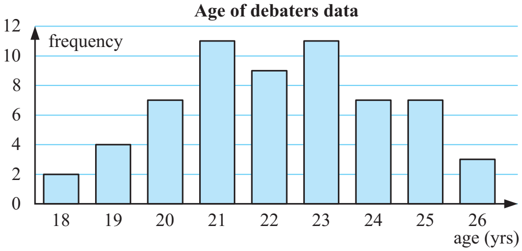
b red

c No, depends on race, fashion, etc. Also the sample is too small.

5 **a**

Age	*Frequency*
18	2
19	4
20	7
21	11
22	9
23	11
24	7
25	7
26	3
Total	61

b



c 21 and 23 **d** 9.84%

6 **a** Group A has mean 7.5
Group B has mean 8.45

b Group B as it has the higher mean.

EXERCISE 21A

1 **a** $A = \{6, 7, 8, 9\}$ **b** $A = \{11, 13, 15, 17, 19\}$

c $A = \{$Ja, Fe, Ma, Ap, My, Ju, Jl, Au, Se, Oc, No, De$\}$

d $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

e $A = \{$Z, X, C, V, B, N, M$\}$

f $A = \{$Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, Pluto$\}$

g $A = \{$B, A, S, E, L$\}$ **h** $A = \{$black, white$\}$ **i** $A = \varnothing$

2 **a** 4 **b** 5 **c** 12 **d** 8 **e** 7 **f** 9 **g** 5
h 2 **i** 0

3 **a** **i** 5 **ii** 6

b **i** True **ii** False **iii** False **iv** True

c No, as $7 \in P$ but $7 \notin Q$

4 **a** $R = \{2, 4, 6, 8\}$
$S = \{1, 2, 3, 4, 6, 8, 12, 24\}$
$T = \{2, 3, 5, 7, 11, 13\}$

b **i** 4 **ii** 8 **iii** 6

c No

d Yes

5 **a** $\varnothing$, $\{5\}$, $\{7\}$, $\{5, 7\}$

b $\varnothing$, $\{c\}$, $\{l\}$, $\{p\}$, $\{c, l\}$, $\{c, p\}$, $\{l, p\}$, $\{c, l, p\}$

6 **a** $A = \{$red, blue, green$\}$
$B = \{$red, yellow, blue, pink, green, orange$\}$

b Yes **c** **i** 3 **ii** 6 **d** Yes

7 $x = 7$ **8** **a** True **b** True

9 $\varnothing$, $\{w\}$, $\{x\}$, $\{y\}$, $\{z\}$, $\{w, x\}$, $\{w, y\}$, $\{w, z\}$, $\{x, y\}$, $\{y, z\}$, $\{x, z\}$, $\{w, x, y\}$, $\{w, x, z\}$, $\{w, y, z\}$, $\{x, y, z\}$, $\{w, x, y, z\}$

EXERCISE 21B

1 **a** $A' = \{2, 3, 5, 6, 8, 9\}$ **b** $B' = \{1, 6, 7, 8\}$

c $C' = \{2, 3, 4, 6, 8\}$ **d** $D' = \{1, 3, 5, 7, 9\}$

2 **a** $P = \{1, 2, 5, 10\}$ **b** $Q = \{2, 3, 5, 7, 11, 13\}$

c $P' = \{3, 4, 6, 7, 8, 9, 11, 12, 13, 14\}$

d $Q' = \{1, 4, 6, 8, 9, 10, 12, 14\}$

3 **a** **i** $U = \{$Sp, Ba, HJ, Hu, SP, Te, LJ$\}$

ii $B = \{$Ba, SP, Te$\}$ **iii** $B' = \{$Sp, HJ, Hu, LJ$\}$

b The sports at sports day where a ball is not used.

4 **a** $A = \{$M, A, T, H, E, I, C, S$\}$

b $B = \{$G, E, O, R, A, P, H, Y$\}$

c $A' = \{$B, D, F, G, J, K, L, N, O, P, Q, R, U, V, W, X, Y, Z$\}$

d $B' = \{$B, C, D, F, I, J, K, L, M, N, Q, S, T, U, V, W, X, Z$\}$

5 **a** **i** $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

ii $A = \{3, 6, 9\}$ **iii** $A' = \{1, 2, 4, 5, 7, 8\}$

iv $B = \{1, 3, 5, 7, 9\}$ **v** $B' = \{2, 4, 6, 8\}$

b **i** 9 **ii** 3 **iii** 6 **iv** 5 **v** 4 **vi** 9 **vii** 9

c $n(A) + n(A') = n(U)$

6 $A' = \varnothing$

EXERCISE 21C

1 **a** $\{12, 14\}$ **b** $\{10, 11, 12, 13, 14, 16\}$

2 **a** $\{6, 11, 13\}$, $\{4, 5, 6, 7, 9, 11, 13, 15\}$

b $\{d, p\}$, $\{a, c, d, f, l, m, p, r, t\}$

c $\{$R, T$\}$, $\{$R, T, L, NY, P, C$\}$

3 **a** $\{1, 4\}$ **b** 2 **c** $\{1, 2, 4, 5, 9, 10, 16, 20\}$ **d** 8

4 **a** $U = \{$Ja, Fe, Ma, Ap, My, Ju, Jl, Au, Se, Oc, No, De$\}$

b $R = \{$Ma, Ap, My, Ju, Jl, Au$\}$
$D = \{$My, Ju, Jl, Au, Se, Oc, No$\}$

c $\{$My, Ju, Jl, Au$\}$. These are the months when both roses and daisies flower in her garden.

d $\{$Ma, Ap, My, Ju, Jl, Au, Se, Oc, No$\}$. These are the months when either roses or daisies flower in her garden.

e $\{$Ja, Fe, De$\}$. These are the months when neither roses nor daisies flower in her garden.

5 **a** $P = \{3, 6, 9\}$ and $Q = \{1, 2, 3, 6, 9, 18\}$

b Yes **c** **i** P **ii** Q

d If $P \subseteq Q$ then $P \cap Q = P$ and $P \cup Q = Q$.

6 **a** $B = \{$f, b, v, e, c$\}$, $C = \{$f, b, s, e, m, l$\}$

b $\{$f, b, e$\}$. These are ingredients common to both recipes.

c $\{$f, b, s, e, m, l, v, c$\}$ **d** 8 as $n(B \cup C) = 8$

EXERCISE 21D

1 **a** True **b** False **c** False **d** True

2 **a** $A = \{4, 8, 12, 16, 20, 24\}$, $B = \{1, 4, 9, 16\}$,
$C = \{1, 3, 7, 21\}$, $D = \{2, 3, 5, 7, 11, 13, 17, 19\}$

b A and C, A and D, B and D

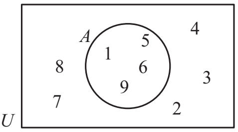
3 **a** $E = \{$P, S, J$\}$, $F = \{$S, Fr$\}$, $G = \{$Eg, Fr$\}$, $H = \{$C, J, Eg$\}$

b **i** $\{$S$\}$ **ii** $\varnothing$ **iii** $\{$J$\}$ **iv** $\{$Fr$\}$ **v** $\varnothing$ **vi** $\{$Eg$\}$

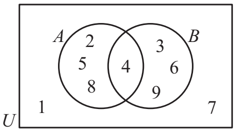
c **i** Japanese **ii** French
d They have no language in common.
e Eduardo and Gordon, Fernando and Huan

EXERCISE 21E

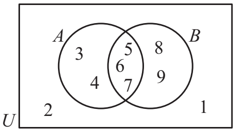
1 **a**



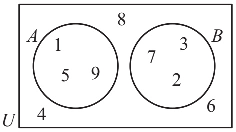
b



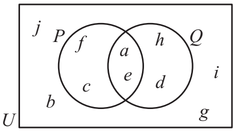
c



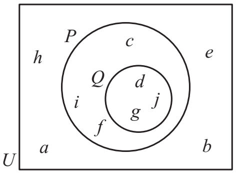
d



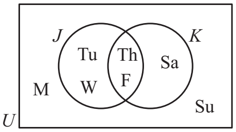
2 **a**



b

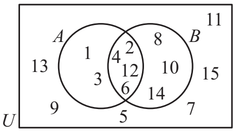


3 **a** $U = \{\text{M, Tu, W, Th, F, Sa, Su}\}$
b



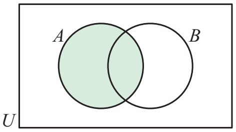
4 **a** **A** **b** **C** **c** **D** **d** **B** **e** **A** **f** **D** **g** **A**
h **D** **i** **C** **j** **B**

5 **a** $A = \{1, 2, 3, 4, 6, 12\}$, $B = \{2, 4, 6, 8, 10, 12, 14\}$
b **i** $\{2, 4, 6, 12\}$ **ii** $\{1, 2, 3, 4, 6, 8, 10, 12, 14\}$
c

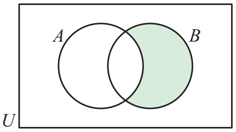


d **i** 2 **ii** 7 **iii** 9

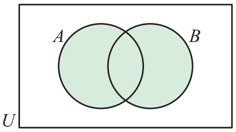
6 **a**



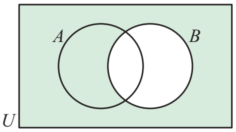
b



c



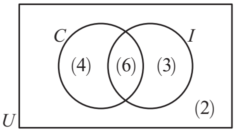
d



7 **a** not in P **b** neither in P nor in Q
c in exactly one of P or Q (P or Q, but not both)

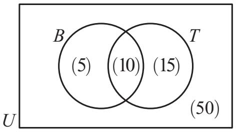
EXERCISE 21F

1 **a**



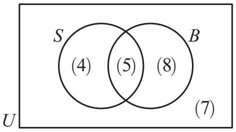
b **i** 3
ii 4
iii 2
iv 13

2 **a**



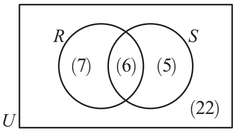
b **i** 15
ii 50
iii 30

3 **a**



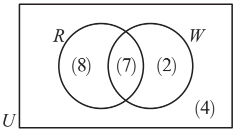
b **i** 8
ii 7
iii 17

4 **a**



b **i** 11
ii 7
iii 18

5 **a**

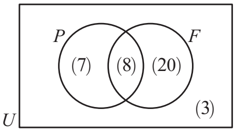


b **i** 7
ii 2
iii 10

EXERCISE 21G

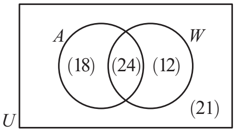
1 **a** 25 members **b** **i** $\frac{7}{25}$ **ii** $\frac{14}{25}$ **iii** $\frac{10}{25}$ $(= \frac{2}{5})$

2 **a**



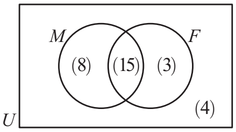
b **i** $\frac{10}{19}$
ii $\frac{35}{38}$
iii $\frac{3}{38}$

3 **a**



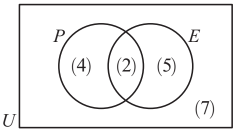
b **i** $\frac{12}{25}$
ii $\frac{4}{25}$
iii $\frac{6}{25}$

4 **a**



b **i** $\frac{4}{15}$
ii $\frac{2}{15}$
iii $\frac{11}{30}$

5 **a**



b $\frac{11}{18}$

REVIEW SET 21A

1 $\{24, 30, 36, 42, 48\}$
2 **a** $P = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$, $Q = \{1, 4, 9, 16\}$
b **i** 9 **ii** 4 **c** No, as $16 \notin P$
d **i** True **ii** False **iii** True
3 **a** **i** $A = \{\text{A, E, F, H, I, K, L, M, N, T, V, W, X, Y, Z}\}$
ii $A' = \{\text{B, C, D, G, J, O, P, Q, R, S, U}\}$
iii $V = \{\text{A, E, I, O, U}\}$

b **i** 15 **ii** 11 **iii** {A, E, I} **iv** {O, U}

4 **a** **i** $S = \{9, 12, 15, 18, 21, 24, 27\}$
ii $T = \{9, 11, 13, 15, 17, 19, 21, 23, 25, 27\}$

b **i** $\{9, 15, 21, 27\}$
ii $\{9, 11, 12, 13, 15, 17, 18, 19, 21, 23, 24, 25, 27\}$
iii 4 **iv** 13

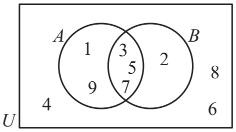
c **i** True **ii** True **iii** False **iv** False

5 False, e.g., $A = \{2, 4\}$, $C = \{4, 6\}$ and $B = \{1, 3, 5\}$

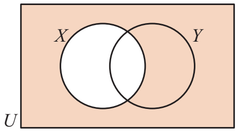
6 A and C, B and D, C and E

7 **a** **i** $\{1, 2, 3, 5, 7, 9\}$
ii $\{3, 5, 7\}$

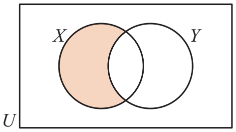
b



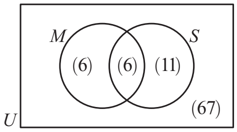
8 **a**



b

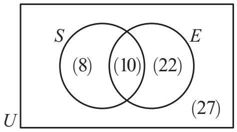


9 **a**



b 23

10 **a**



b **i** $\frac{8}{67}$
ii $\frac{27}{67}$

REVIEW SET 21B

1 **a** $\{7, 14, 21, 28, 35, 42, 49\}$ **b** $\{1, 5, 25\}$

2 **a** True **b** True **c** False **d** True **e** False

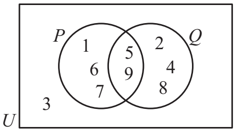
3 **a** **i** $A' = \{1, 3, 4, 5, 6, 7, 8\}$ **ii** $B' = \{3, 4, 5, 6, 7\}$
b Yes **c** Yes

4 $\{6, 12, 24\}$, $\{1, 2, 3, 4, 6, 8, 12, 16, 18, 24, 30, 36, 48\}$

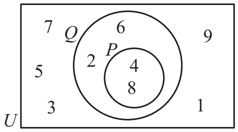
5 **a** $R = \{\text{M, B, C, Sa, A, K, E, N}\}$,
$S = \{\text{W, N, K, D, Sm, C, L}\}$
b {C, K, N}. These are invitees on both lists.
c $R \cup S = \{\text{M, B, C, Sa, A, K, E, N, W, D, Sn, L}\}$
$n(R \cup S) = 12$; 12 guests will be invited

6 **a** No **b** Yes **c** No **d** Yes

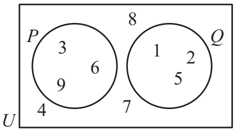
7 **a**



b

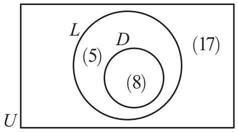


c



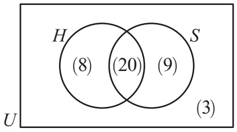
8 **a** in G but not F **b** in both F and G

9 **a**



b **i** 5
ii 17

10 **a**



b **i** $\frac{1}{5}$
ii $\frac{9}{40}$
iii $\frac{29}{40}$

EXERCISE 22A

1 **a** 8 metres in 1 second **b** 13 litres in 1 second
c 4 km in 1 hour **d** 2.5 kL in 1 minute
e 32 grams in 1 hour **f** 2.7 kg in 1 minute
g 74 dollars in 1 day **h** 23 cents in 1 second

2 **a** 27 **b** 2.76 **c** 14.5 **d** 27 **e** 24 **f** 12

3 **a** Trevor 15 km per L, Jane 17 km per L **b** Jane's

4 **a** Mandy \$18 per hr, Dzung \$15 per hr **b** Mandy

EXERCISE 22B

1 **a** 1.5 cents per gram **b** £1.40 per L
c €0.89 per kg **d** RM 5.13 per toothbrush
e €3.30 per metre **f** 94 pence per L

2 **a** 500 g at \$0.658 per 100 g
b 4 pack at 44.75 pence per pack **c** 2 L at €1.945 per L
d 10 m at \$0.42 per m **e** 5 L at €13.00 per L
f 25 kg at £1.40 per kg

EXERCISE 22C

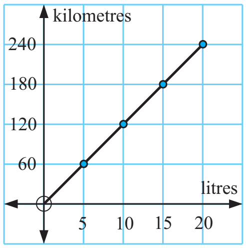
1 600 litres **2** £445.20 **3** **a** 6300 L **b** 11 minutes

4 **a** 684 km **b** 35 L **5** **a** \$4095 **b** 116 m

6 **a** **i** 8.8 km per L **ii** 11.4 L per 100 km
b 34.1 L **c** \$63.07

7 **a** \$48 **b** 1.5 hours **c** \$12 per hour

8 **a**



b 240 km
c 15 L
d straight line
e 216 km
f 12 km per L

9 **a** 100 km **b** 2 hours **c** 50 km per h **d** 100 km
e 1 hour **f** 100 km per h **g** change in slope

EXERCISE 22D

1 **a** 20 km per h **b** 15 km per h **c** 75 km per h
d 900 km per h

2 Yes, his average speed is 75 km per h.

3 a greyhound (17.5 m per sec compared with 15.7 m per sec for the horse)

4 200 km **5** **a** 105 km **b** $4\frac{2}{3}$ km **c** 5.6 km

6 **a** 1 hour 20 min **b** 1 hour 40 min **c** 48 min

EXERCISE 22E

1 **a** 4 g per cm^3 **b** 8 g per cm^3 **c** 0.9 g per cm^3

2 **a** 19.3 times **b** 2.89 times **c** 1.43 times

3 Gold weighs 19.3 g per cm^3 whereas lead weighs only 11.3 g per cm^3. ∴ gold is heavier.

4 **a** 648 g **b** 11.7 kg **c** 624 kg

5 Petrol as it is less dense.

6 **a** 26 550 cm^3 or 0.026 55 m^3
b 27 830 cm^3 or 0.027 83 m^3

7 weighs 171.2 g ∴ costs £7190

EXERCISE 22F.1

1 12 000 L per hour

2 **a** ≈ 104 000 beats per day **b** ≈ 37 800 000 beats per year
c 1.2 beats per second

3 **a** 3000 mL per h **b** 3 L per h

4 **a** 32 cm per year **b** 0.0877 cm per day
c 0.877 mm per day

5 ≈ 122 days per year **6** **a** 0.8 g per cm **b** 21.6 g

EXERCISE 22F.2

1 **a** 72 km per h **b** 180 km per h **c** 18 km per h

2 **a** 20 m per s **b** 40 m per s **c** 2.5 m per s

3 **a** 360 km per h **b** 54 km per h **c** 99.36 km per h
d 1800 km per h

4 **a** 16.7 m per s **b** 27.8 m per s **c** 23 m per s
d 175 m per s

5 **a** 30 km per h **b** 62.1 km per h **c** 57.6 km per h
d 6 km per h

REVIEW SET 22A

1 15 L per min

2 **a** 115 200 beats per day **b** 64.8 km per h

3 250 g at 0.876 cents per g **4** **a** €2.50 per kg **b** €8.50

5 the train (154 km per h compared with 78 km per h)

6 **a** 19 km per h **b** 5.07 km per L

7 No, he will be 5 m late. **8** 0.88 g per cm^3

9 1460 cm^3 **10** 30.3 km per h

REVIEW SET 22B

1 €13.50 per hour **2** 1 L at 0.259 pence per mL

3 **a** 13.9 m per s **b** 18.25 m per year

4 Li (RM 1.19 compared with RM 1.21)

5 60 seconds **6** **a** 150 km **b** 2 hours **c** 50 km per h

7 **a** $32\frac{1}{2}$ km **b** 1 h 15 min **8** 4.71 g per cm^3

9 **a** Iron cube 3.99 kg, gold cube 4.17 kg.
So, the gold cube is heavier.
b 81 500 cm^3

10 45.6 days per year

EXERCISE 23A

1 **a** $\frac{5}{x}$ **b** $2a$ **c** 2 **d** $\frac{1}{2}$

2 **a** c **b** $\frac{1}{d}$ **c** $\frac{b}{n}$ **d** a **e** $\frac{1}{a}$ **f** $\frac{2}{a}$
g 1 **h** t^2 **i** $5a$ **j** $2b$ **k** $\frac{ab}{2}$ **l** $\frac{bc}{2a}$

EXERCISE 23B

1 **a** $\frac{x}{4}$ **b** $\frac{1}{x}$ **c** 1 **d** $\frac{x^2}{4}$ **e** $\frac{a}{3}$ **f** $\frac{3}{a}$
g $\frac{a^2}{6}$ **h** $\frac{3}{2}$ or $1\frac{1}{2}$

2 **a** 2 **b** $\frac{a^2}{8}$ **c** $4n$ **d** $\frac{16}{n}$ **e** $\frac{3a}{2}$ **f** $2m$
g $\frac{b}{2}$ **h** $\frac{6}{a}$ **i** $\frac{2c}{3d}$ **j** 1 **k** 2 **l** $\frac{ab}{2}$
m $\frac{4}{m^2}$ **n** $\frac{a}{b^2}$ **o** 1 **p** $\frac{2}{a}$

3 **a** $\frac{c^2}{9}$ **b** $\frac{2}{b}$ **c** $\frac{5}{x}$ **d** $\frac{4}{n}$ **e** 1 **f** $\frac{1}{k^2}$
g 2 **h** $\frac{m^2}{18}$ **i** $\frac{1}{2}$ **j** $\frac{18}{m^2}$ **k** 4 **l** 3
m 2 **n** $\frac{a^2}{18}$ **o** $\frac{1}{3}$ **p** 1

EXERCISE 23C

1 **a** a **b** $\frac{a}{b^2}$ **c** $\frac{b^2}{a}$ **d** a **e** $\frac{3}{2}$ **f** $\frac{a^2}{6}$
g $\frac{2}{3}$ **h** $\frac{6}{a^2}$ **i** a^2 **j** $2x$ **k** $\frac{1}{5x}$ **l** $\frac{1}{b^2}$

2 **a** $\frac{3}{2a}$ **b** $\frac{3}{2t}$ **c** $\frac{c}{b^2}$ **d** $\frac{12a}{b}$

EXERCISE 23D.1

1 **a** $\frac{4b}{5}$ **b** $\frac{a+3}{4}$ **c** $\frac{2x+1}{3}$ **d** $\frac{5b}{6}$
e $\frac{a}{4}$ **f** $\frac{x}{2}$ **g** $\frac{13b}{18}$ **h** $\frac{3a-4b}{12}$

2 **a** $\frac{n+4}{4}$ **b** $\frac{10+y}{5}$ **c** $\frac{6-c}{2}$ **d** $\frac{4n}{3}$
e $\frac{3x}{2}$ **f** $\frac{m+3}{3}$ **g** $\frac{4-3x}{4}$ **h** $\frac{6x}{5}$

3 **a** $\frac{5x+2}{6}$ **b** $\frac{9x+10}{20}$ **c** $\frac{6x+1}{6}$ **d** $\frac{4x+7}{9}$
e $\frac{3x+3}{10}$ **f** $\frac{3x+12}{14}$ **g** $\frac{x+7}{3}$ **h** $\frac{11+x}{4}$
i $\frac{2x+13}{6}$ **j** $\frac{14x+11}{20}$ **k** $\frac{7x+4}{4}$ **l** $\frac{8x+2}{9}$
m $\frac{5x-1}{10}$ **n** $\frac{11x-49}{15}$ **o** $\frac{14x+5}{6}$ **p** $\frac{8x-1}{14}$
q $\frac{27x+26}{12}$ **r** $\frac{19x-4}{8}$ **s** $\frac{49x-18}{10}$ **t** $\frac{35x-25}{18}$
u $\frac{34x-11}{28}$

EXERCISE 23D.2

1 **a** $\frac{13-x}{4}$ **b** $\frac{34-x}{7}$ **c** $\frac{x+23}{6}$ **d** $\frac{2x-2}{3}$
e $\frac{3x+1}{2}$ **f** $\frac{13x-2}{4}$

2 **a** $\frac{x-2}{6}$ **b** $\frac{-x-10}{20}$ **c** $\frac{2x-1}{6}$ **d** $\frac{2x-1}{9}$
e $\frac{3-x}{10}$ **f** $\frac{x}{14}$ **g** $\frac{17x-7}{21}$ **h** $\frac{9x+11}{20}$
i $\frac{5-5x}{6}$ **j** $\frac{2x+13}{10}$ **k** $\frac{13-4x}{6}$ **l** $\frac{13x+40}{14}$
m $\frac{4x+18}{9}$ **n** $\frac{18x+7}{24}$ **o** $\frac{27x+5}{30}$

REVIEW SET 23A

1 **a** $3c$ **b** cannot be simplified **c** $\frac{1}{3k^2}$ **d** $\frac{a}{2}$

2 **a** $\frac{1}{4}$ **b** $\frac{m^2}{12}$ **c** $\frac{cd}{20}$ **d** $\frac{ac}{9}$ **e** $\frac{p^2}{16}$ **f** $\frac{3t^2}{2}$

3 **a** $\frac{d}{6}$ **b** $\frac{1}{2}$ **c** $\frac{10j}{k^2}$ **d** $\frac{6m}{n^2}$ **e** $\frac{2}{cd}$ **f** $5t^2$

4 **a** $\frac{7x}{12}$ **b** $\frac{3y}{4}$ **c** $\frac{n+12}{3}$ **d** $\frac{5k}{12}$ **e** $\frac{10-2t}{5}$ **f** $\frac{5s}{4}$

5 **a** $\frac{5x+15}{6}$ **b** $\frac{x+2}{4}$ **c** $\frac{4x-1}{12}$ **d** $\frac{5x-2}{12}$
e $\frac{x+4}{6}$ **f** $\frac{11x-25}{15}$

REVIEW SET 23B

1 **a** $\frac{a}{3}$ **b** $\frac{k}{2}$ **c** $\frac{2m^2}{5n}$ **d** $\frac{1}{3t^2}$

2 **a** 2 **b** $\frac{m^2}{9}$ **c** $\frac{x}{24}$ **d** $\frac{3t^2}{2s^2}$ **e** $\frac{d^2}{10}$ **f** $\frac{x^2}{3y}$

3 **a** $\frac{3}{4}$ **b** $\frac{a^2}{2}$ **c** $\frac{d^3}{3}$ **d** $\frac{l^2m}{10}$ **e** $\frac{3k}{2}$ **f** $\frac{t^3}{2}$

4 **a** $\frac{5x}{12}$ **b** $\frac{10-3l}{2}$ **c** $\frac{-3m}{4}$ **d** $\frac{13c}{15}$
e $\frac{-7p}{4}$ **f** $\frac{-k}{45}$

5 **a** $\frac{-x-2}{4}$ **b** $\frac{4x+5}{10}$ **c** $\frac{5-x}{9}$
d $\frac{11x-5}{12}$ **e** $\frac{17-7x}{10}$ **f** $\frac{25x-10}{24}$

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